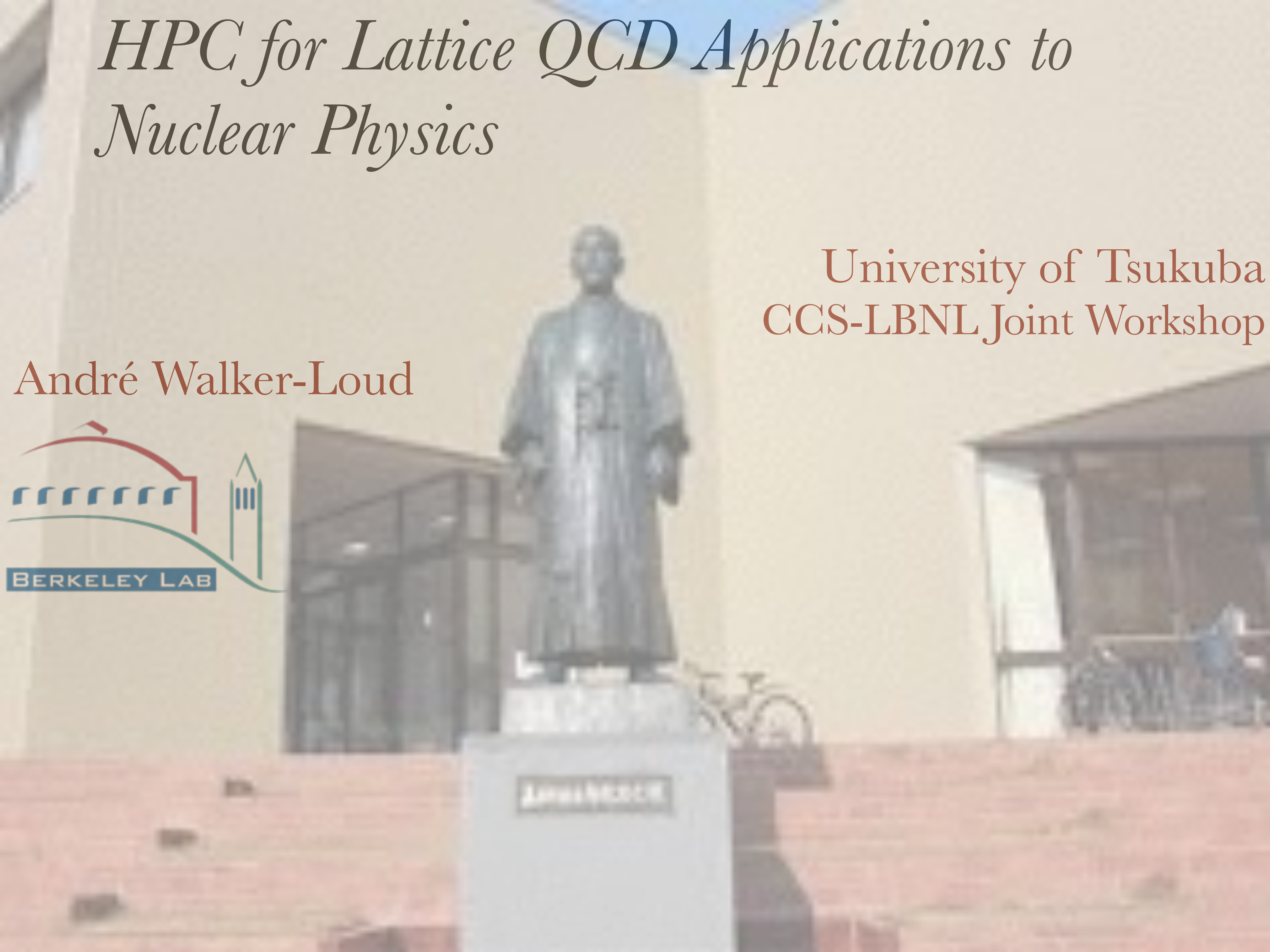

HPC for Lattice QCD Applications to Nuclear Physics

University of Tsukuba
CCS-LBNL Joint Workshop

André Walker-Loud

BERKELEY LAB

HPC for Lattice QCD Applications to NP

- Science Goals and need for HPC
- Some details of calculations & challenges
- Some progress: Big Bang Nucleosynthesis and QCD
- Possible collaborative ideas

