Light nuclei and nucleon form factors from $N_f = 2 + 1$ lattice QCD

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for PACS Collaboration

CCS LBNL Collaborative Workshop 2016

1. Light nuclei

collaboration with

K.-I. Ishikawa, Y. Kuramashi, and A. Ukawa for PACS Collaboration

Refs: PRD81:111504(R)(2010); PRD84:054506(2011); PRD86:074514(2012)
PRD92:014501(2015); PoS(LATTICE 2015)081

2. Nucleon form factors

collaboration with

K.-I. Ishikawa, Y. Kuramashi, S. Sasaki and A. Ukawa
for PACS Collaboration

Ref: PoS(LATTICE 2015)081

Outline

- Introduction
- Calculation method of nuclei in lattice QCD
- Results of light nuclei
  - Simulation parameters
  - ^{4}He and ^{3}He channels
  - NN channels
- Preliminary result of nucleon form factors
- Summary and future work

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} \;\rightarrow\; \text{nuclei} \\ \text{quarks and gluons} \;\rightarrow\; \text{protons and neutrons} \end{cases}$$

both from fundamental strong interaction of quark and gluon

well known, but hard to prove

quark and gluon $\rightarrow$ proton and neutron $\rightarrow$ nucleus

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Spectrum of proton and neutron (nucleons)

success of non-perturbative lattice QCD calculation

degrees of freedom of quarks and gluons

$$\overbrace{\text{quark and gluon} \rightarrow \text{proton and neutron}} \rightarrow \text{nucleus}$$

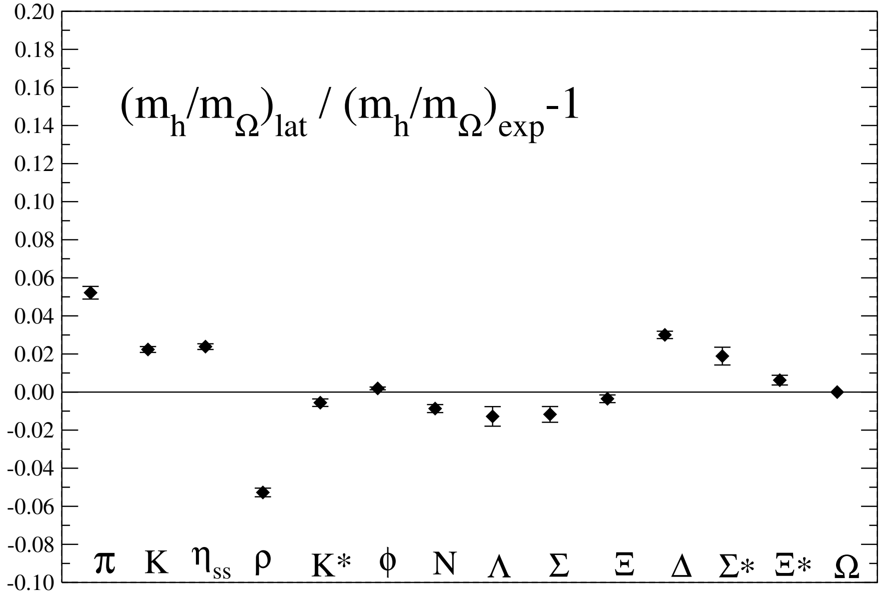
Hadron spectrum in $N_f = 2+1$ QCD

Lattice 2015, Ukita for PACS Collaboration PoS(LATTICE2015)075

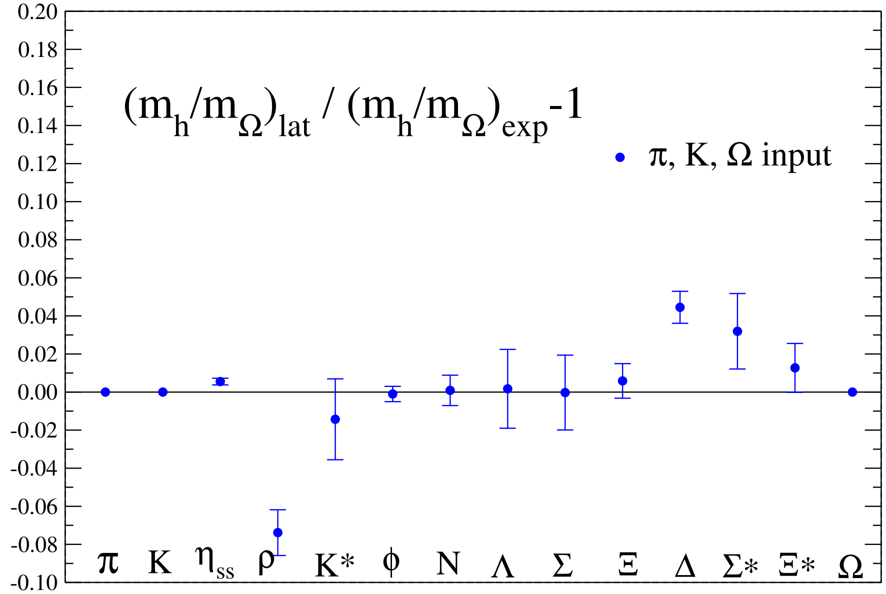
$m_\pi \sim 0.145$ GeV on $L \sim 8$ fm at $a^{-1} = 2.33$ GeV (SPIRE Field 5)

using reweighting m_{ud}, m_s + extrapolation $\rightarrow$ physical m_π and m_K

$m_\pi \sim 0.145$ GeV



physical point



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using reweighting m_{ud}, m_s + extrapolation $\to$ physical m_π and m_K

$$\bar{l}_3 = 2.87(62), \quad \bar{l}_4 = 4.38(33)$$

FLAG2013: $\bar{l}_3 = 3.05(99)$, $\bar{l}_4 = 4.02(28)$ at $\mu = m_\pi^{\text{phys}}$

$$m_{ud}^{\overline{\text{MS}}} = 3.142(26)(35)(28)\text{MeV}, \quad m_s^{\overline{\text{MS}}} = 88.59(61)(98)(79)\text{MeV}$$

FLAG2013: $m_{ud}^{\overline{\text{MS}}} = 3.42(6)(7)$MeV, $m_s^{\overline{\text{MS}}} = 93.8(1.5)(1.9)$MeV

$$f_\pi = 131.79(80)(90)(1.25)\text{MeV}, \quad f_K = 155.55(68)(1.06)(1.48)\text{MeV}$$

FLAG2013: $f_\pi = 130.2(1.4)$MeV, $f_K = 156.3(0.9)$MeV

reasonably consistent

investigation of $a \to 0$ limit necessary

Introduction

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both from fundamental strong interaction of quark and gluon

