Energy spectrum of global atmosphere

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Papers for 5 years

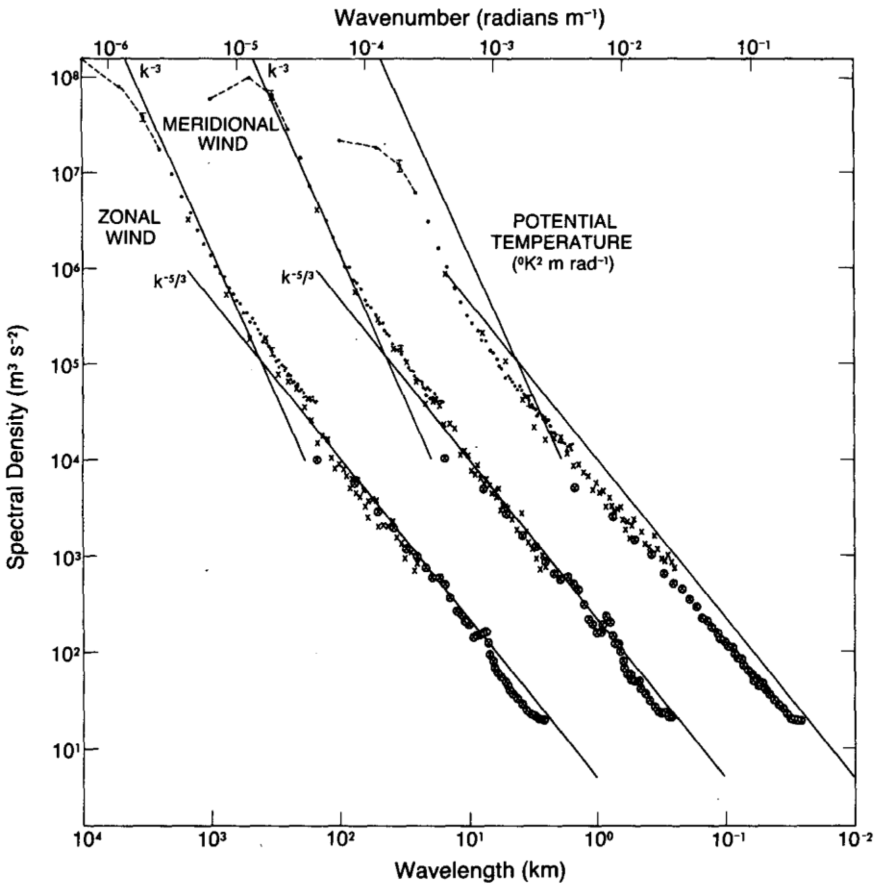
- Terasaki, K., H. L. Tanaka, and Masaki Satoh, 2009: Characteristics of the Kinetic Energy Spectrum of NICAM Model Atmosphere. SOLA, **5**, 180-183.
- Terasaki, K., H. L. Tanaka, and N. Žagar, 2011: Energy Spectra of Rossby and Gravity Waves. SOLA, **7**, 45-48.
- Žagar, N., K. Terasaki, and H. L. Tanaka, 2012: Impact of the vertical discretization of analysis data on the estimates of atmospheric inertio-gravity energy. Mon. Wea. Rev, **140**, 2297-2307.

Contents

- ✓ Kinetic energy spectrum of vertical wind using global nonhydrostatic atmospheric model NICAM
  - Kinetic energy spectrum of vertical wind shows white noise spectrum in glevel-11 (Δx=3.5km)
- ✓ Transition of energy spectrum from -3 spectrum to -5/3 spectrum based on 3D normal modes energetics
- ✓ Development of a method to compute very high resolution 3D normal modes energetics using GPGPU

