



Time-Dependent Density Functional Theory in Optical Sciences

K. YABANA

Center for Computational Sciences, University of Tsukuba

Collaborators:

G.F. Bertsch	Univ. Washington
T. Otobe	JAEA
J.-I. Iwata	Univ. Tokyo
S. Shinohara	Univ. Tsukuba/MPI
T. Sugiyama	Univ. Tsukuba
S.A. Sato	Univ. Tsukuba

First-principles electron dynamics simulation based on Time-Dependent Density Functional Theory (TDDFT)

Real-time and real-space solution of time-dependent Kohn-Sham equation

$$\left\{ -\frac{\hbar^2}{2m} \vec{\nabla}^2 + \sum_a V_{ion}(\vec{r} - \vec{R}_a) + e^2 \int d\vec{r}' \frac{n(\vec{r}', t)}{|\vec{r} - \vec{r}'|} + \mu_{xc}(n(\vec{r}, t)) + V_{ext}(\vec{r}, t) \right\} \psi_i(\vec{r}, t) = i\hbar \frac{\partial}{\partial t} \psi_i(\vec{r}, t)$$

$$n(\vec{r}, t) = \sum_i |\psi_i(\vec{r}, t)|^2$$

Development of methodology and our own softwear
ARTED = Ab-initio Real-Time Electron Dynamics simulator

1996 Isolated system (molecules, clusters)

K. Yabana, G.F. Bertsch,
Phys. Rev. B54, 4484 (1996).

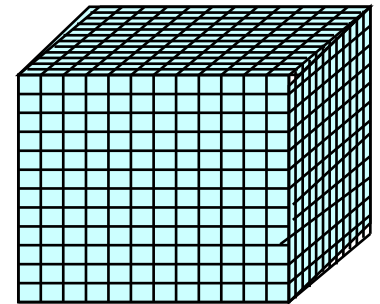
2000 Infinitely periodic system (crystalline solids)

G.F. Bertsch, J.-I. Iwata, A. Rubio, K. Yabana,
Phys. Rev. B62, 7998 (2000).

2012 Coupling to macroscopic Maxwell equation

K. Yabana, T. Sugiyama, Y. Shinohara, T. Otobe, G.F. Bertsch,
Phys. Rev. B85, 045134 (2012).

$$\psi_m(x_i, y_j, z_k, t_l)$$



2008 - 2013

Linear optical absorption in molecules

K. Yabana, Y. Kawashita, T. Nakatsukasa, J.-I. Iwata,
Charged Particle and Photon Interactions with Matter:
Recent Advances, Applications, and Interfaces Chapter 4, Taylor & Francis, 2010.

Electron dynamics in crystalline solids under femtosecond laser pulses

- Optical breakdown of dielectrics

T. Otobe, M. Yamagiwa, J.-I. Iwata, K. Yabana, T. Nakatsukasa, G.F. Bertsch
Phys. Rev. B77, 165104 (2008).

- Coherent phonon generation

Y. Shinohara, K. Yabana, Y. Kawashita, J.-I. Iwata, T. Otobe, G.F. Bertsch
Phys. Rev. B82, 155110 (2010)

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J. Chem. Phys. 137, 22A527 (2012).

Coupled dynamics of macroscopic electromagnetic fields and microscopic electron dynamics

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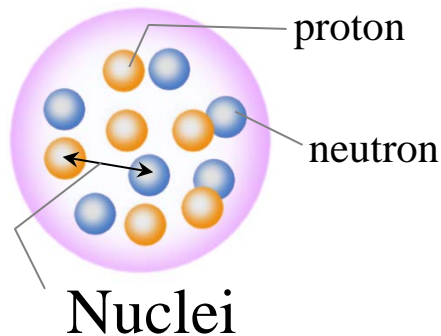
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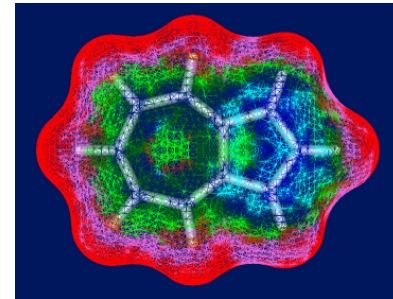
K. Yabana, T. Sugiyama, Y. Shinohara, T. Otobe, G.F. Bertsch
Phys. Rev. B85, 045134 (2012).

Working in two fields
Nuclear Physics and Condensed Matter Physics
has brought us original developments



Nuclei

Nucleon many-body system



Atoms, Molecules, Solids

Electron many-body systems

Common theories and computational methods are useful.

In nuclear physics, 3D simulation solving TDHF eq. has long history.

One of the oldest 3D quantum-mechanical simulation

Nuclear fusion reaction of $^{16}\text{O}-^{16}\text{O}$

Spatial grid: $30 \times 28 \times 16$ (10^{-15}m), Time-step 4×10^2 (10^{-22}s)

17 H. Flocard, S.E. Koonin, M.S. Weiss, Phys. Rev. 17(1978)1682.

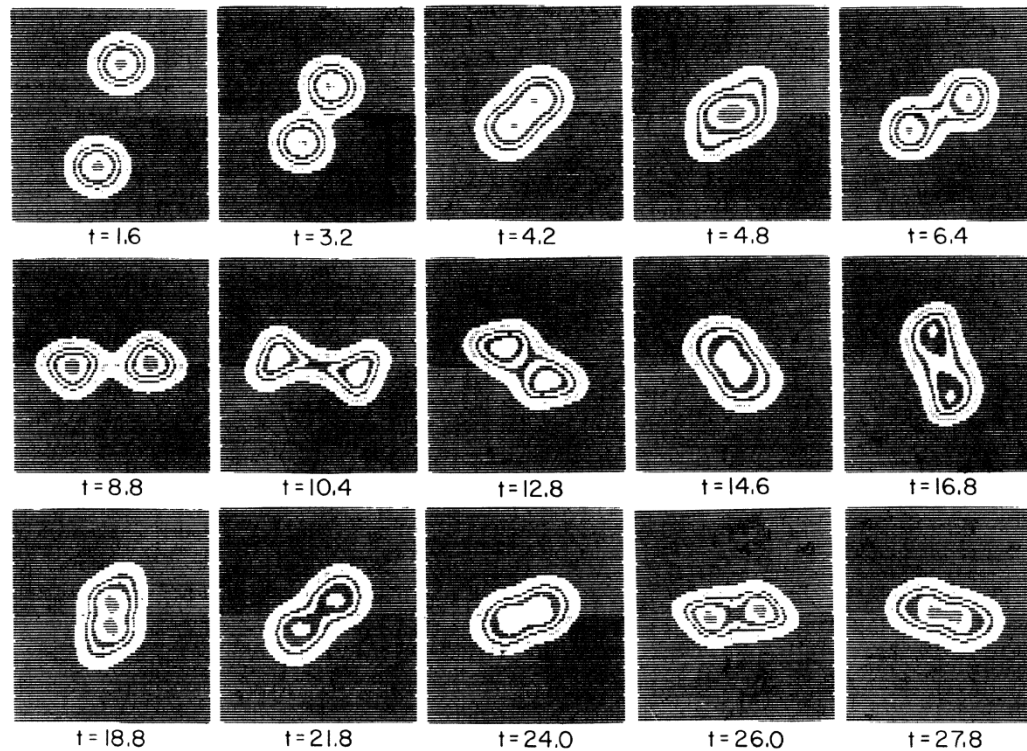


FIG. 2. Contour lines of the density integrated over the coordinate normal to the scattering plane for an $^{16}\text{O}+^{16}\text{O}$ collision at $E_{\text{lab}}=105$ MeV and incident angular momentum $L=13\hbar$. The times t are given in units of 10^{-22} sec.

Electron dynamics in metallic clusters by TDDFT

K. Yabana, G.F. Bertsch, Phys. Rev. B54, 4484 (1996).

Assume
Icosahedral shape

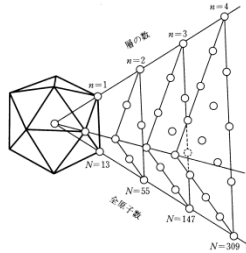
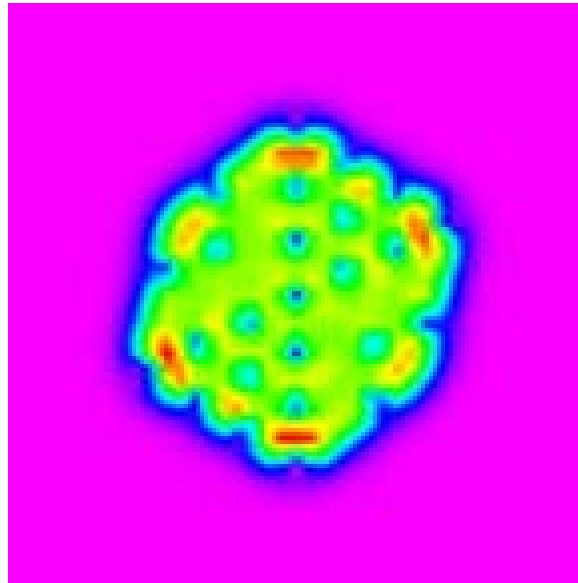
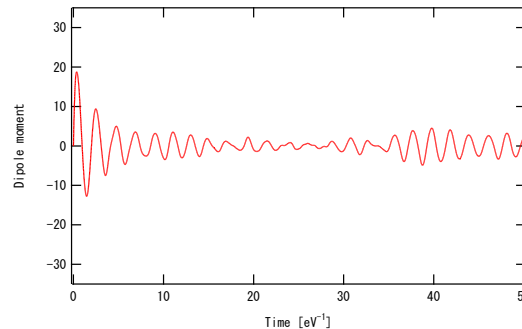