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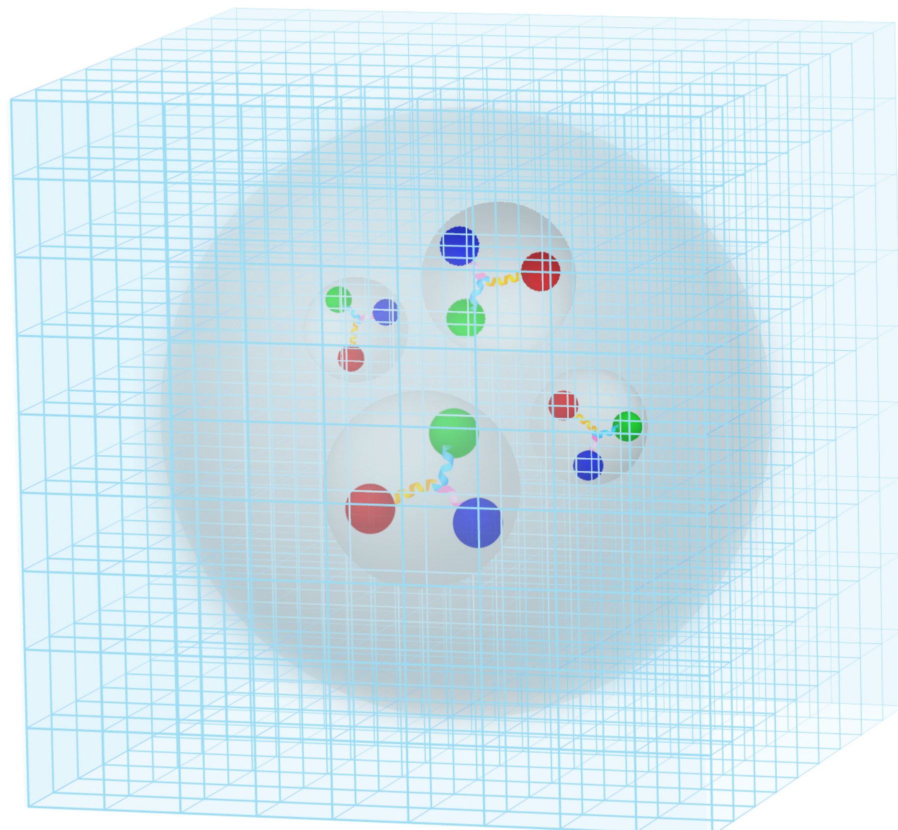
$\underbrace{\overbrace{\text{quark and gluon} \rightarrow \text{proton and neutron}} \rightarrow \text{nucleus}}$

goal: quantitatively understand property of nucleus from QCD

So far not many studies for multi-baryon bound states

$\rightarrow$ Can we reproduce binding energy of light nuclei?

Ultimate goal of lattice QCD



http://www.jicfus.jp/jp/promotion/pr/mj/2014-1/ ; figure from Irie-san

quantitatively understand property of nuclei from QCD

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} \;\to\; \text{nuclei} \\ \text{quarks and gluons} \;\to\; \text{protons and neutrons} \end{cases}$$

both from fundamental strong interaction of quark and gluon

well known, but hard to prove

Spectrum of proton and neutron (nucleons)

success of non-perturbative lattice QCD calculation

degrees of freedom of quarks and gluons

2nd part: Nucleon form factors not well understood

$$\underbrace{\overbrace{\text{quark and gluon} \to \text{proton and neutron}}\to \text{nucleus}}$$

1st part: nucleus from lattice QCD

Multi-baryon bound state from lattice QCD

1. ^{4}He and ^{3}He

   '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)

   '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He

   '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV

   '12 TY *et al.* $N_f = 2+1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)

   '15 TY *et al.* $N_f = 2+1$ $m_\pi = 0.30$ GeV PRD92:014501(2015)

2. H dibaryon in ΛΛ channel (S=−2, I=0)

   '11, '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV, $N_f = 3$ $m_\pi = 0.81$ GeV

   '11, '12 HALQCD $N_f = 3$ $m_\pi = 0.47$–1.02 GeV

   '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV

   '14 Mainz $N_f = 2$ $m_\pi = 0.45, 1.0$ GeV

3. NN

   '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)

   '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV (Possibility)

   '12 NPLQCD, '15 CalLat $N_f = 3$ $m_\pi = 0.81$ GeV

   '12 TY *et al.* $N_f = 2+1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)

   '15 TY *et al.* $N_f = 2+1$ $m_\pi = 0.30$ GeV PRD92:014501(2015)

   '15 NPLQCD $N_f = 2+1$ $m_\pi = 0.45$ GeV

Other states: ΞΞ, '12 NPLQCD; spin-2 $N\Omega$, ^{16}O and ^{40}Ca, '14 HALQCD, $\cdots$

Calculation method of multi-nucleon bound state

Traditional method

nucleon

$$C_N(t) \;=\; \langle 0|N(t)\overline{N}(0)|0\rangle = \sum_n \langle 0|N|n\rangle\langle n|\overline{N}|0\rangle e^{-E_n^N t} \xrightarrow[t\gg 1]{} A_0^N\, e^{-m_N t}$$

NN channel

$$\begin{aligned} C_{NN}(t) \;&=\; \langle 0|O_{NN}(t)\overline{O}_{NN}(0)|0\rangle = \sum_n \langle 0|O_{NN}|n\rangle\langle n|\overline{O}_{NN}|0\rangle e^{-E_n t} \\ &\xrightarrow[t\gg 1]{} A_0\, e^{-m_{NN} t} \end{aligned}$$

Ratio of correlation functions

$$R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \xrightarrow[t\gg 1]{} A_0'\, e^{-\Delta E t}, \quad \Delta E = m_{NN} - 2m_N$$

Important condition

$C_N(t)$ and $C_{NN}(t)$ are written by each ground state in $t \gg 1$

Simulation parameters

$N_f = 2+1$ QCD $\beta = 1.90$, $a^{-1} = 2.194$ GeV with $m_\Omega = 1.6725$ GeV, '10 PACS-CS

Iwasaki gauge + non-perturbative $O(a)$-improved Wilson fermion actions

$m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV PRD86:074514(2012)

$m_\pi = 0.30$ GeV and $m_N = 1.05$ GeV PRD92:014501(2015)

$m_s \sim$ physical strange quark mass

^{4}He, ^{3}He, NN(^{3}S$_1$ and ^{1}S$_0$)

		$m_\pi = 0.5$ GeV		$m_\pi = 0.3$ GeV		R
L	L [fm]	N_{conf}	N_{meas}	N_{conf}	N_{meas}	
32	2.9	200	192			
40	3.6	200	192			
48	4.3	200	192	400	1152	12
64	5.8	190	256	160	1536	5

$R = (N_{\text{conf}} \cdot N_{\text{meas}})_{0.3\text{GeV}} / (N_{\text{conf}} \cdot N_{\text{meas}})_{0.5\text{GeV}}$

Smear source and point sink (N with $p = 0$) operators

$\rightarrow$ after some tests in $N_f = 0$, consider large overlap to ground state