Low-Energy Nuclear Physics

- *Understanding Nuclear Physics from QCD*
- *Testing the Standard Model at low-energy in nuclear environments*

Nuclear Physics from QCD

- *QCD is The fundamental theory of the strong interactions*

$$\mathcal{L}_{QCD} = \bar{q}_{a,\alpha,f}(x)\left[D_\mu\gamma_\mu + m\right]^{b,\beta,f'}_{a,\alpha,f} q_{b,\beta,f'}(x) - \frac{1}{4}G_{\mu\nu}G_{\mu\nu}$$

$q_{b,\beta,f'}(x)$ Quark of *color* b, *spin* β, *flavor* f

flavors = up (u), strange (s), top (t)
down (d), charm (c), bottom (b)

colors = red, green, blue

quarks transform under the fundamental representation of SU(3) color (unitary 3x3)

spin = 4 spin states, 2 particle, 2 anti-particle

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For Nuclear Physics: flavors = up (u), down (d), (strange (s))

$$M_p = 938.272046 \text{ MeV} \qquad M_n = 939.565379 \text{ MeV}$$

$$M_n - M_p = 1.29333217(42) \text{ MeV}$$

$$M_e = 0.511 \text{ MeV}$$

Nuclear Physics from QCD

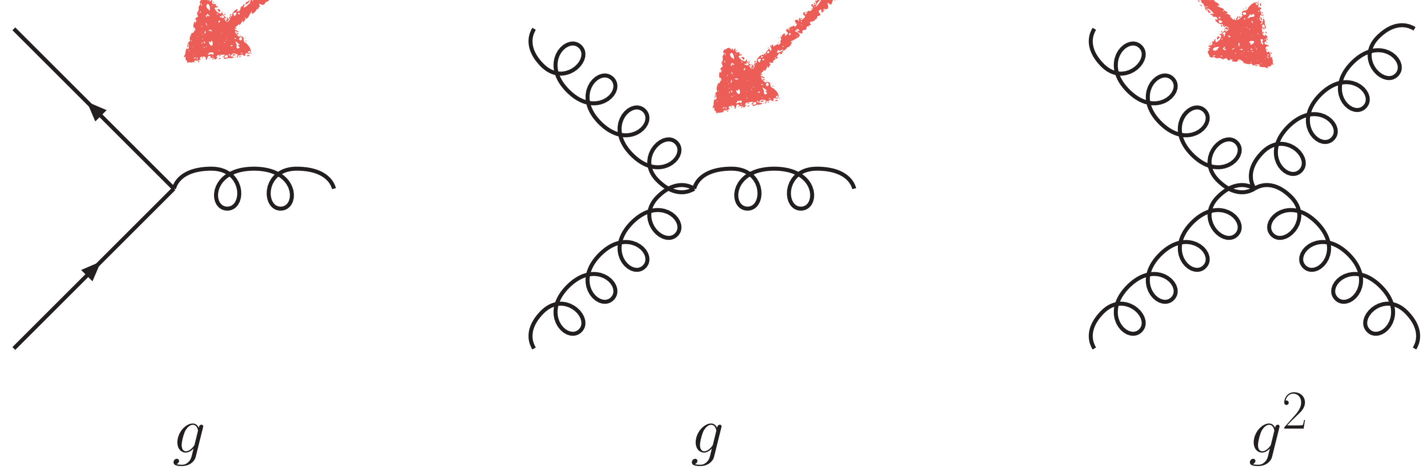
- *QCD is The fundamental theory of the strong interactions*

$$\mathcal{L}_{QCD} = \bar{q}_{a,\alpha,f}(x)\,[D_\mu\gamma_\mu + m]_{a,\alpha,f}^{b,\beta,f'}\,q_{b,\beta,f'}(x) - \frac{1}{4}G_{\mu\nu}G_{\mu\nu}$$