Energy Spectrum

Observation by aircraft

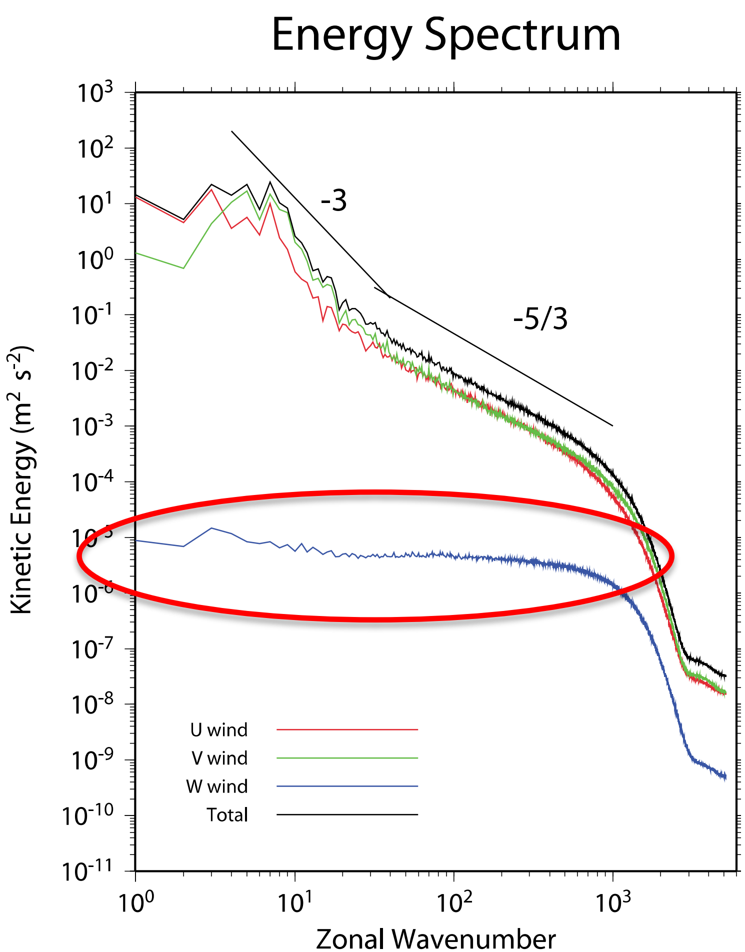


Nastrom and Gage (1985)

-3 power law in synoptic scale

-5/3 power law in meso-scale

Kinetic energy of vertical wind



Terasaki et al. (2009)

- 3.5 km horizontal resolution with NICAM
- Only Fourier expansion to zonal direction
- Average over 40 degree N to 50 degree N on 200 hPa surface
- Kinetic energy spectrum for horizontal wind shifts from -3 power law to -5/3 power law
- Kinetic energy spectrum of vertical wind shows white noise spectrum.

Data used in this analysis was provided by Prof. Masaki Satoh at the University of Tokyo.

3D Normal Mode Energetics

- A method to convert atmospheric variables in physical space to 3D spectral space.

Zonal	Fourier series
Meridional	Hough Functions
Vertical	Vertical structure functions

$$\frac{\partial u}{\partial t} - 2\Omega \sin\theta v + \frac{1}{a\cos\theta}\frac{\partial \phi}{\partial \lambda} = -\mathbf{V}\cdot\nabla u - \omega\frac{\partial u}{\partial \sigma} + \frac{\tan\theta}{a}uv + F_u,$$

$$\frac{\partial v}{\partial t} + 2\Omega \sin\theta u + \frac{1}{a}\frac{\partial \phi}{\partial \theta} = -\mathbf{V}\cdot\nabla v - \omega\frac{\partial v}{\partial \sigma} - \frac{\tan\theta}{a}uu + F_v,$$

$$\frac{\partial c_p T}{\partial t} + \mathbf{V}\cdot\nabla c_p T + \omega\frac{\partial c_p T}{\partial \sigma} = \omega p_s \alpha + Q,$$