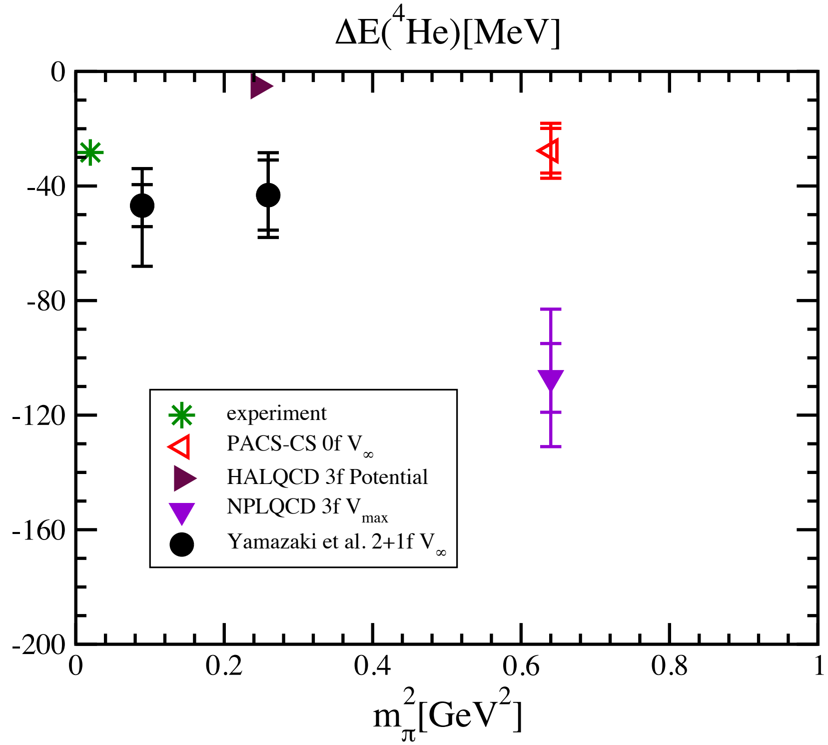
Computational resources

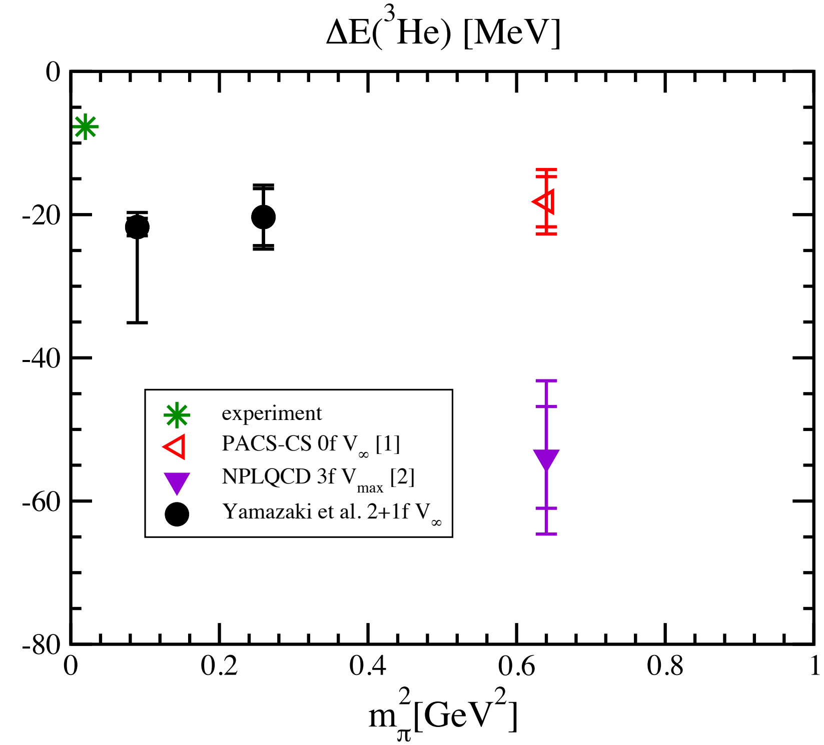
PACS-CS, T2K-Tsukuba, HA-PACS, COMA at Univ. of Tsukuba

T2K-Tokyo and FX10 at Univ. of Tokyo, and K at AICS

Result: ^{4}He and ^{3}He channels

$$\Delta E(^4\text{He}) = m_{^4\text{He}} - 4m_N \qquad \Delta E(^3\text{He}) = m_{^3\text{He}} - 3m_N$$





Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

Same order of ΔE to experiments

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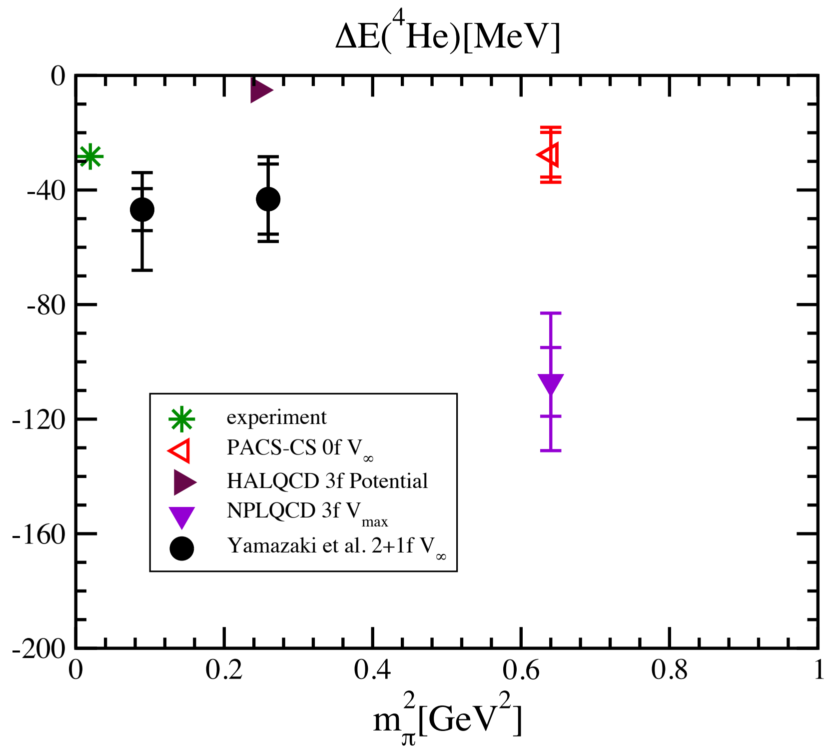

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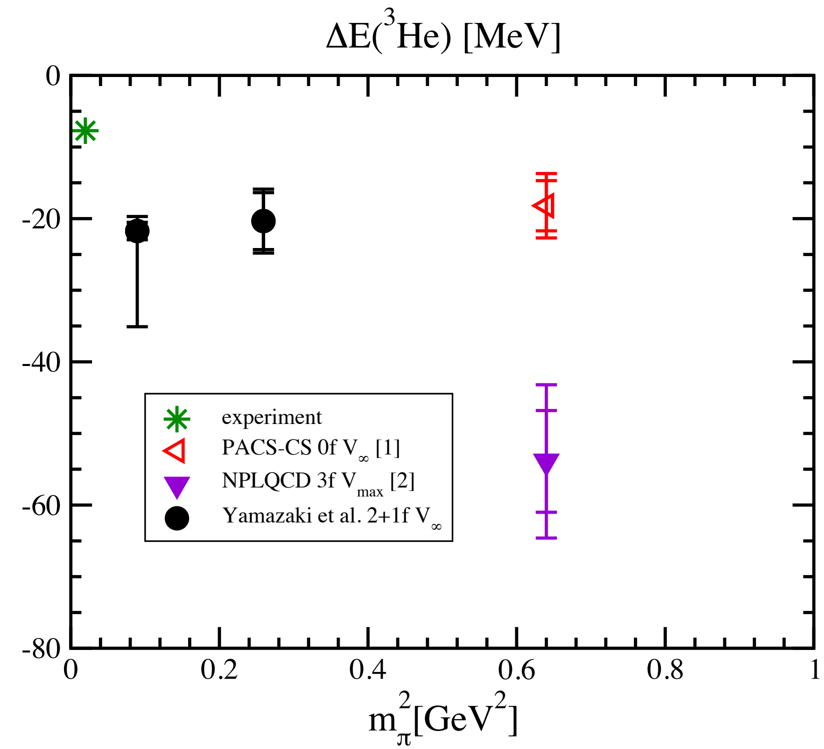


Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

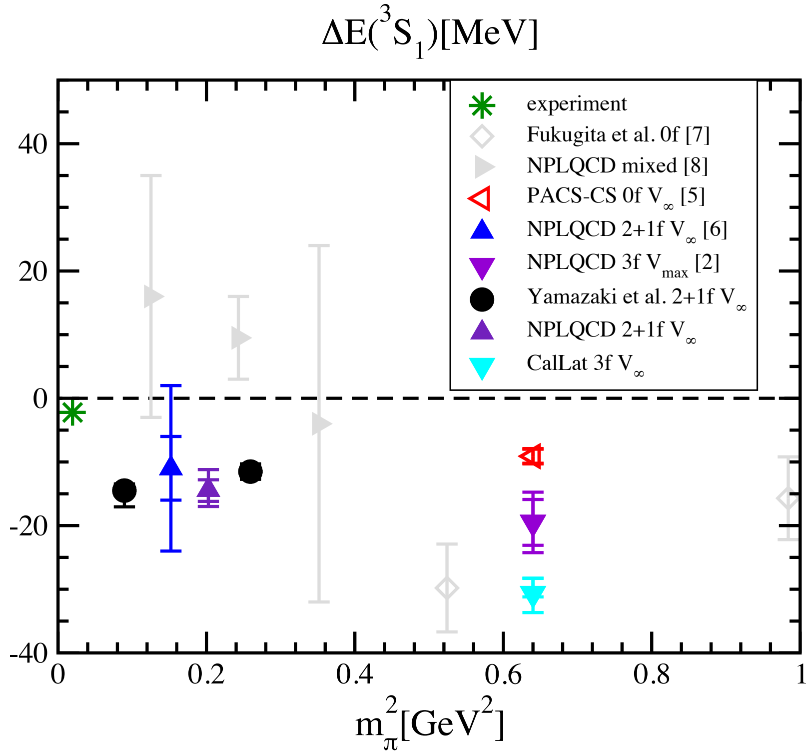
Same order of ΔE to experiments $\rightarrow$ relatively easier than NN

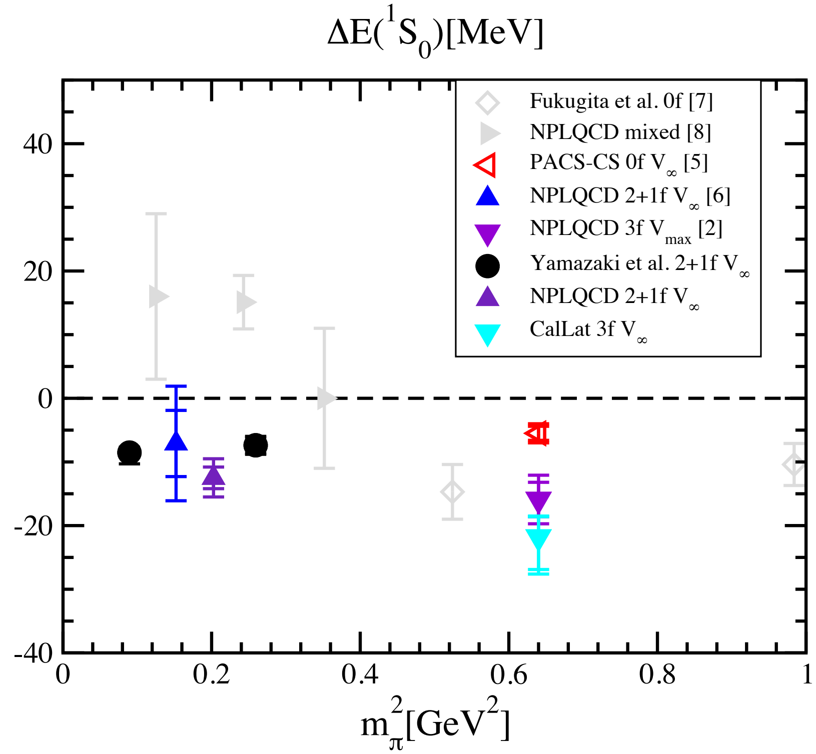
large $|\Delta E|$ makes less V dependence at physical m_π

touchstone of quantitative understanding of nuclei from lattice QCD

Investigations of m_π dependence $\rightarrow$ $m_\pi \sim$ 0.145 GeV on $L \sim 8$ fm