図 5-61 多層正二十面体構造 (MIC) の概念図⁶⁴⁾
n は MIC の層の数であり、N は全原子数である。1 層構造 (n=1) では原子数 N=13 に対応し、2 層構造 (n=2) では N=55、3 層構造 (n=3) では N=147...となる。

Li_{147}^+



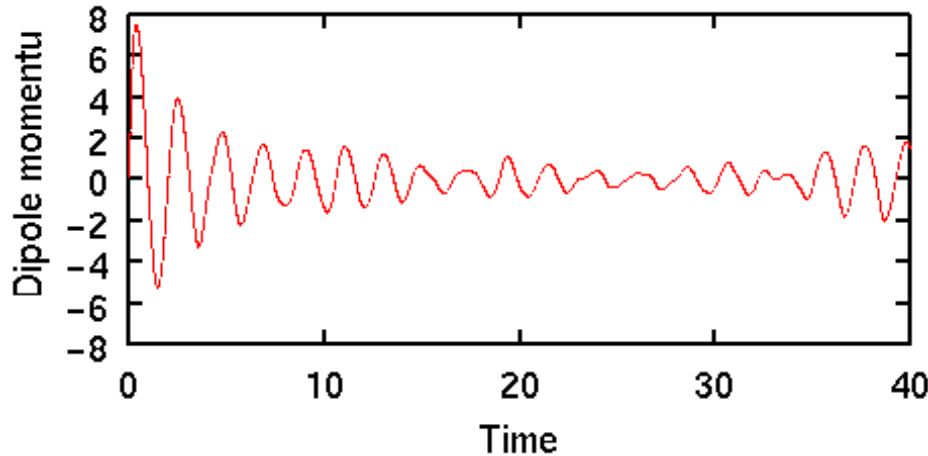
Density change induced by impulsive force



Dipole moment as function of time

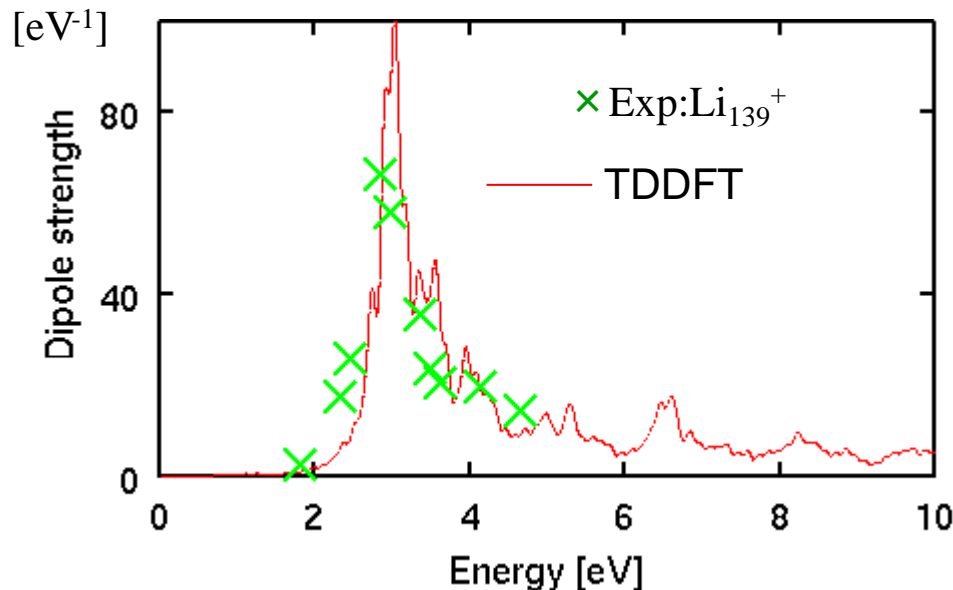
Real-time calculation for optical absorption spectrum of Li_{147}^+

K. Yabana, G.F. Bertsch, Phys. Rev. B54, 4484 (1996).



Real-time calculation
for autocorrelation function

$$\langle \hat{z}(t)\hat{z}(0) \rangle$$



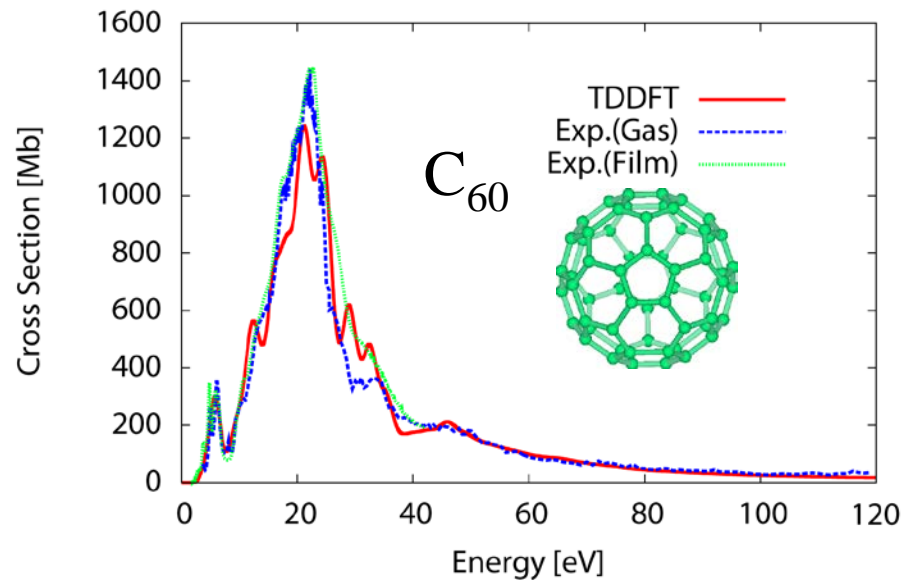
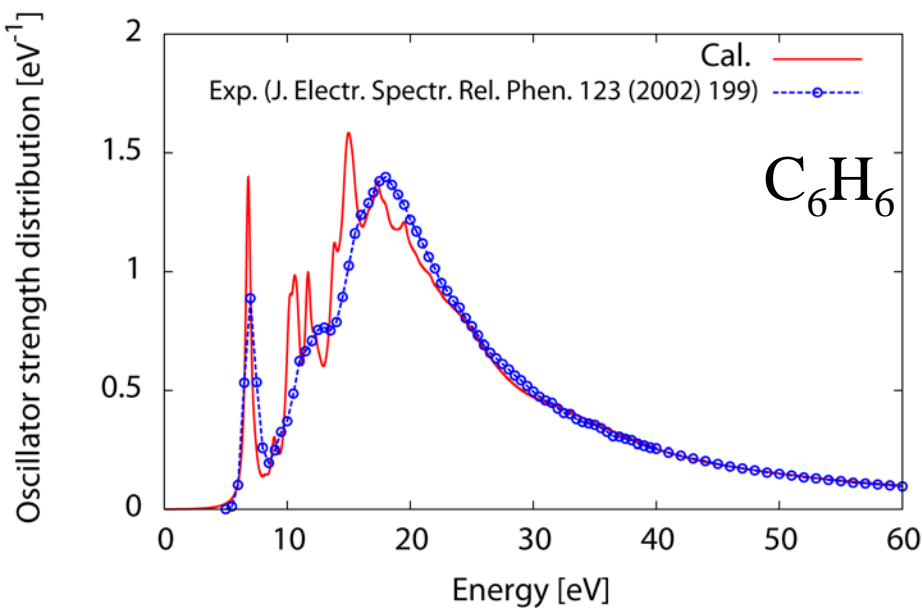
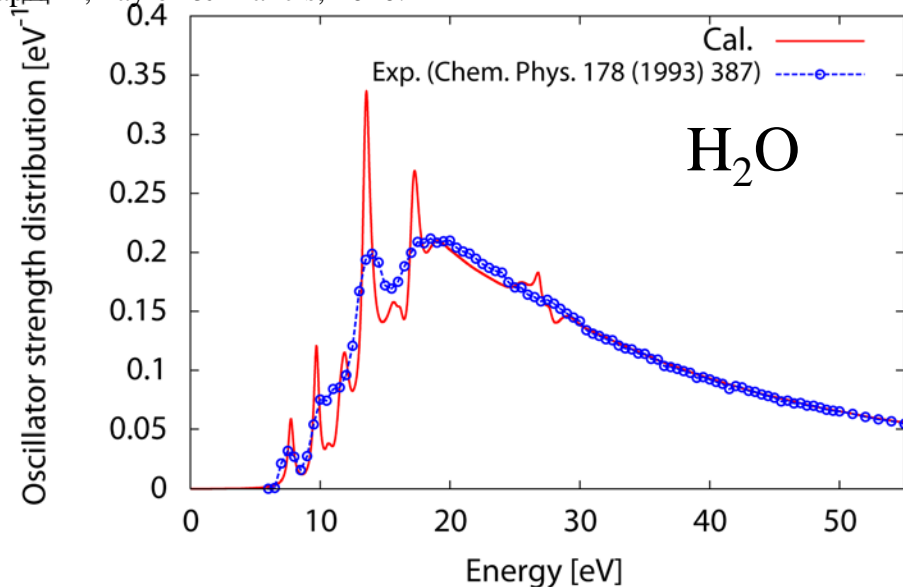
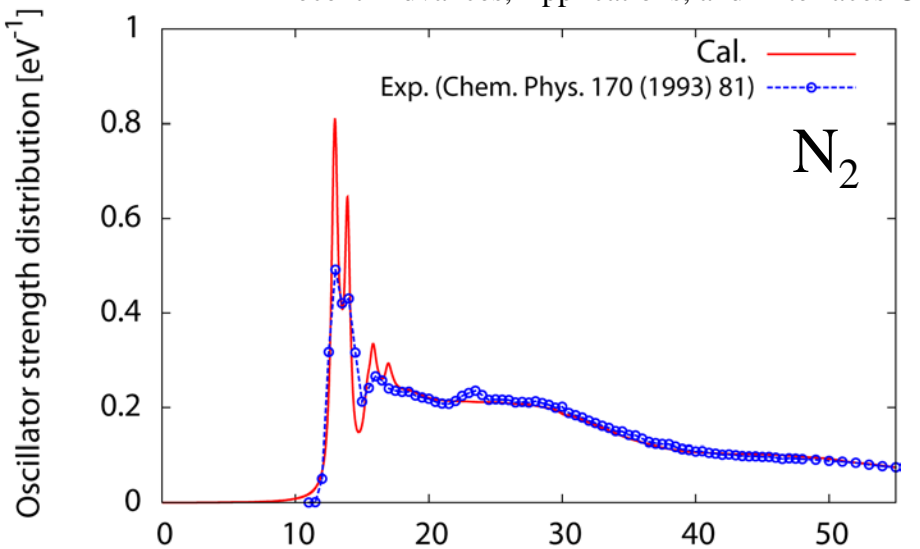
Fourier transformation
→ oscillator strength distribution