$$\frac{1}{a\cos\theta}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\theta}\frac{\partial v\cos\theta}{\partial \theta} + \frac{\partial \omega}{\partial \sigma} = 0,$$

$$p_s \sigma \alpha = RT,$$

$$\frac{\partial \phi}{\partial \sigma} = -\frac{\alpha}{p_s},$$

3D Normal Mode Energetics

$$\begin{pmatrix} u \\ v \\ \phi' \end{pmatrix} = \sum_i w_i \begin{pmatrix} \sqrt{gh_i} & U_i \\ \sqrt{gh_i} & (-iV_i) \\ gh_i & Z_i \end{pmatrix} G_i e^{i n_i \lambda}.$$

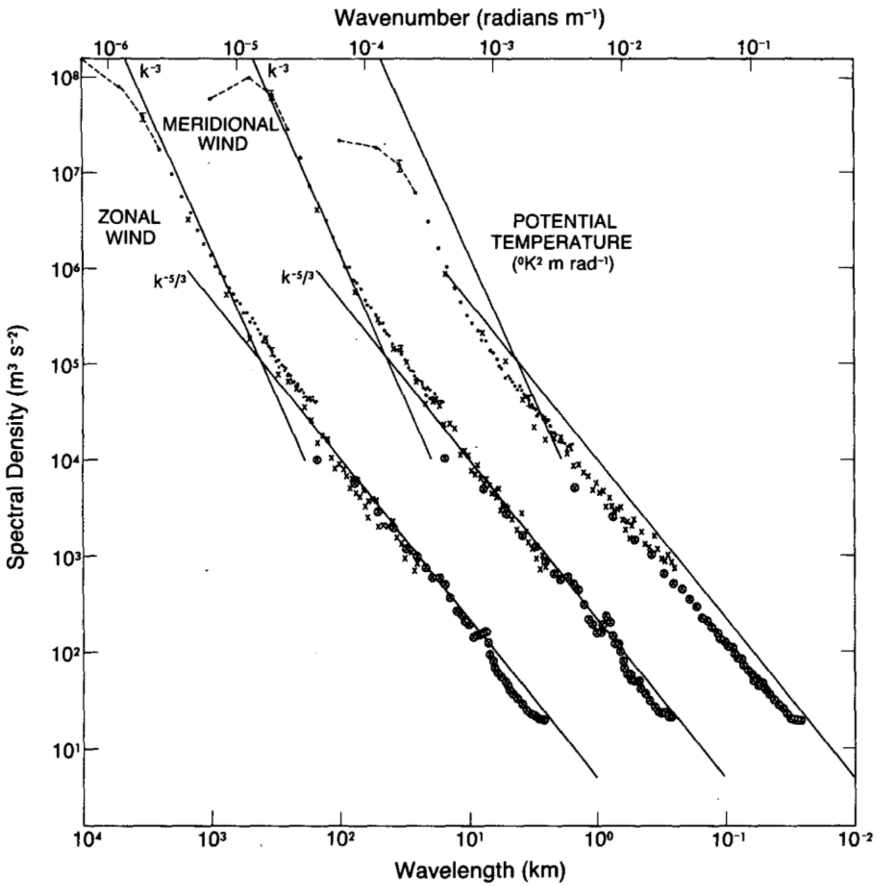
- Hough Functions
- Vertical structure functions
- Fourier expansion