$$[D_\mu]_a^b q_b(x) = \delta_{a,b}\partial_\mu q_b(x) + ig[A_\mu]_a^b q_b(x)$$

gluons adjoint rep. of SU(3) color - 8 gluons

$$\frac{1}{4}G_{\mu\nu}G_{\mu\nu}$$



Nuclear Physics from QCD

- *QCD is The fundamental theory of the strong interactions*

$$\mathcal{L}_{QCD} = \bar{q}_{a,\alpha,f}(x)\left[D_\mu\gamma_\mu + m\right]_{a,\alpha,f}^{b,\beta,f'} q_{b,\beta,f'}(x) - \frac{1}{4}G_{\mu\nu}G_{\mu\nu}$$

QCD is a remarkably simple theory to write down. At low energies (will define) QCD is a theory of only 3 or 4 parameters:

m_u	mass of the up quark (dimensionfull)
m_d	mass of the down quark (dimensionfull)
(m_s)	mass of the strange quark (dimensionfull)
g	*gauge* coupling between quarks and gluons (dimension-less)

Once these parameters are fixed - everything else is a prediction! - proton mass, He binding energy, neutron star equation of state (maximum neutron star mass), …

Lattice QCD

low energy → high energy

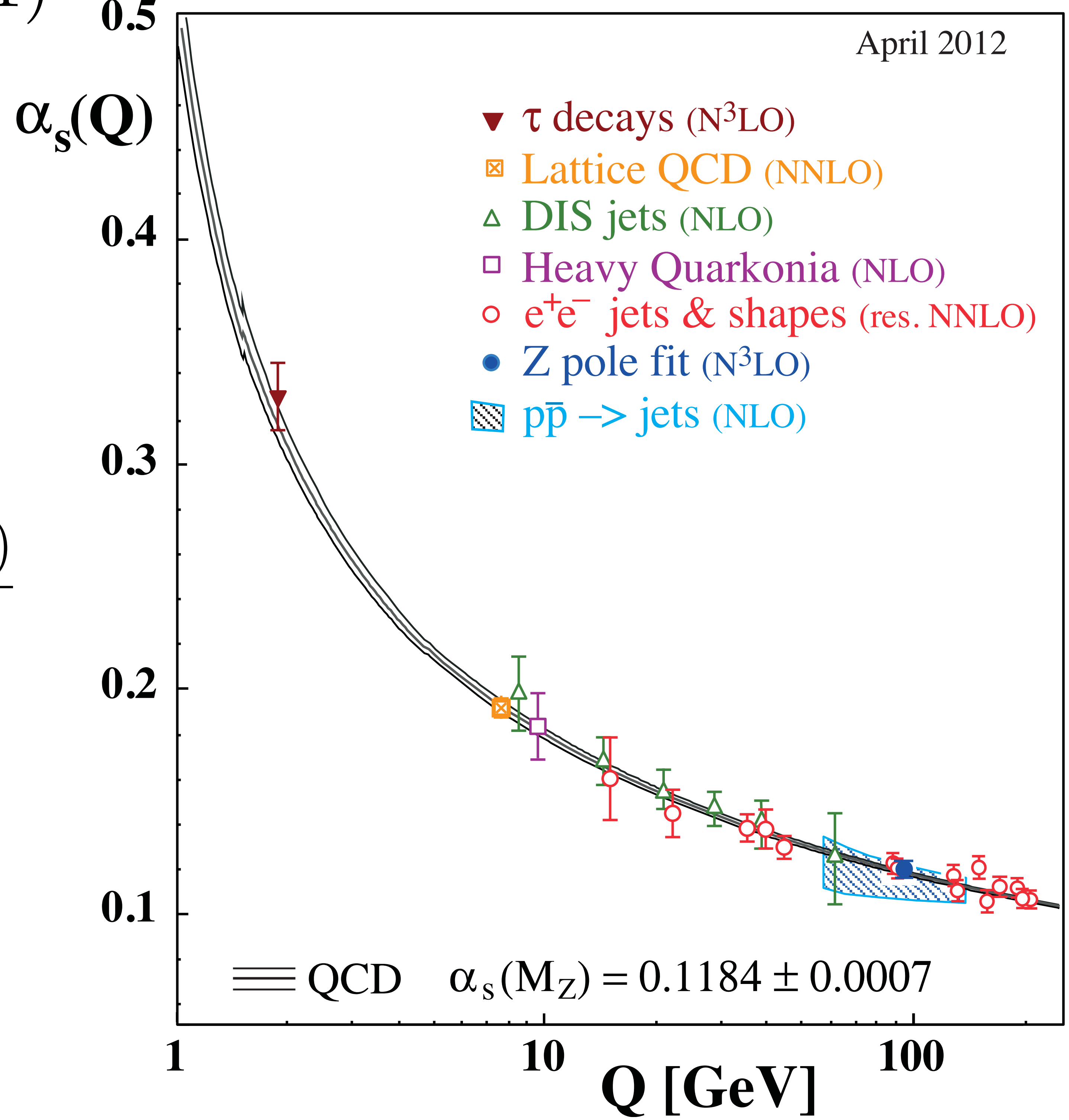
$g \sim \mathcal{O}(1)$

Strong Coupling

$g \to 0$

Asymptotic Freedom

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$



2004 Nobel Prize
David Gross
David Politzer
Frank Wilczek

Lattice QCD

low energy → high energy

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Strong Coupling

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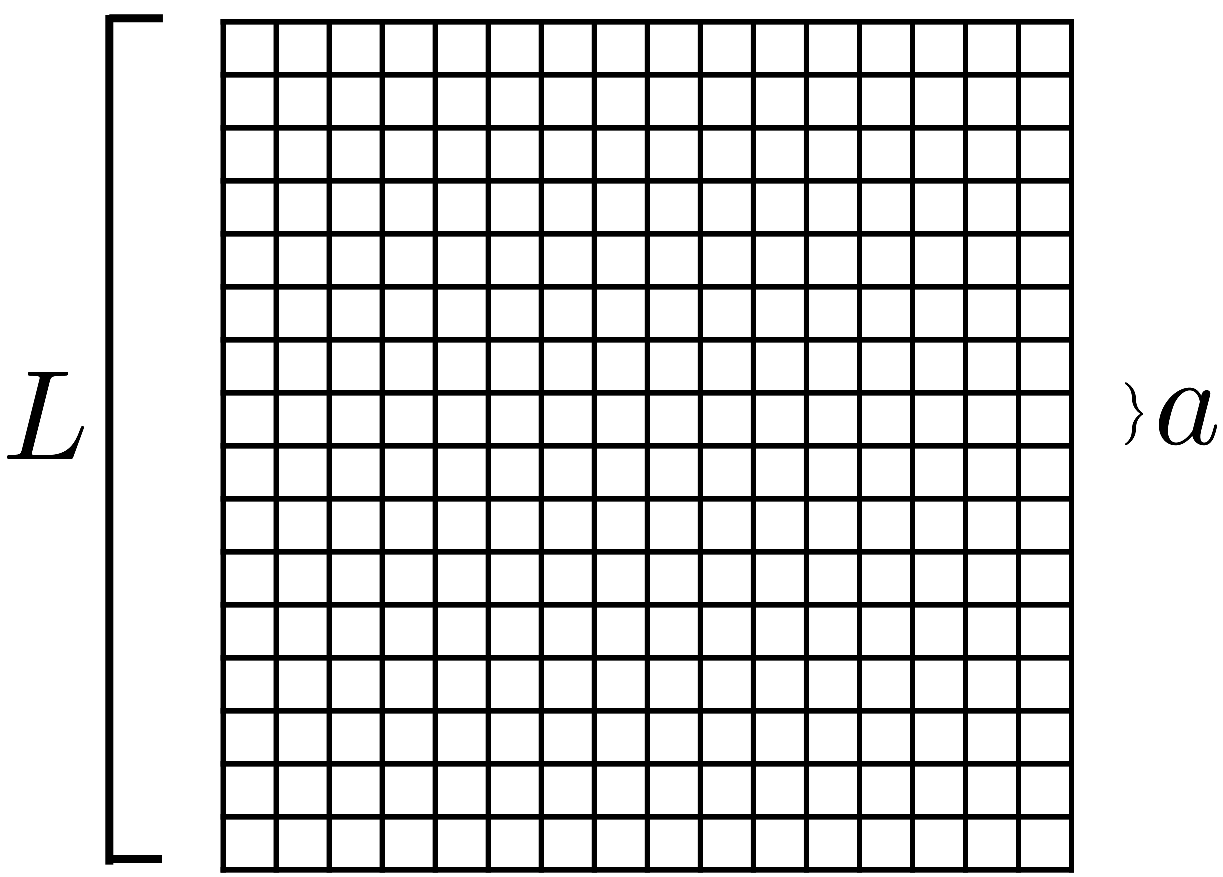
Asymptotic Freedom