$$\sigma(\omega) \propto \frac{1}{k} \int dt e^{i\omega t} \langle \hat{z}(t)\hat{z}(0) \rangle$$

Photoabsorption of molecules by TDDFT (LB94 functional)

“Continuum RPA calculation for deformed system”

K. Yabana, Y. Kawashita, T. Nakatsukasa, J.-I. Iwata, Charged Particle and Photon Interactions with Matter: Recent Advances, Applications, and Interfaces Chapter 4, Taylor & Francis, 2010.



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Time-dependent extension of Bloch's band theory

$$i\hbar \frac{\partial}{\partial t} u_{n\vec{k}}(\vec{r}, t) = \left[\frac{1}{2m} \left(\vec{p} + \vec{k} + \frac{e}{c} \vec{A}(t) \right)^2 + \int d\vec{r}' \frac{e^2}{|\vec{r} - \vec{r}'|} n(\vec{r}', t) + \mu_{xc} [n(\vec{r}, t)] \right] u_{n\vec{k}}(\vec{r}, t)$$

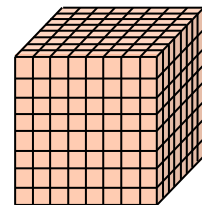
$$n(\vec{r}, t) = \sum_{nk} |u_{n\vec{k}}(\vec{r}, t)|^2$$

$$u_{nk}(\vec{r} + \vec{a}, t) = u_{nk}(\vec{r}, t)$$

Electron dynamics in crystalline solid (atomic positions are fixed)

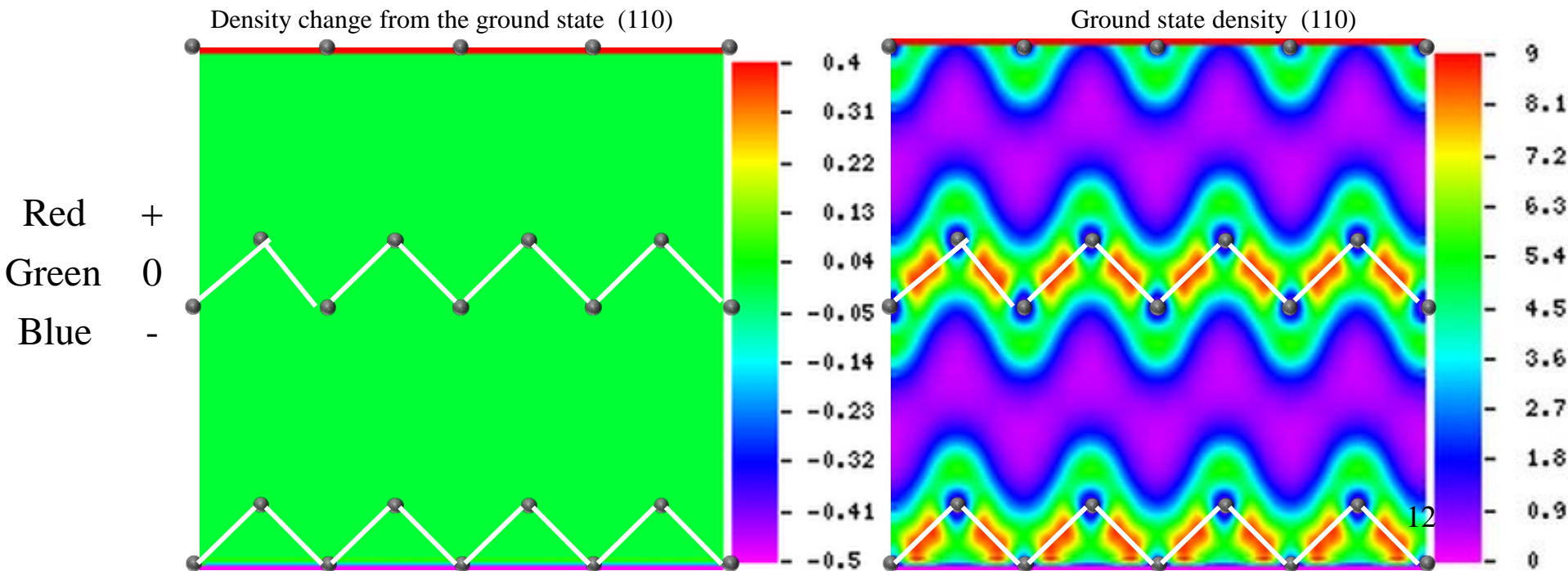
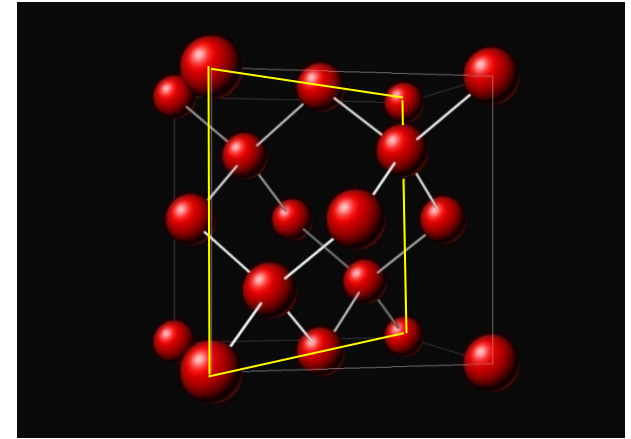
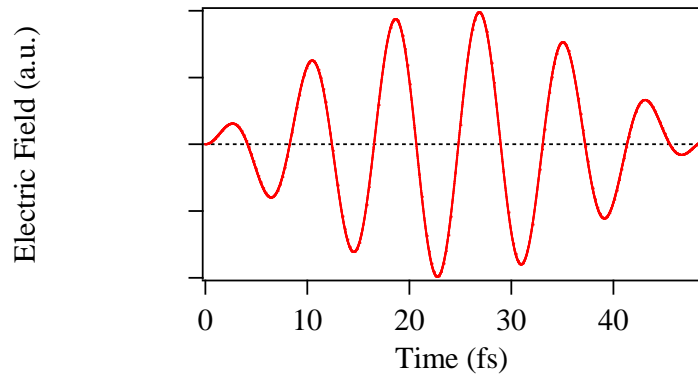
Computational aspects

- 3D uniform grid for space, high-order finite difference for differentiation
- Taylor expansion for time evolution