$$\frac{dw_i}{d\tau} + i\sigma w_i = -i \sum_{j,k}^{N} r_{ijk} w_j w_k + f_i$$

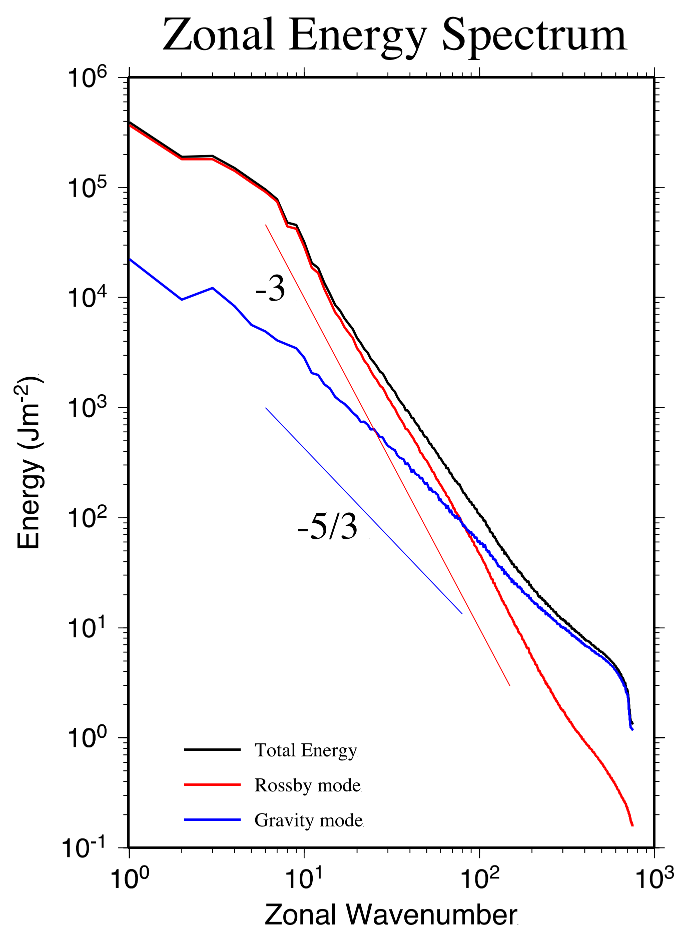
$$E_i = \frac{1}{2} p_s h_m |w_i|^2$$

Hough Functions can divide the atmospheric state variables into Rossby mode and Gravity wave mode in 3D spectral space.

Observation by aircraft



Nastrom and Gage (1985)



Terasaki, et. al., (2011)