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

small distance
=
large energy

Asymptotic Freedom
Feynman Path Integrals
Wilson Lattice Field Theory
Monte Carlo methods

allows numerical solution

Nuclear Physics from QCD

- $$\mathcal{L}_{QCD} = \bar{q}_{a,\alpha,f}(x)\left[D_\mu\gamma_\mu + m\right]_{a,\alpha,f}^{b,\beta,f'} q_{b,\beta,f'}(x) - \frac{1}{4}G_{\mu\nu}G_{\mu\nu}$$

$$E^{(i)}_{N,Z,S} = \Lambda_{QCD} \times f^{(i)}_{N,Z,S}\left(\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}}, \frac{m_s}{\Lambda_{QCD}}, \alpha_{f.s.}, \Lambda^2_{QCD}G_F\right)$$

these energy levels range from a few KeV to MeV to many GeV (eV = atomic energy scales)

Λ_{QCD} QCD dynamics generates a mass scale (even if all quark masses were 0) $\Lambda_{QCD} \sim 250$ MeV

to normalize this: proton mass = 938 MeV

$\alpha_{f.s.}$ electromagnetic coupling constant: how strong does photon couple to quarks

G_F Fermi's weak coupling

Nuclear Physics from QCD

- $$\mathcal{L}_{QCD} = \bar{q}_{a,\alpha,f}(x)\left[D_\mu\gamma_\mu + m\right]^{b,\beta,f'}_{a,\alpha,f} q_{b,\beta,f'}(x) - \frac{1}{4}G_{\mu\nu}G_{\mu\nu}$$

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these energy levels range from a few KeV to MeV to many GeV (eV = atomic energy scales)

Λ_{QCD}	QCD binds quarks/gluons into protons and neutrons, it binds protons and neutrons into nuclei - strong nuclear force - fusion
$\alpha_{f.s.}$	E&M + QCD responsible for fission
G_F	*Weak* decay: $n \rightarrow p$ $\qquad \tau_n = 881.5s$

Nuclear Physics from QCD

- *There are well known fine-tunings in nature that have a significant impact on our existence*

  $M_n - M_p, \quad B_d, \quad \text{triple alpha process and } ^{12}C, \quad \cdots$

How sensitive are these fine-tunings to variations of fundamental parameters in the Standard Model?

How sensitive is the Universe as we know it to variations in these fundamental parameters?

→ *need a solution to QCD*

Nuclear Physics from QCD

- *What is the weak fusion rate*

$$p + p \rightarrow d + \nu_e + e^+$$

  as a function of parameters in the Standard Model?

- *What is the composition and equation of state of dense nuclear matter in neutron stars?*

- $\ldots$

Testing the Standard Model at low-energy

- *With the discovery of the Higgs boson, the Standard Model (SM) is now complete*
- *However, the LHC has turned up only one hint of any physics beyond the Standard Model (BSM)*
- *Further, there is almost NO terrestrial experimental hints for any physics BSM*