Electron dynamics in bulk Si under strong laser pulse

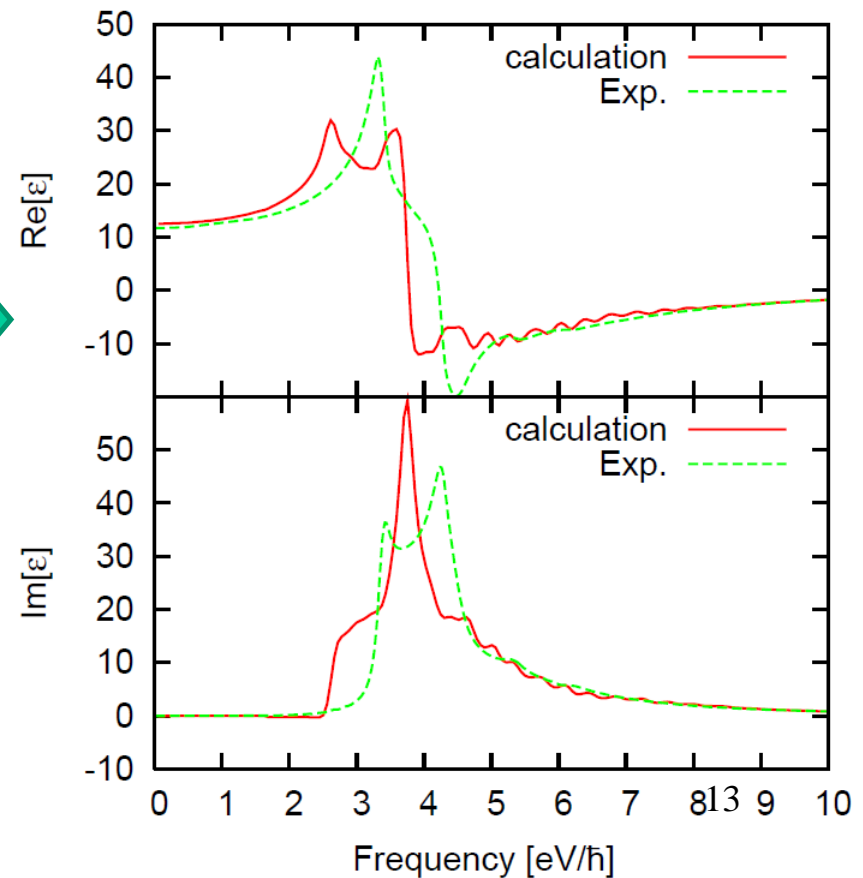
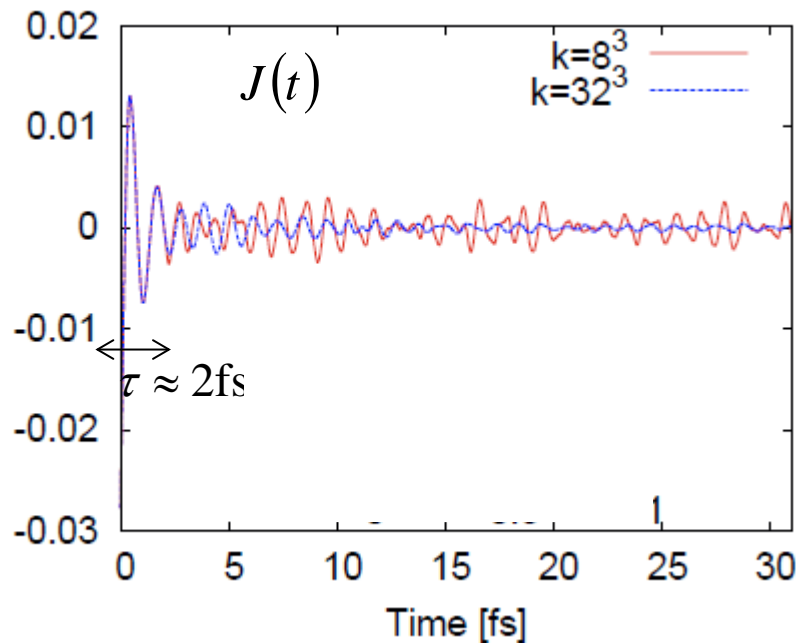
$I = 3.5 \times 10^{14} \text{ W/cm}^2$, $T = 50 \text{ fs}$, $\hbar\omega = 0.5 \text{ eV}$
Laser photon energy is much lower than direct bandgap.



Dielectric function of Si from real-time TDDFT-ALDA

Instantaneous kick at $t=0$, then calculate current $J(t)$

$$\sigma(\omega) = \frac{1}{k} \int dt e^{i\omega t} J(t), \quad \varepsilon(\omega) = 1 + \frac{4\pi i \sigma(\omega)}{\omega}$$



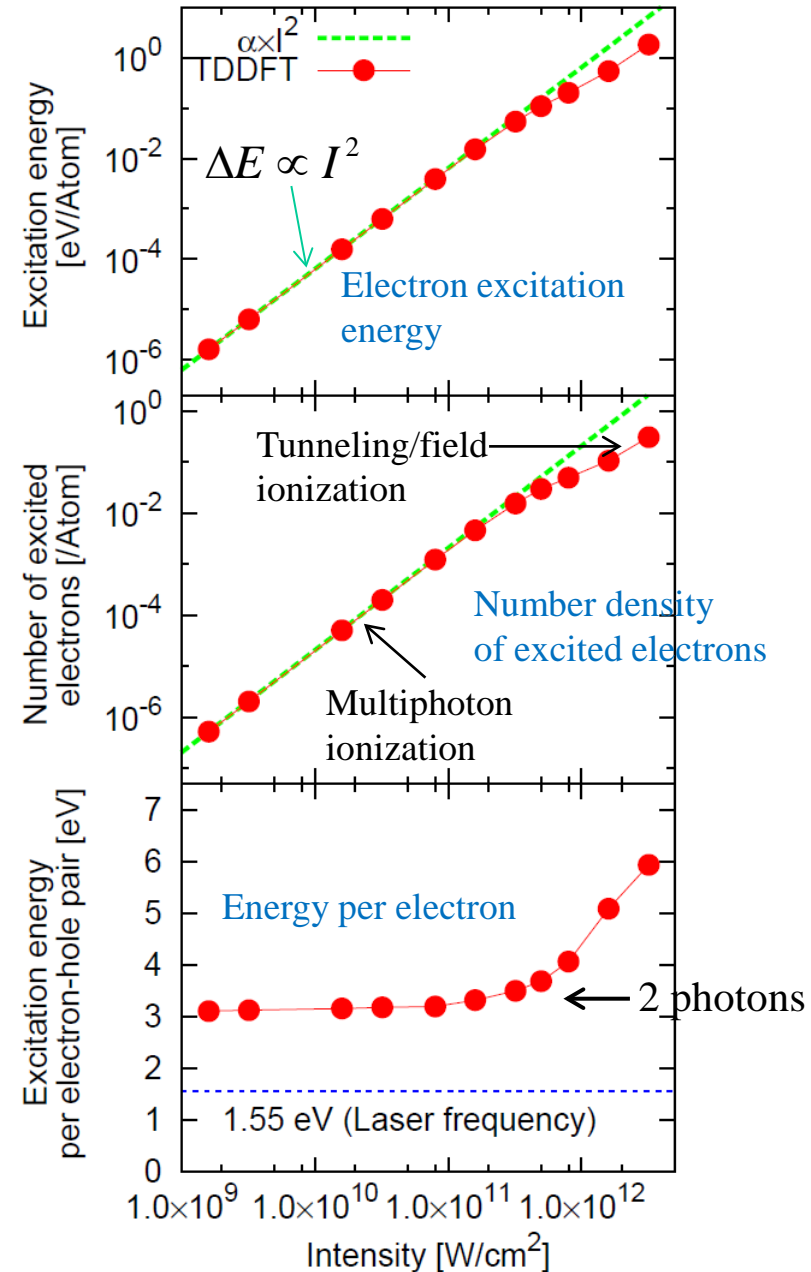
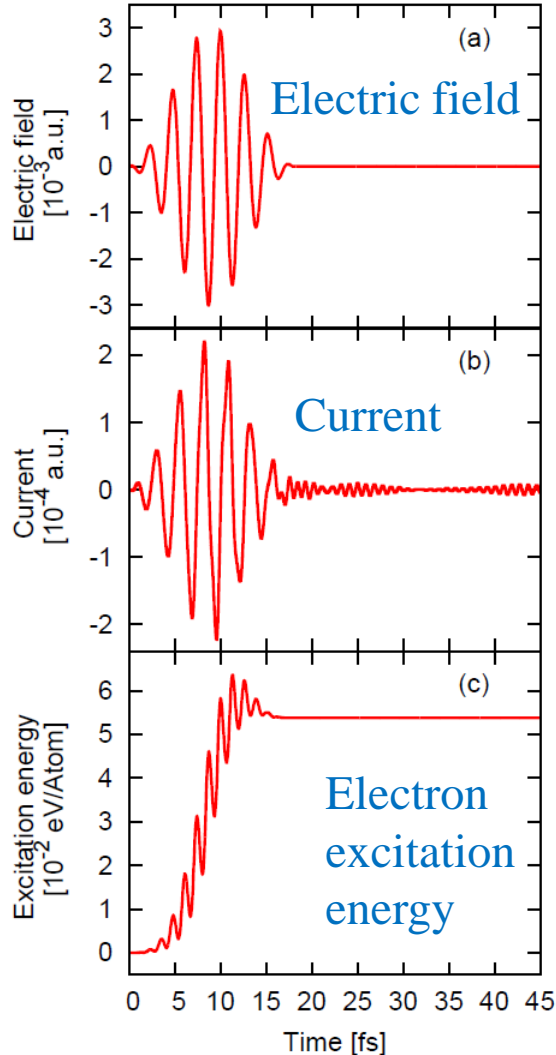
Not very good in quality.