Result: NN channels $\Delta E = m_{NN} - 2m_N$





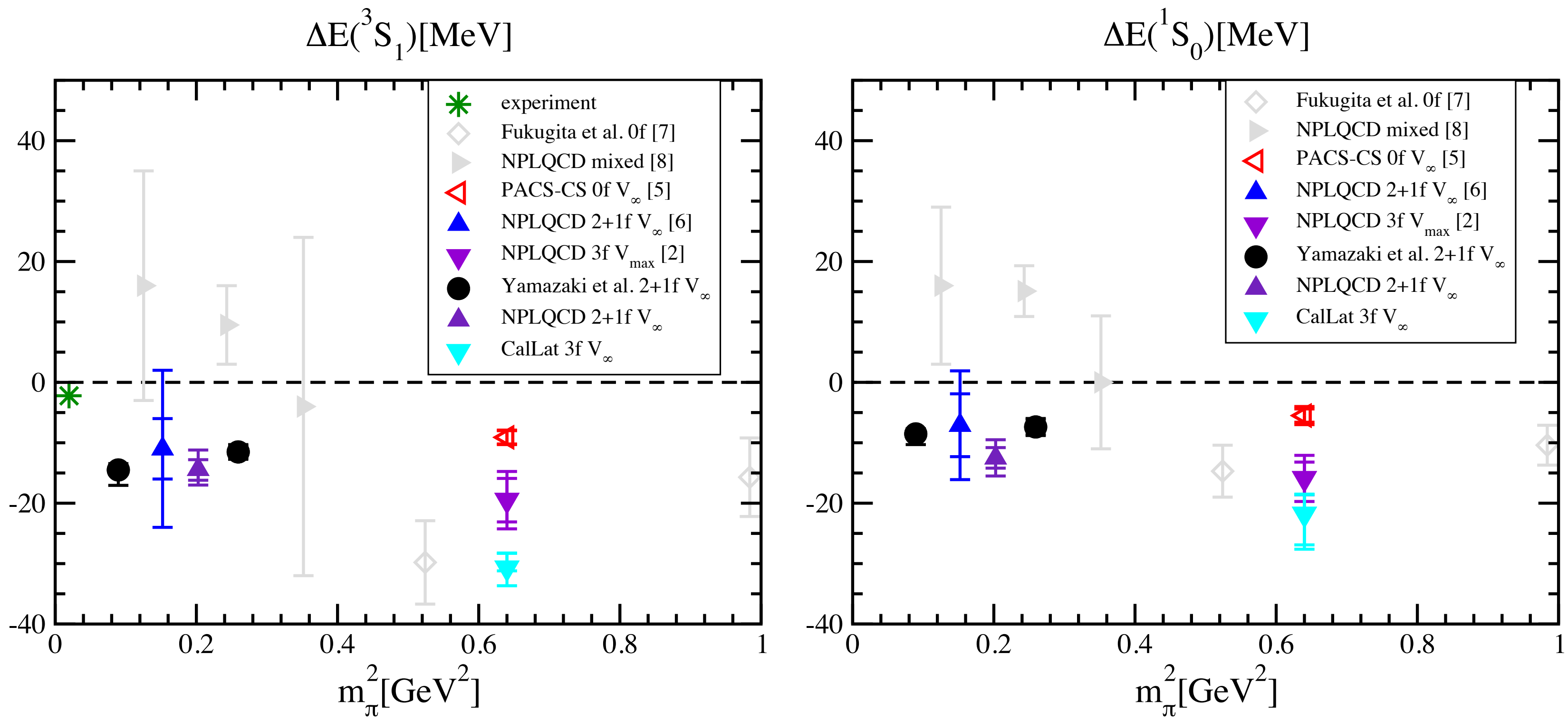
gray data: single volume calculation

$L^3 \to \infty$ data: existence of bound states in $^3S_1$ and $^1S_0$

inconsistent with experiment due to larger m_π(?)

Investigation of m_π dependence $\to m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Result: NN channels $\Delta E = m_{NN} - 2m_N$



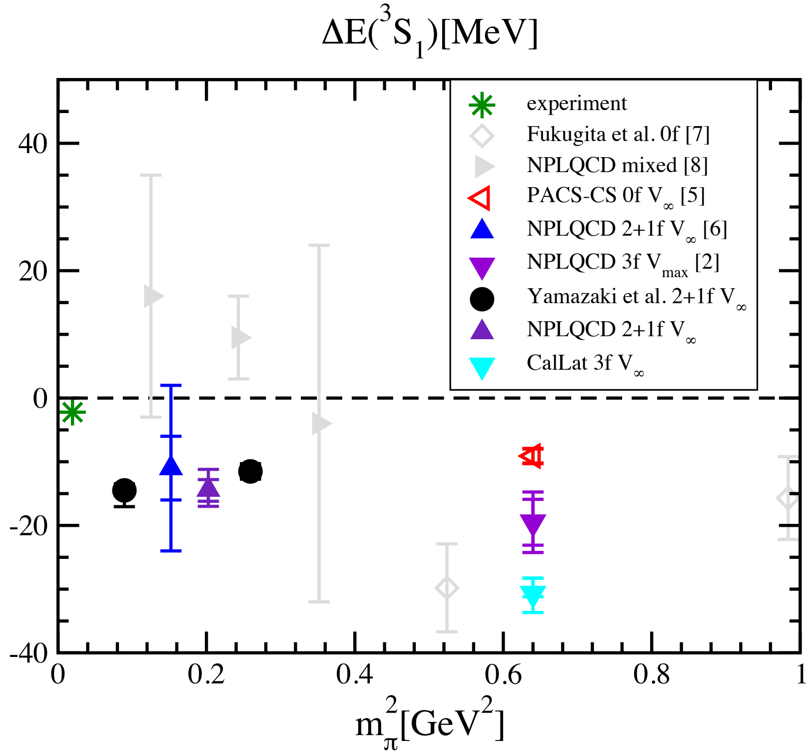
gray data: single volume calculation

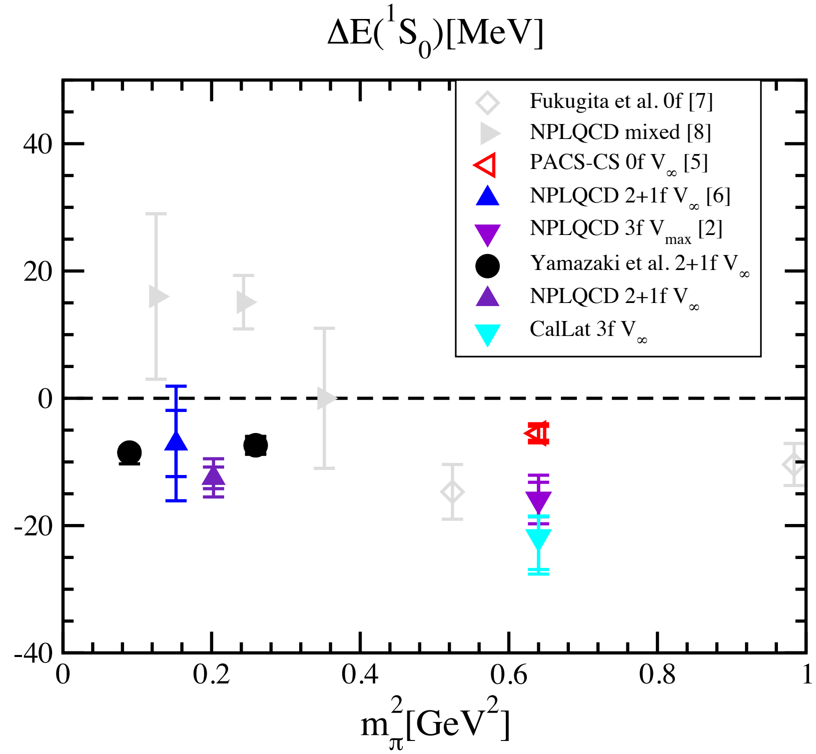
Investigations of m_π dependence $\rightarrow$ $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Large finite volume effect expected even on $L \sim 8$ fm