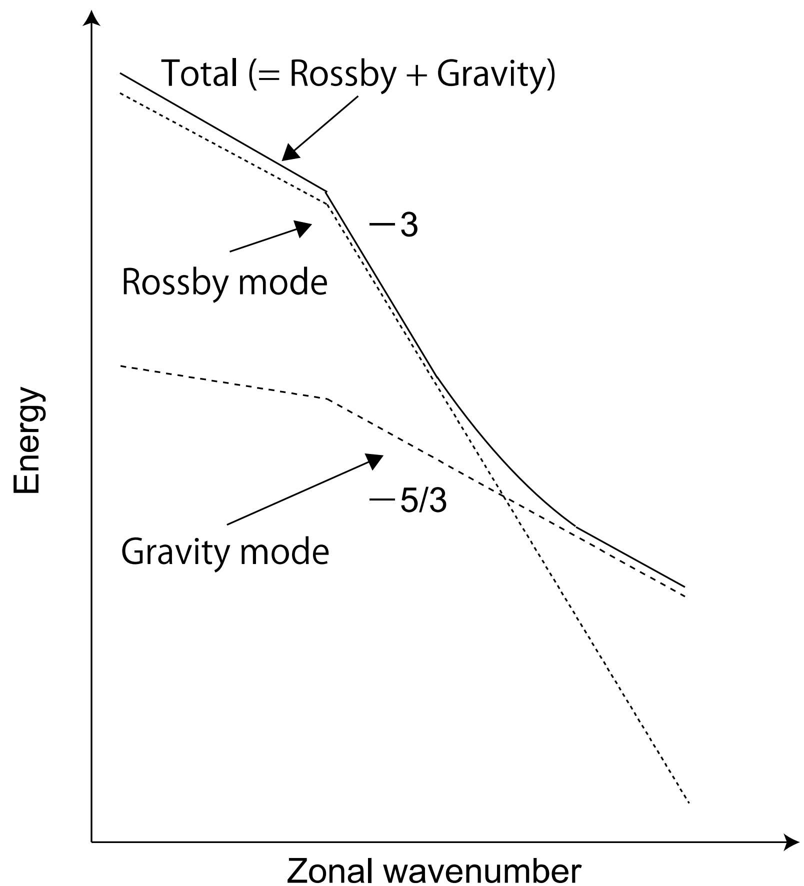


Fig. 4. Schematic diagram of energy spectrum for baroclinic atmosphere. The dotted and dashed lines show the energy spectra for Rossby and gravity modes, respectively. The solid line shows the total (Rossby + gravity) energy spectrum.

Rossby mode => -3 power law

Gravity mode => -5/3 power law

Energy slope changes from -3 power law to -5/3 power law gradually.

Very high resolution 3D normal mode energetics with GPGPU

- Vertical direction ・・・ Vertical structure functions

  Eigenvalue problem for square matrix with number of vertical grid

- Zonal direction ・・・ Fourier expansion (cuFFT)

  FFT can reduce the computational cost of Fourier expansion from $O(N^2)$ to O(NLogN)

- Meridional direction ・・・ Hough functions

  Large computational cost for associate Legendre functions and solving eigenvalue problem

Associate Legendre Functions

Associate Legendre Equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\left\{n(n+1)-\frac{m^2}{1-x^2}\right\}y=0$$

Associate Legendre Functions

$$P_l^m(x)=(-1)^m(1-x^2)^{m/2}\frac{d^m}{dx^m}(P_l(x))$$

$$P_l^0(x)=(P_l(x))$$

Recurrence formula

$$(l-m+1)P_{l+1}^m(x)=(2l+1)xP_l^m(x)-(l+m)P_{l-1}^m(x)$$

$$2mxP_l^m(x)=-\sqrt{1-x^2}\left[P_l^{m+1}(x)+(l+m)(l-m+1)P_l^{m-1}(x)\right]$$

Associate Legendre Functions

Recurrence formula

$$(l-m+1)P_{l+1}^{m}(x) = (2l+1)xP_{l}^{m}(x) - (l+m)P_{l-1}^{m}(x)$$

$$2mxP_{l}^{m}(x) = -\sqrt{1-x^2}\left[P_{l}^{m+1}(x) + (l+m)(l-m+1)P_{l}^{m-1}(x)\right]$$

Advantage

- Low computational cost

Disadvantage

- Overflow occurs when the order increases.
- Round error affects the high order computations.

Associate Legendre Functions

- Yu et al. (2012)
  Integral method is used to avoid the problems with recurrence formula