Intense and ultrashort laser pulse on Si: Multi-photon to tunnel/field ionizations

$$\hbar\omega = 1.55\text{eV}$$

(direct bandgap 2.4 eV in LDA)

$$I = 3.2 \times 10^{12} \text{ W/cm}^2$$



Laser-Matter interaction: Strong and Ultra-Short Laser Pulse

Strong light field

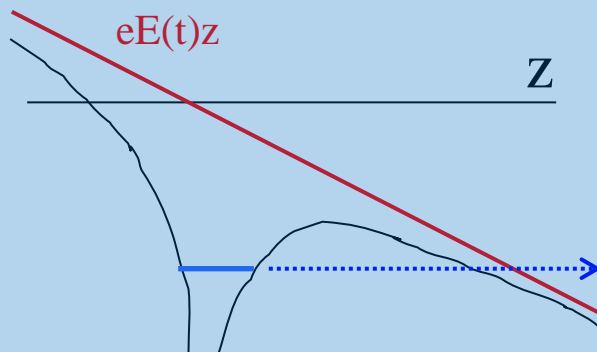
Magnitude of light electric field comparable to that bound electrons in matters.

Nonlinear, nonequilibrium
Electron Dynamics

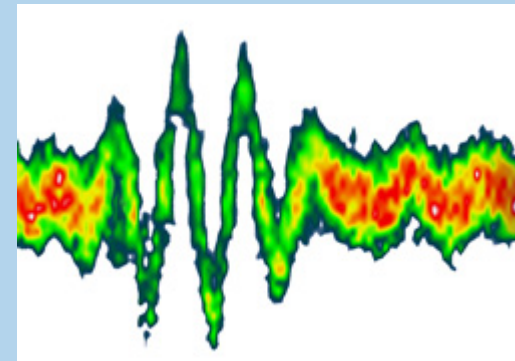
Ultra-short pulse

Pulse time duration comparable to a period of electron motion in matters.

Femto-technology
Atto-second science



Real-time observation of laser electric field using atto-second streaking technique



Joint LMU-MPQ Laboratory of Attosecond

Nonthermal Laser Machinery

Melting, ablation, filamentation on bulk surface

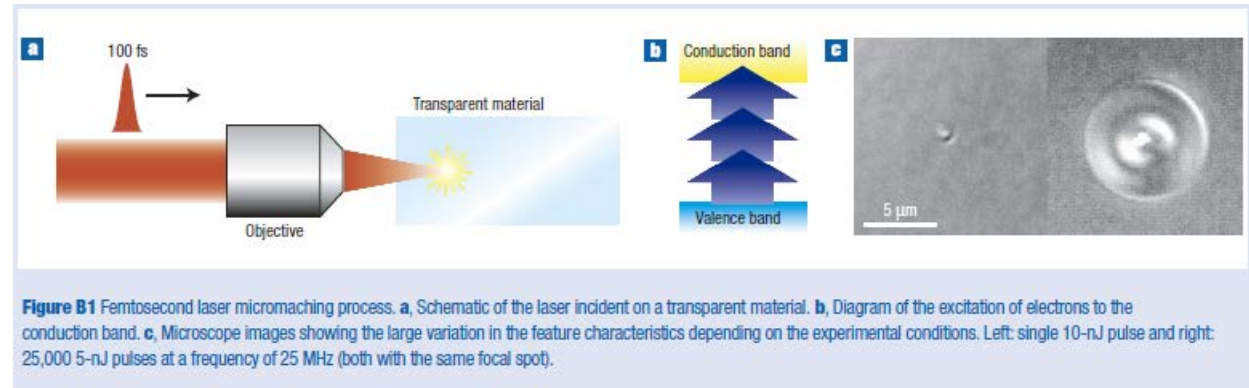
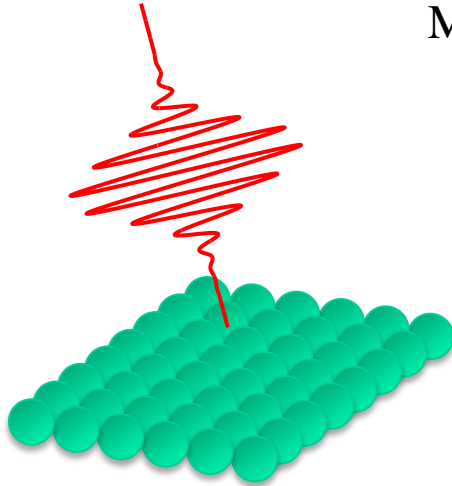
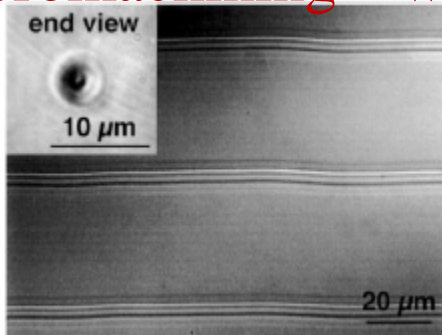


Figure B1 Femtosecond laser micromachining process. **a**, Schematic of the laser incident on a transparent material. **b**, Diagram of the excitation of electrons to the conduction band. **c**, Microscope images showing the large variation in the feature characteristics depending on the experimental conditions. Left: single 10-nJ pulse and right: 25,000 5-nJ pulses at a frequency of 25 MHz (both with the same focal spot).

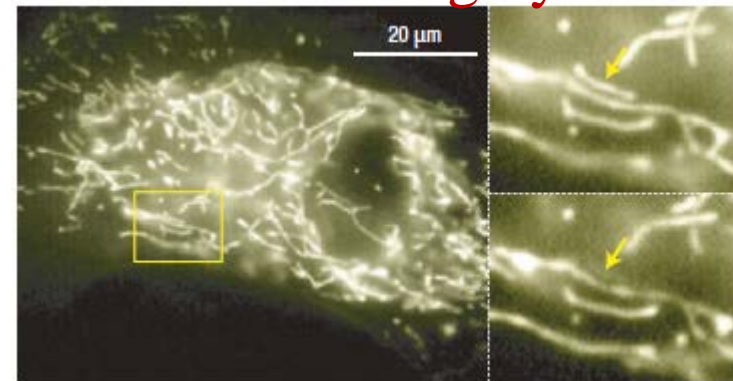
R.R. Gattass, E. Mazur, Nature Photonics 2, 220 (2008).

Micromachining – waveguide-



Optical microscope image of waveguides written inside bulk glass by a 25-MHz train of 5-nJ sub-100-fs pulses, C.B. Schaffer et.al, OPTICS LETTERS 26, 93 (2001)

Nanosurgery



Ablation of a single mitochondrion in a living cell, N. Shen et.al, Mech. Chem. Biosystems, 2, 17 (2003).

2008 - 2013

Linear optical absorption in molecules

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Ordinary macroscopic electromagnetism

Electromagnetism:

Maxwell equation for macroscopic fields, E, D, B, H

Linear constitutive relation



$$D = D[E] = \varepsilon(\omega)E$$



Quantum Mechanics:

Perturbation theory to calculate linear susceptibilities, $\varepsilon(\omega)$

As the field strength becomes large,
“nonlinear optics” becomes important.

$$D_\alpha(\vec{r}, t) = \int^t dt' \varepsilon_{\alpha\beta}(t-t') E_\beta(\vec{r}, t') + 4\pi \int^t dt' \int^t dt'' \chi_{\alpha\beta\gamma}^{(2)}(t-t', t-t'') E_\beta(\vec{r}, t') E_\gamma(\vec{r}, t'') + \dots$$

At extreme intense limit, EM and QM no more separate.



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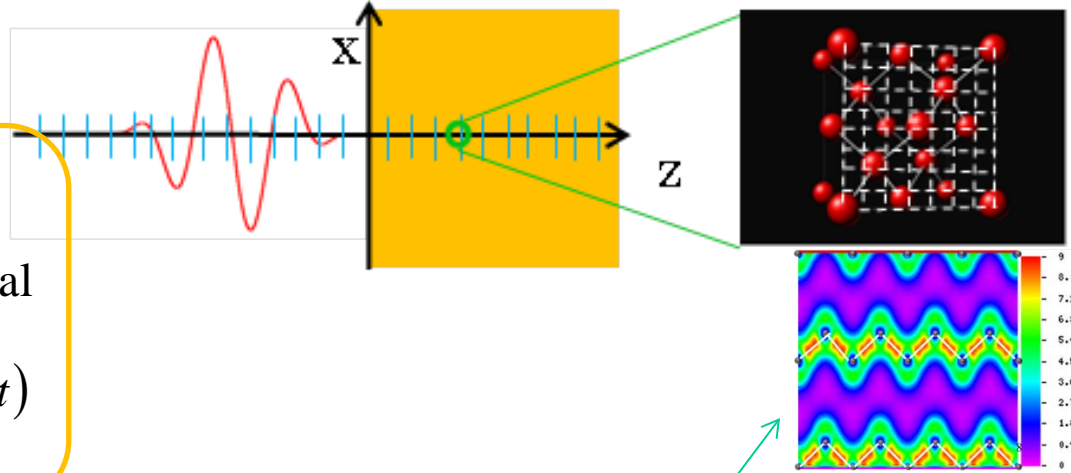
Multiscale simulation

K. Yabana, T. Sugiyama, Y. Shinohara, T. Otobe,
G.F. Bertsch, Phys. Rev. B85, 045134 (2012).

At each macroscopic grid point,
We consider a unit cell and prepare microscopic grid.