'86 Lüscher, '04 Beane *et al.*, '14 Briceño *et al.*

Result: NN channels $\Delta E = m_{NN} - 2m_N$





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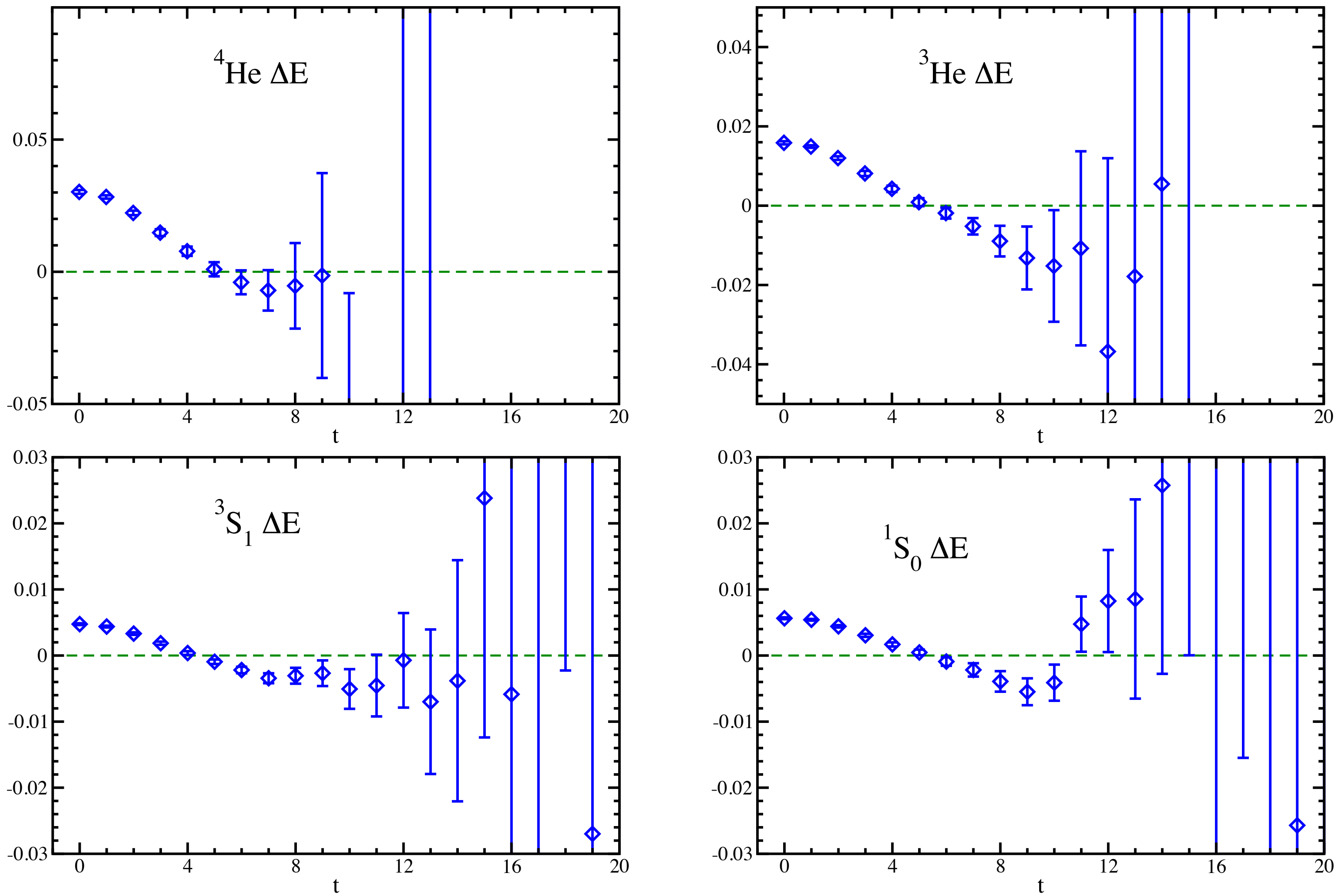
$^3\mathrm{S}_1$: $\Delta E_{\mathrm{exp}} = 2.2$ MeV

$$\Delta E_L = -\left(\Delta E_{\mathrm{exp}} + \mathcal{O}(\exp(-L\sqrt{m_N \Delta E_{\mathrm{exp}}}))\right) \lesssim -4 \text{ MeV}$$

$^1\mathrm{S}_0$: $a_0^{\mathrm{exp}} = 23.7$ fm

$$\Delta E_L = -\frac{4\pi a_0^{\mathrm{exp}}}{m_N L^3} + \mathcal{O}(1/L^4) \lesssim -2 \text{ MeV}$$

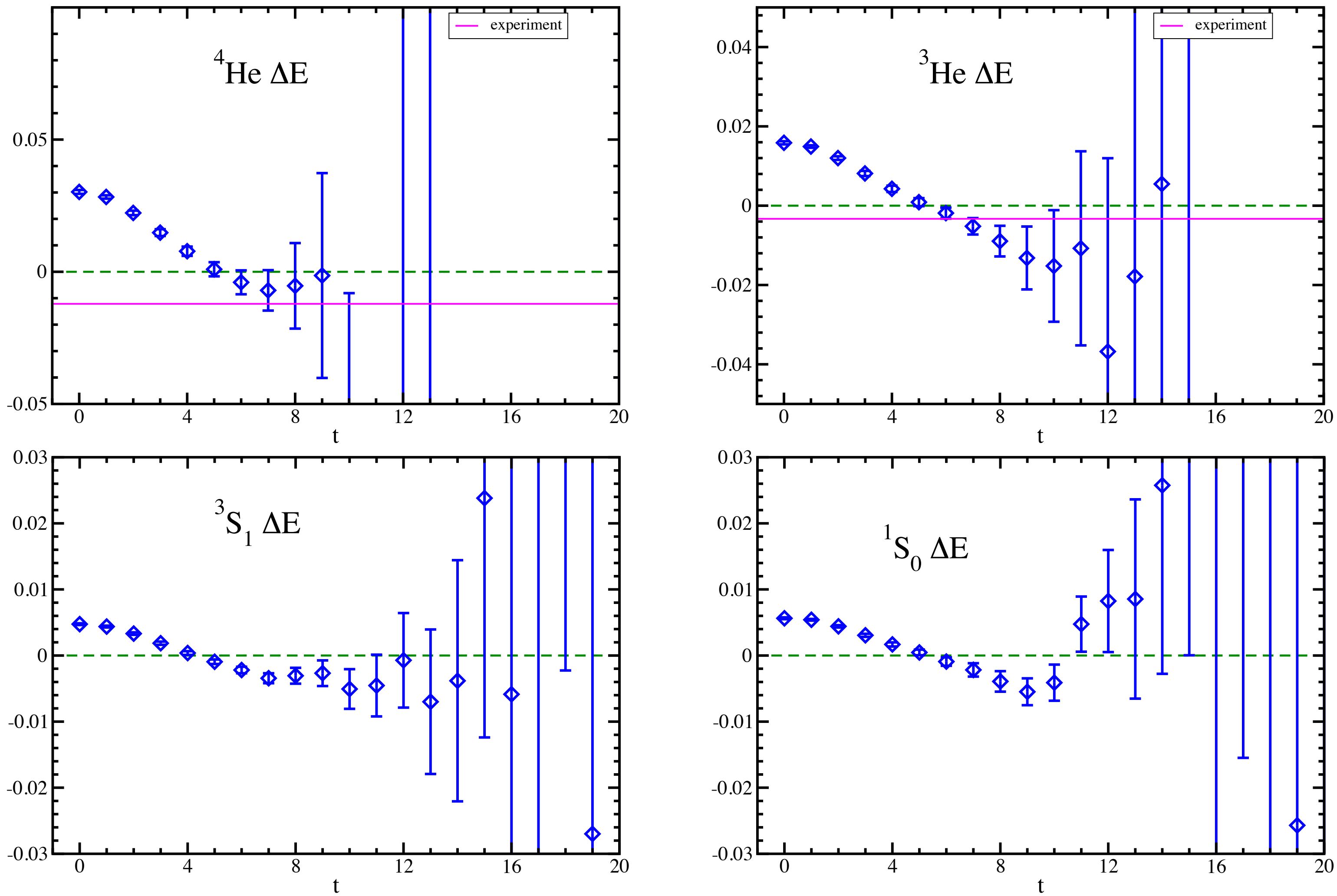
Preliminary results of effective ΔE at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm



need more statistics to obtain clear signal

Computational resources (the HPCI System Research Project: hp160124)
HA-PACS, COMA @Univ. of Tsukuba, K @AICS, FX100 @RIKEN