When l-m is even number

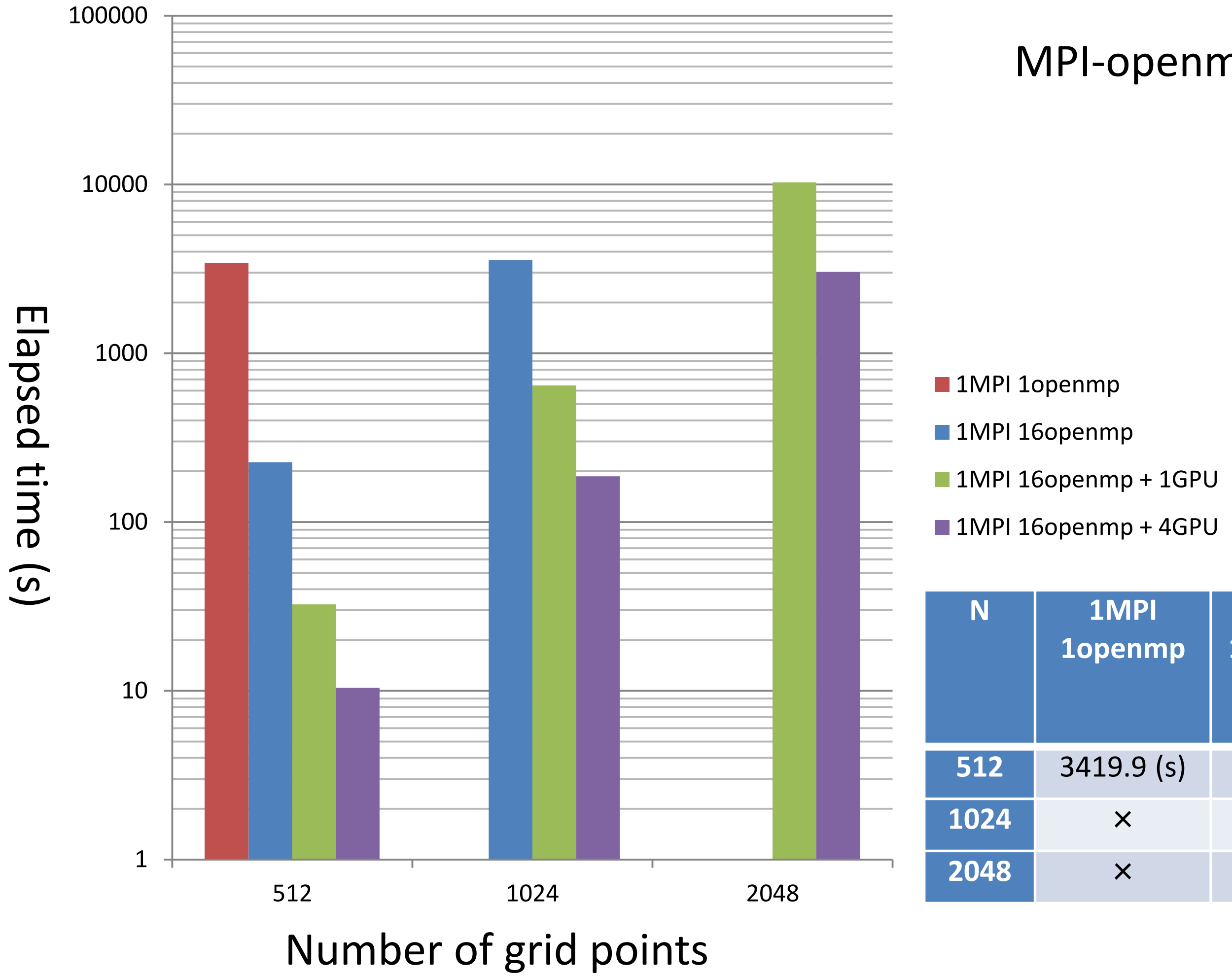
$$P_l^m(x) = \frac{1}{\pi}\frac{2n+1}{P_l^m(0)}\int_0^{\pi/2} \boxed{P_l(\sqrt{1-x^2}\cos\lambda)} \cos m\lambda d\lambda$$

When l-m is odd number

$$\begin{aligned} P_l^m(x) &= \frac{\sqrt{1-x^2}}{x}\left[\frac{\sqrt{(1+\delta_{l,m})((n-m+1)(n+m)}}{2m} \times P_l^{m-1} \right. \\ &\left. + \frac{\sqrt{(n+m-1)(n-m)}}{2m} \times P_l^{m+1}\right.\end{aligned}$$

Computational cost is much higher than recurrence method.
The computation of Legendre functions is hot spot in this method.
$O(N^4)$ of computational cost is required for integral method.