Macroscopic grid points (μm)
to describe macroscopic vector potential

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} A(Z, t) - \frac{\partial^2}{\partial Z^2} A(Z, t) = \frac{4\pi}{c} J(Z, t)$$



$J(Z, t)$



Exchange of information by
macroscopic current and
macroscopic vector potential.

$A(Z, t)$

At each macroscopic points, Kohn-Sham orbitals $\psi_{i,Z}$
are prepared, and described in microscopic grids.

$$J(Z, t) = \int_{\Omega} d\vec{r} \vec{j}_{e,Z}$$

$$\vec{j}_{e,Z} = \frac{\hbar}{2mi} \sum_i (\psi_{i,Z}^* \vec{\nabla} \psi_{i,Z} - \psi_{i,Z} \vec{\nabla} \psi_{i,Z}^*) - \frac{e}{4\pi c} n_{e,Z} \vec{A}$$

$$i\hbar \frac{\partial}{\partial t} \psi_{i,Z} = \frac{1}{2m} \left(-i\hbar \vec{\nabla} + \frac{e}{c} \vec{A} \right)^2 \psi_{i,Z} - e\phi_Z \psi_{i,Z} + \frac{\delta E_{xc}}{\delta n} \psi_{i,Z}$$

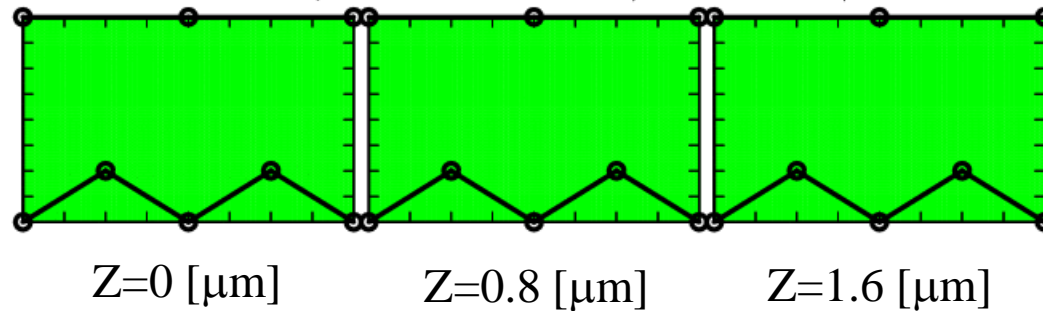
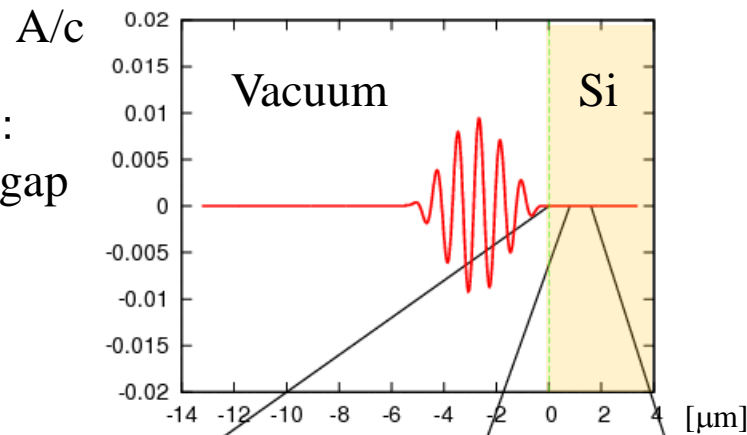
$$\vec{\nabla}^2 \phi_Z = -4\pi \{ en_{ion} - en_{e,Z} \}$$

Propagation of weak pulse

Ordinary electromagnetism is OK.

$$I=10^{10}\text{W}/\text{cm}^2$$

Laser frequency : 1.55eV :
lower than direct bandgap
2.4eV(LDA)

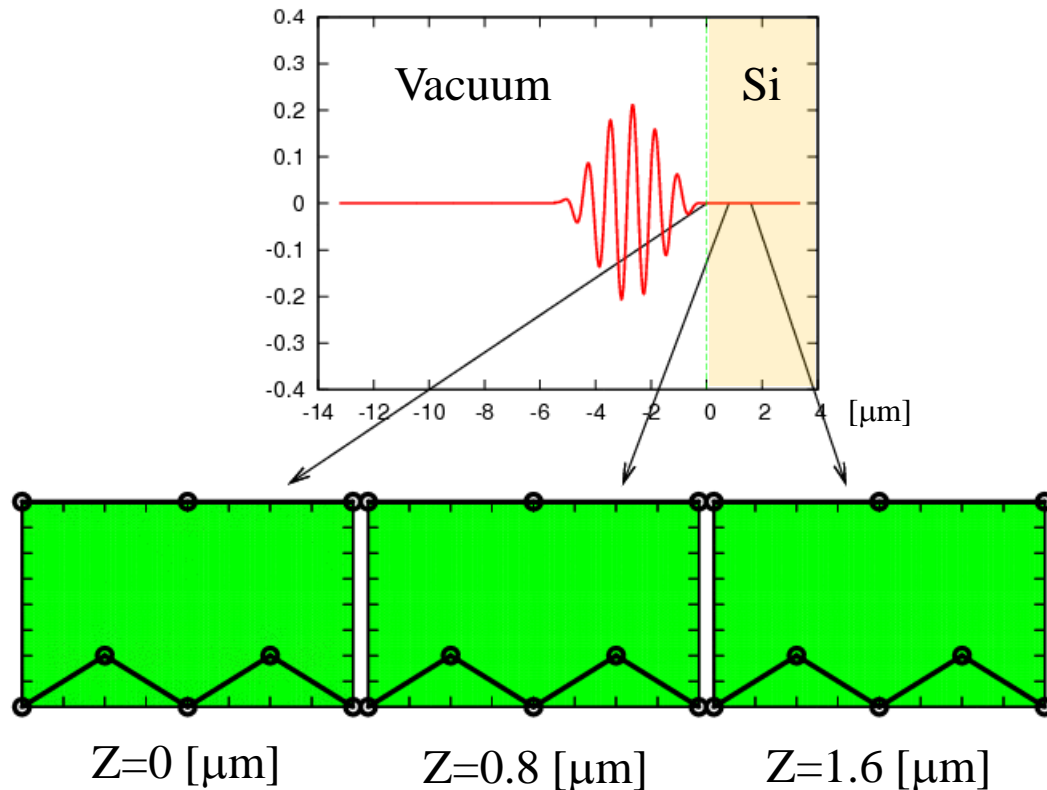


Coupled Maxwell + TDDFT simulation

More intense laser pulse

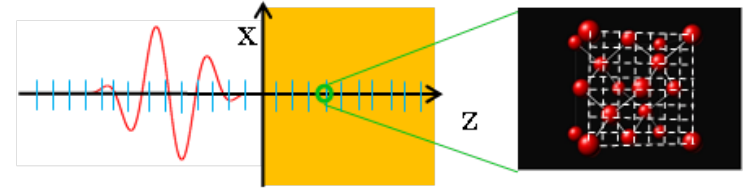
Dynamics of electrons and macroscopic EM fields are no more separable.

$$I = 5 \times 10^{12} \text{ W/cm}^2$$

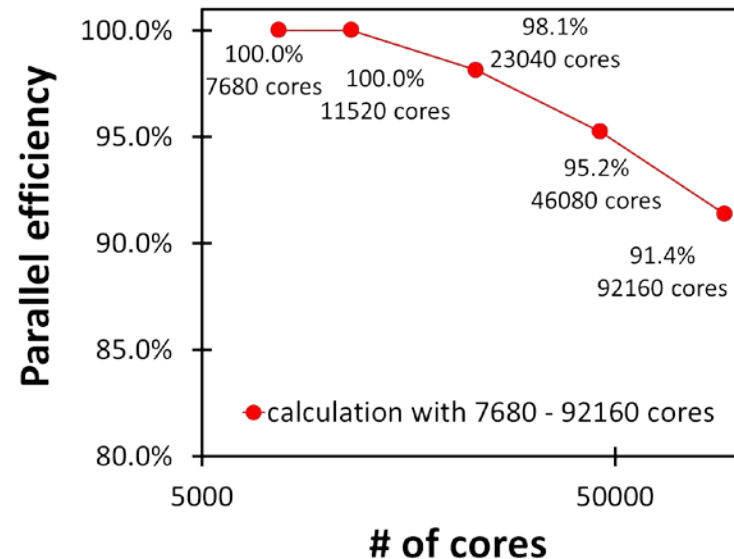
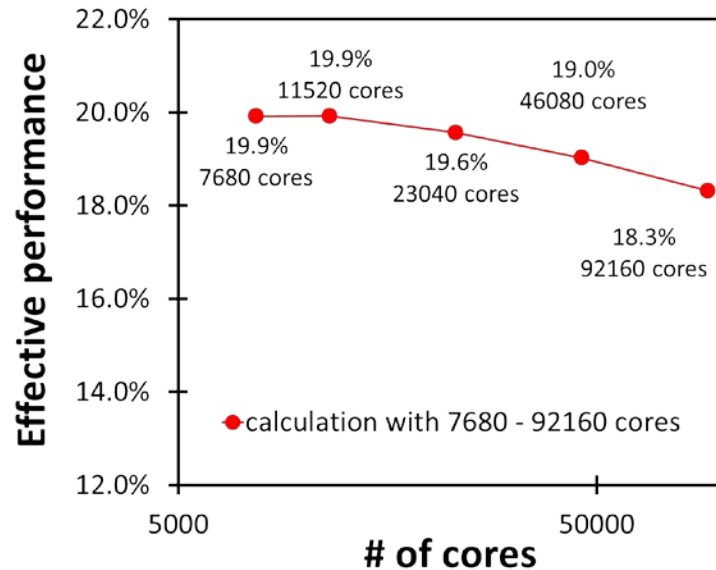