Preliminary results of effective ΔE at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm



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Computational resources (the HPCI System Research Project: hp160124)

HA-PACS, COMA @Univ. of Tsukuba, K @AICS, FX100 @RIKEN

Nucleon form factors at almost physical m_π

in collaboration with

K.-I. Ishikawa, Y. Kuramashi, S. Sasaki, and A. Ukawa

for PACS Collaboration

Computational resources

(the HPCI System Research Project: hp140155, hp150135, hp160125)

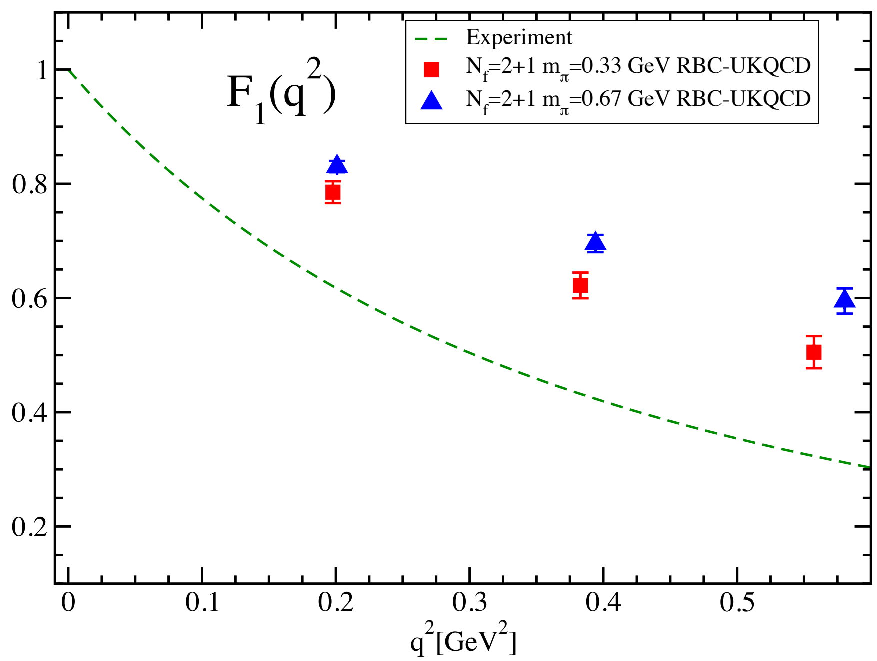
COMA @Univ. of Tsukuba, FX10 @Univ. of Tokyo,

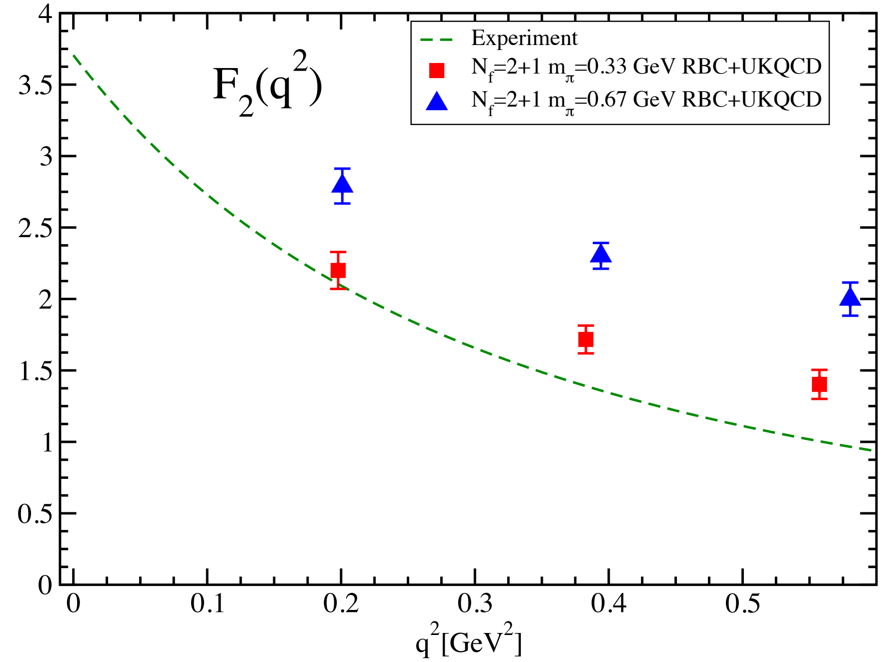
FX100 @RIKEN, System E @Kyoto Univ., FX100 @Nagoya Univ.

Isovector F_1 and F_2 form factors related to size of proton and neutron

not well understood from lattice QCD calculation

largest systematic error in previous calculations: heavier m_π





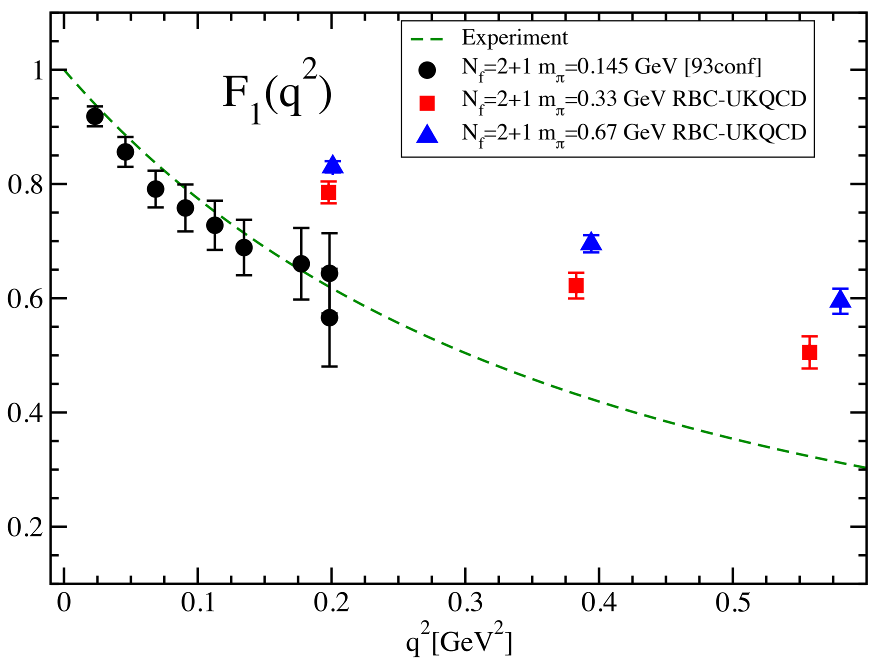
Isovector F_1 and F_2 form factors related to size of proton and neutron