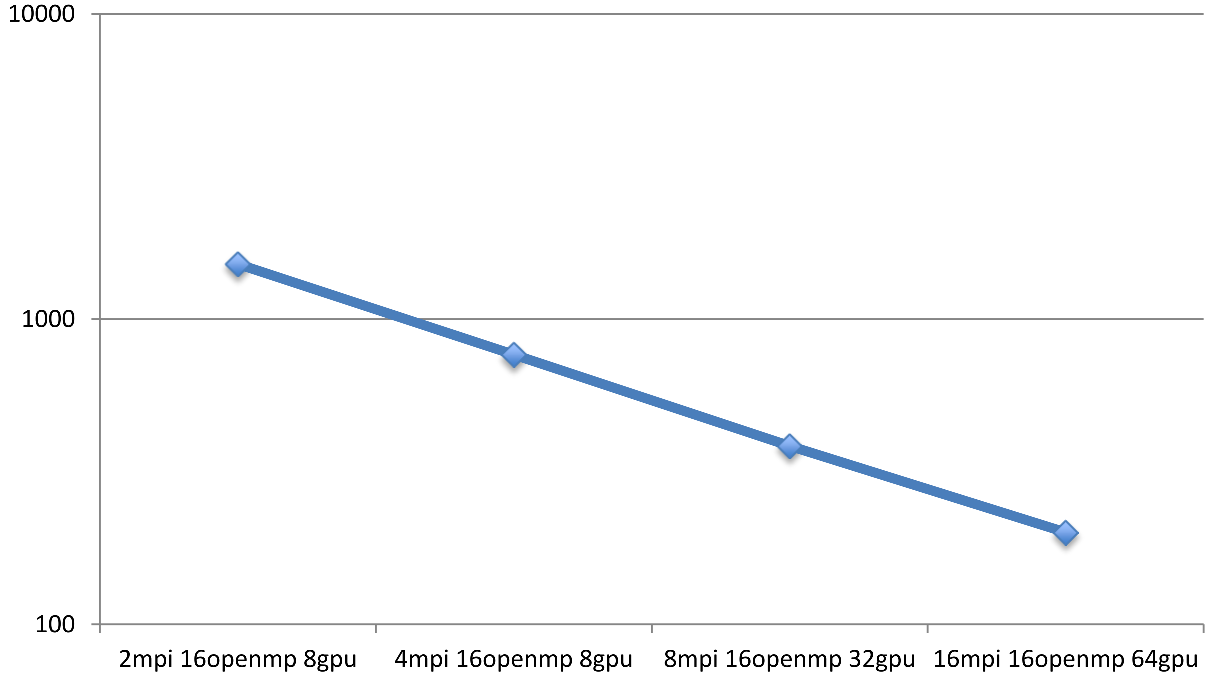
Associate Legendre Functions



MPI-openmp hybrid & multi GPU

N	1MPI 1openmp	1MPI 16openmp	1MPI 16openmp +1GPU	1MPI 16openmp +4GPU
512	3419.9 (s)	225.8 (s)	32.5 (s)	10.4 (s)
1024	×	3562.6 (s)	643.9 (s)	186.7 (s)
2048	×	×	10281.5 (s)	3026.6 (s)

Strong scaling



	2mpi 16openmp 8GPU	4mpi 16openmp 16GPU	8mpi 16openmp 32GPU	16mpi 16openmp 64GPU
2048	1516.2	765.1	384.4	200.2
5120				11636.2

Data Size of Hough Functions

Num. of grid point	Zonal wavenumber	Meridional wavenumber	Vertical wavenumber	Data (TB)
1024	1024	512	40	0.46875
2048	2048	1024	40	3.75
4096	4096	2048	40	30
5120	5120	2560	40	58.59375
8192	8192	4096	40	240

glevel-11 (3.5km)

When the data size is small, it is not a problem to read the data of Hough functions when 3D normal mode expansion.

But it is impossible to read them for high resolution data analysis.

Conclusion

What I found are that

1. Kinetic energy of vertical wind becomes white noise spectrum.
2. Energy spectra of Rossby mode and gravity mode explain -3 and -5/3 power laws, respectively.
3. The codes with CUDA for computing ALFs and expanding to 3D normal mode space have developed.