Computationally scalable simulation

1,000 cores, 10 hours
30,000 cores, 20 min (K-computer, Kobe)



Performance at K-Computer in Kobe (in early access)

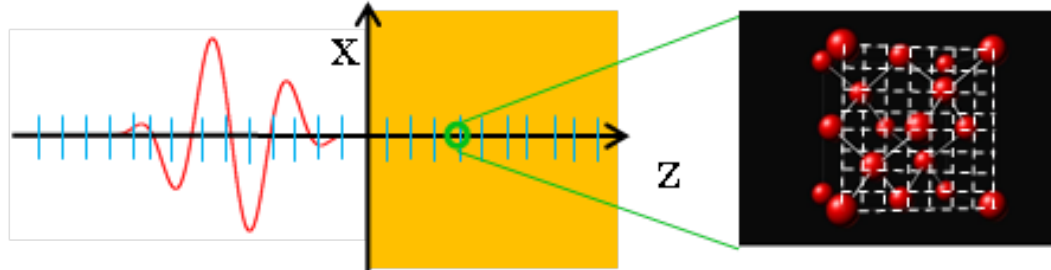


Computational time granted at K-computer for 2014 year.

- General use is admitted (4M node-hours)
- Use in strategic program is planned

We also have computational time at SuperMUC (LRZ, Germany)

Large-scale computation is indispensable



At present, 1-dim propagation (macroscopic grid)

Si, diamond:

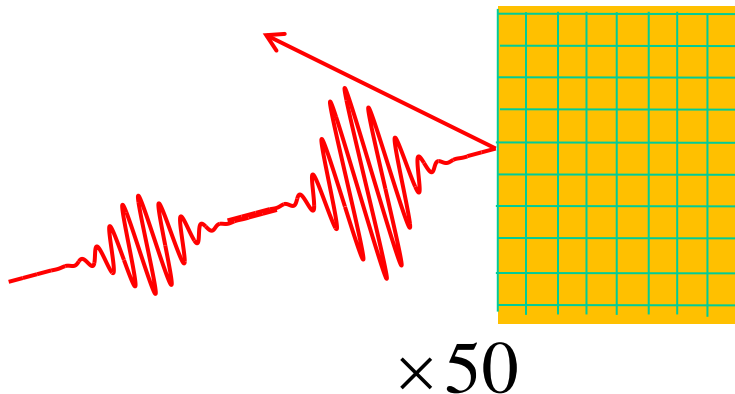
1,000 cores, 10 hours

20,000 cores, 20 min (K-computer, Kobe)

SiO₂ (α -quartz)

30,000 cores, 2 hours

Oblique incidence, 2-dim



3-dim

- Self focusing
- Circular polarization

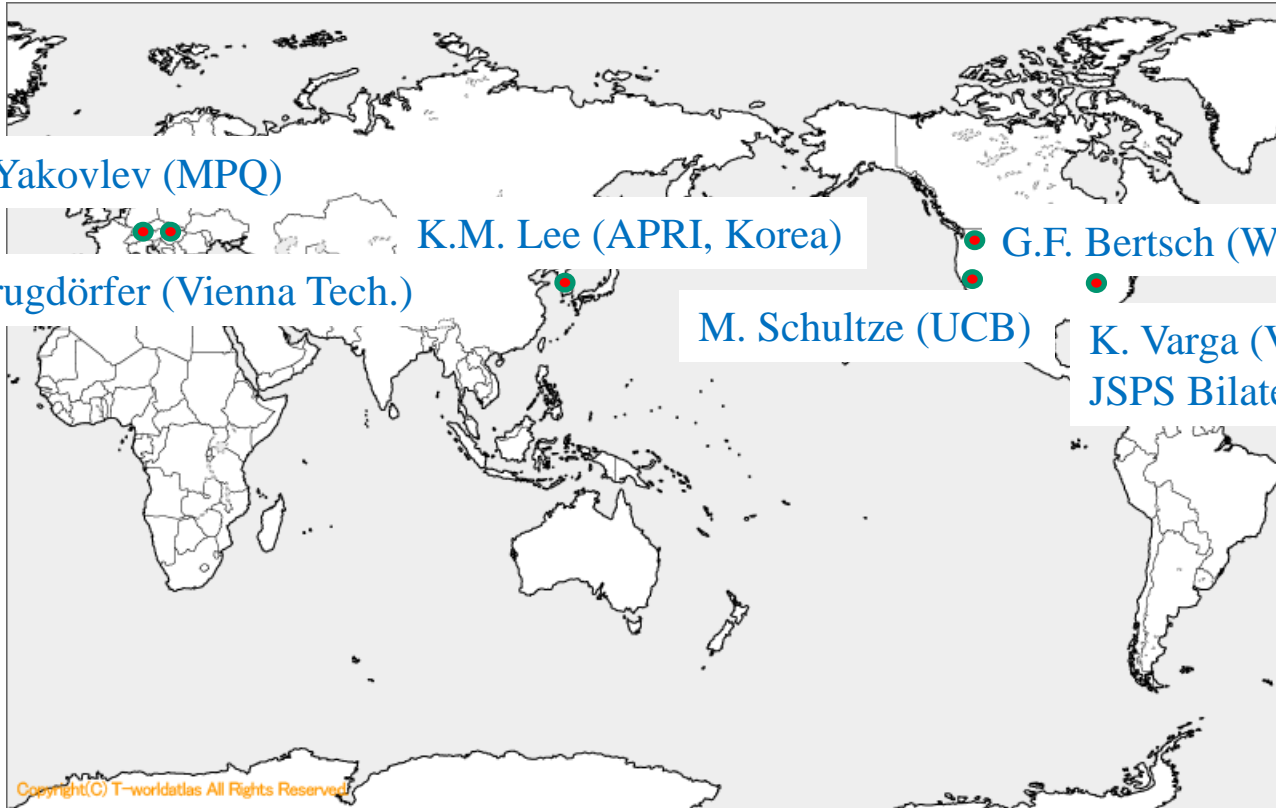
A million of macro-grid points

$\times 1,000$

need to wait next generation
supercomputers

International/Domestic Collaborations

F. Krausz, V. Yakovlev (MPQ)



J. Brugdörfer (Vienna Tech.)

K.M. Lee (APRI, Korea)

G.F. Bertsch (Washington)

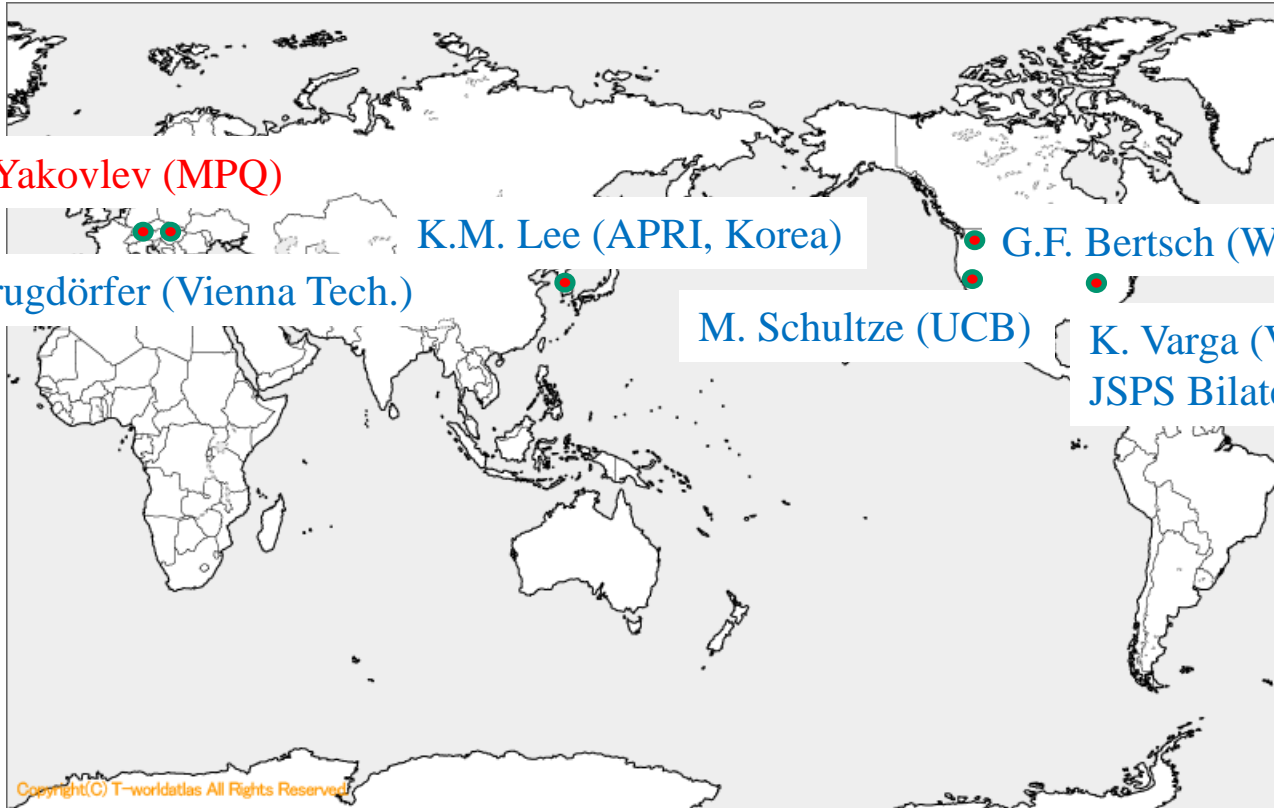
M. Schultze (UCB)