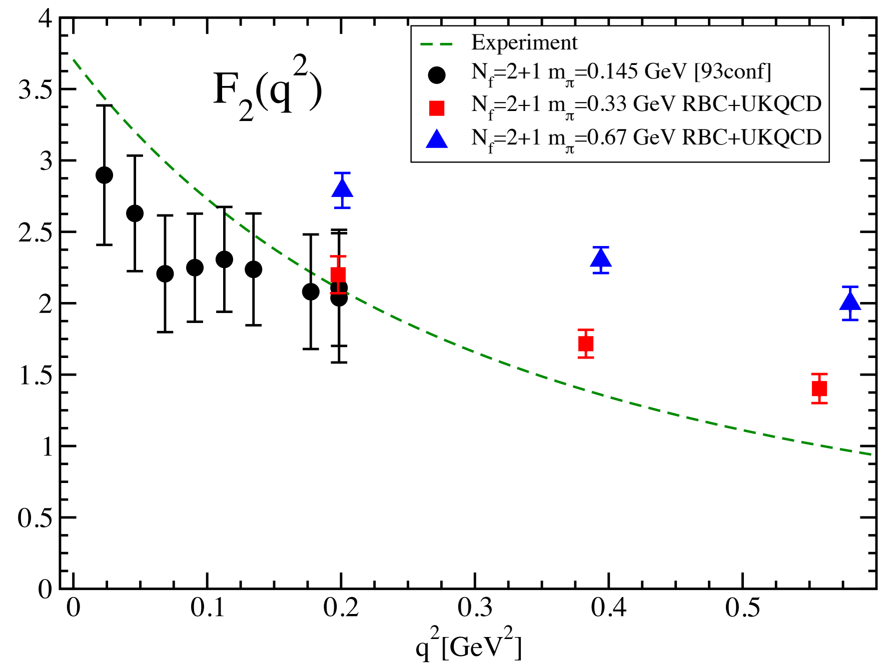
not well understood from lattice QCD calculation

largest systematic error in previous calculations: heavier m_π

$\rightarrow$ almost physical m_π calculation

Preliminary result at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm





smaller results than previous calculation at heavier m_π

roughly consistent with experiment

need much more statistics, but encouraging signal

Axial charge $g_A = Z_A g_A^{\text{bare}}$

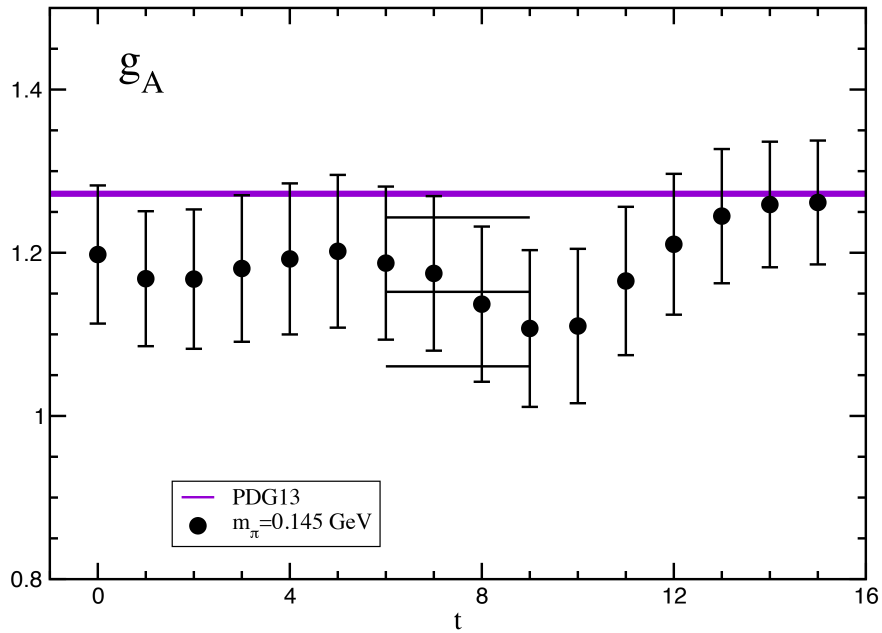
Preliminary results at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

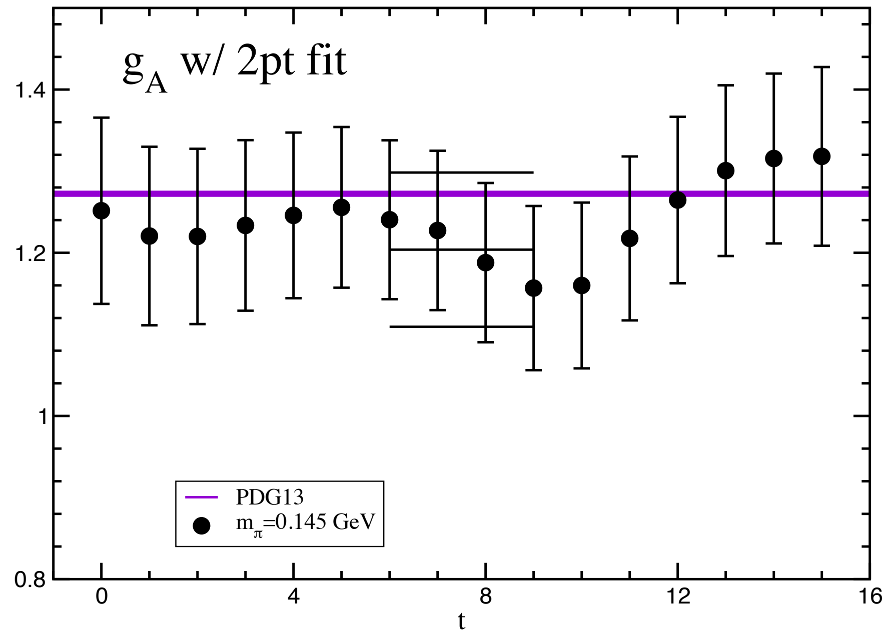
Z_A from SF scheme (Lattice 2015, Ishikawa for PACS Collaboration)

consistent with $Z_V = 1/G_E(0)$ within 1–2%

$g_A^{\text{bare}} = C_{A_3}(t)/C_N(t_{\text{sink}})$

$g_A^{\text{bare}} = C_{A_3}(t)/(Z_N^2 \exp(-M_N t_{\text{sink}}))$
Z_N and M_N from fit of $C_N(t)$





Discrepancy of two results → systematic error

roughly consistent with experiment,
but need much more statistics for stringent test

Summary

$N_f = 2+1$ lattice QCD at $m_\pi = 0.5$ and 0.3 GeV

- bound state in ^{4}He, ^{3}He, ^{3}S$_1$ and ^{1}S$_0$ at $m_\pi = 0.5$ and 0.3 GeV
- ΔE larger than experiment and small m_π dependence
- Bound state in ^{1}S$_0$ not observed in experiment

  $N_f = 3$ at $m_\pi = 0.8$ GeV by NPLQCD and CalLat
  $N_f = 2+1$ $m_\pi = 0.45$ GeV by NPLQCD
- HALQCD's claim $\leftarrow$ Wall source not reach plateau region
  more sophisticated method could give hint for better understanding

Need further investigations

e.g. systematic error from large m_π and finite lattice spacing

$N_f = 2+1$ $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

ΔE for nuclei and Isovector form factors of nucleon