K. Varga (Vanderbilt)
JSPS Bilateral joint research

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K. Varga (Vanderbilt)
JSPS Bilateral joint research

First-principles calculation of ablation depth of α -quartz (SiO_2)

$$\hbar\omega = 1.55\text{eV} (\lambda = 800\text{nm}), \quad T = 10\text{fs}$$

$$E_{\text{gap}} = 6.5\text{eV} (\text{LDA})$$

Energy transfer from
Laser pulse to electrons

Absorbed energy (SiO_2)

Ablation threshold intensity

$$2.5 \times 10^{14} \text{ W/cm}^2$$

Ablation depth

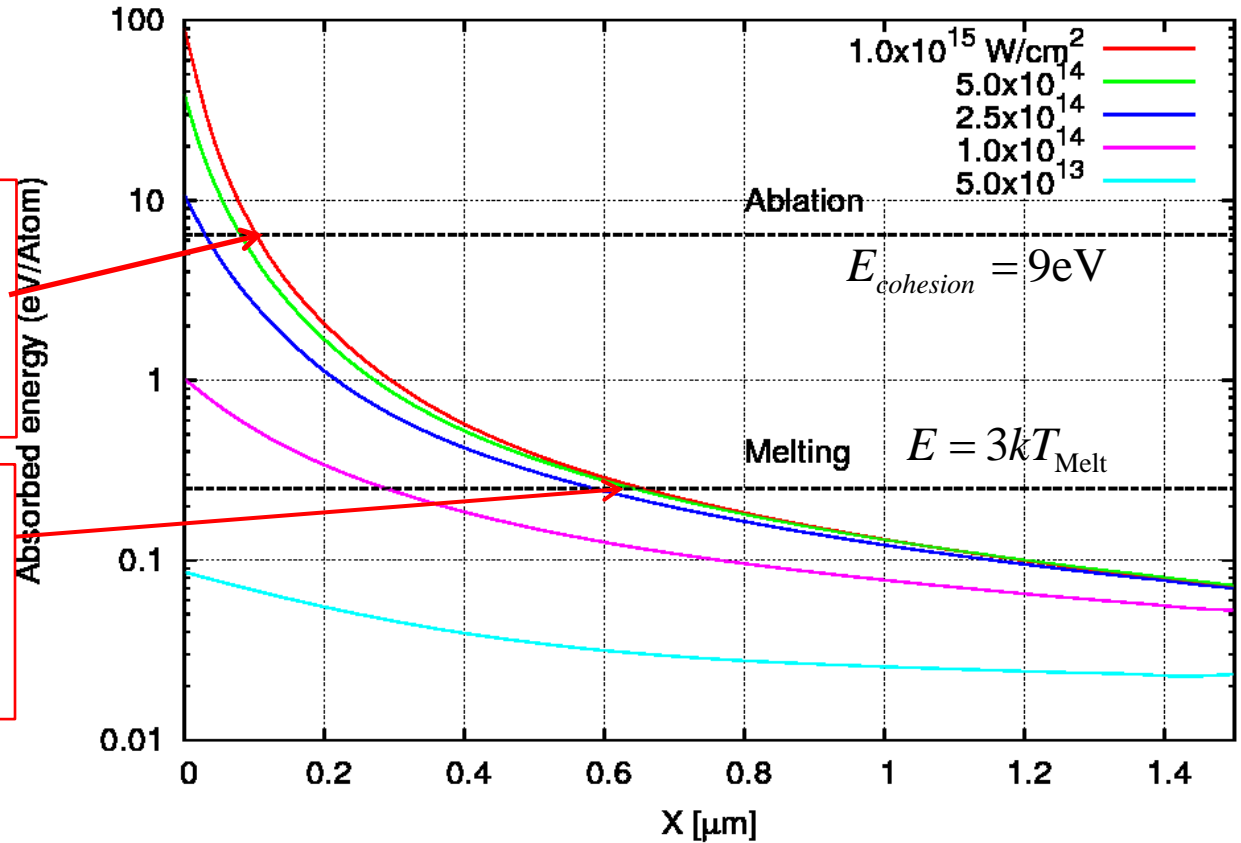
: 100nm

Melting threshold intensity

$$1 \times 10^{14} \text{ W/cm}^2$$

Melting depth

600nm



Typical computational time: 10,000 node-hours

HA-PACS: Multi-GPU calculation for hybrid functional

Hybrid functional provides a better performance for solids.
Calculation of the nonlocal exchange is the bottle neck.

$$\int d\vec{r}' \frac{e^2}{|\vec{r} - \vec{r}'|} \sum_{n'\vec{k}'} u_{n'\vec{k}'}(\vec{r}) u_{n'\vec{k}'}^*(\vec{r}') w_{n\vec{k}}(\vec{r}') = - \sum_{n'\vec{k}'} u_{n'\vec{k}'}(\vec{r}) e^{i\vec{K}\vec{r}} f_{n'\vec{k}',n\vec{k}}^{\vec{K}} \frac{4\pi e^2}{|\vec{K} + \vec{k} - \vec{k}'|^2}$$
$$f_{n'\vec{k}',n\vec{k}}^{\vec{K}} = \frac{1}{\Omega} \int_{\Omega} d\vec{r} e^{-i\vec{K}\vec{r}} u_{n'\vec{k}'}^*(\vec{r}) w_{n\vec{k}}(\vec{r})$$

We accelerate it using HA-PACS multi-GPU machine:

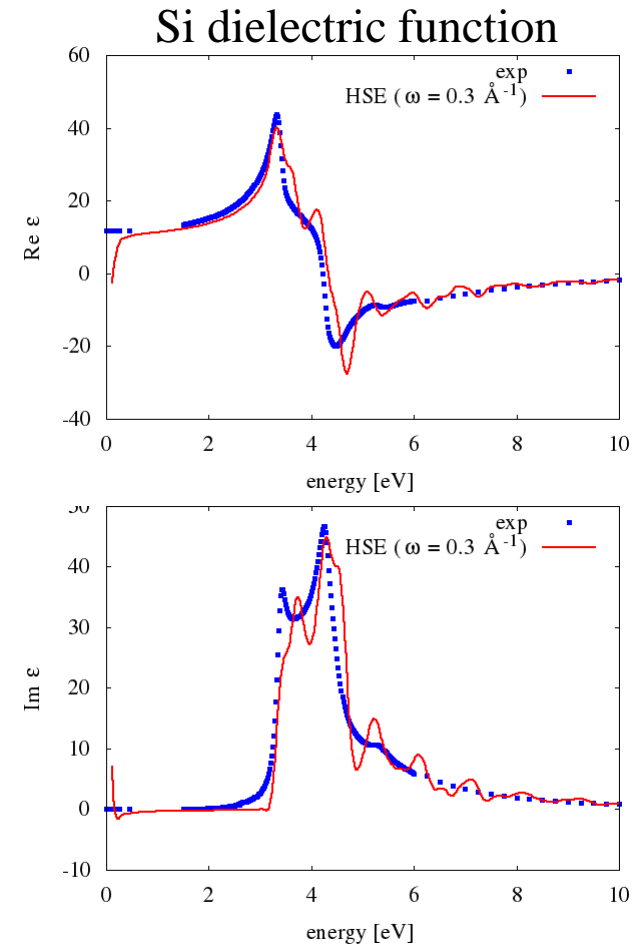
Unit cell: cubic, 8 atoms/cell

k-points: 12^3 downsampled k-points: 4^3 spatial grid: 16^3

108 GPU(K2090), cuFFT \Rightarrow 15 hours

Comparison with CPU (FFTW)

In one node, 4 GPUs is 4 times faster than 16 CPU cores



Code developments (ARTED):

Y. Shinohara (Ph.D, 2013, now JSPS fellowship, MPI-Halle)

S.A. Sato (M2, JSPS fellowship from April)

Y. Taniguchi (PD supported by the Center, developing GPU code)

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