the exceptions: muon anomalous magnetic moment
proton radius puzzle

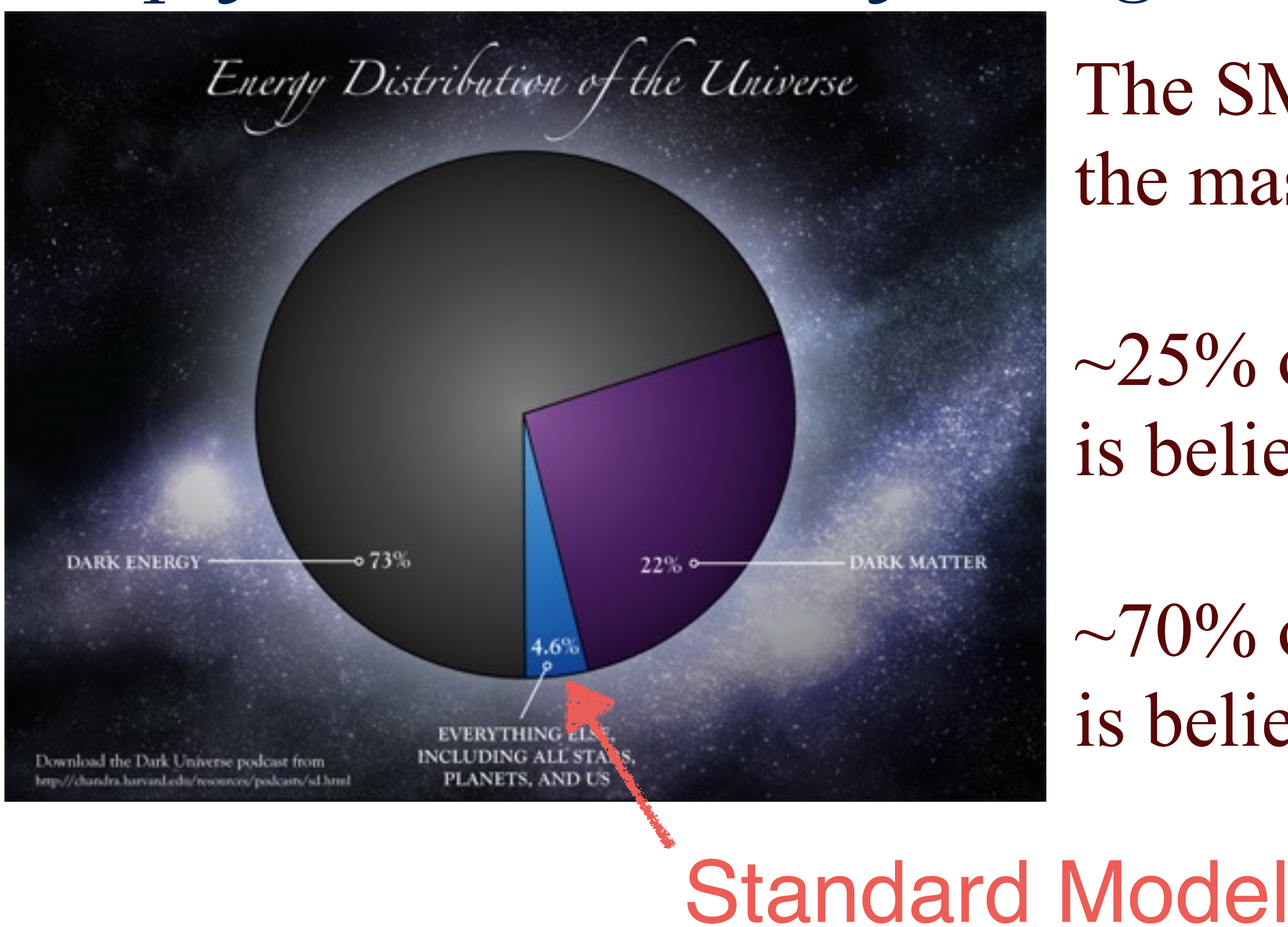
Testing the Standard Model at low-energy

- high-energy physics colliders are one way to search for BSM physics - but it is not clear this will be possible in the near future

  this helps emphasize the important role low-energy precision nuclear physics can play in searching for new physics (in addition to muon g-2 and proton size)

What are important roles for QCD, nuclear physics and current experimental efforts?

- *While we have no direct confirmation of any BSM physics - we have very strong indirect evidence:*



The SM describes only ~5% of the mass of the Universe

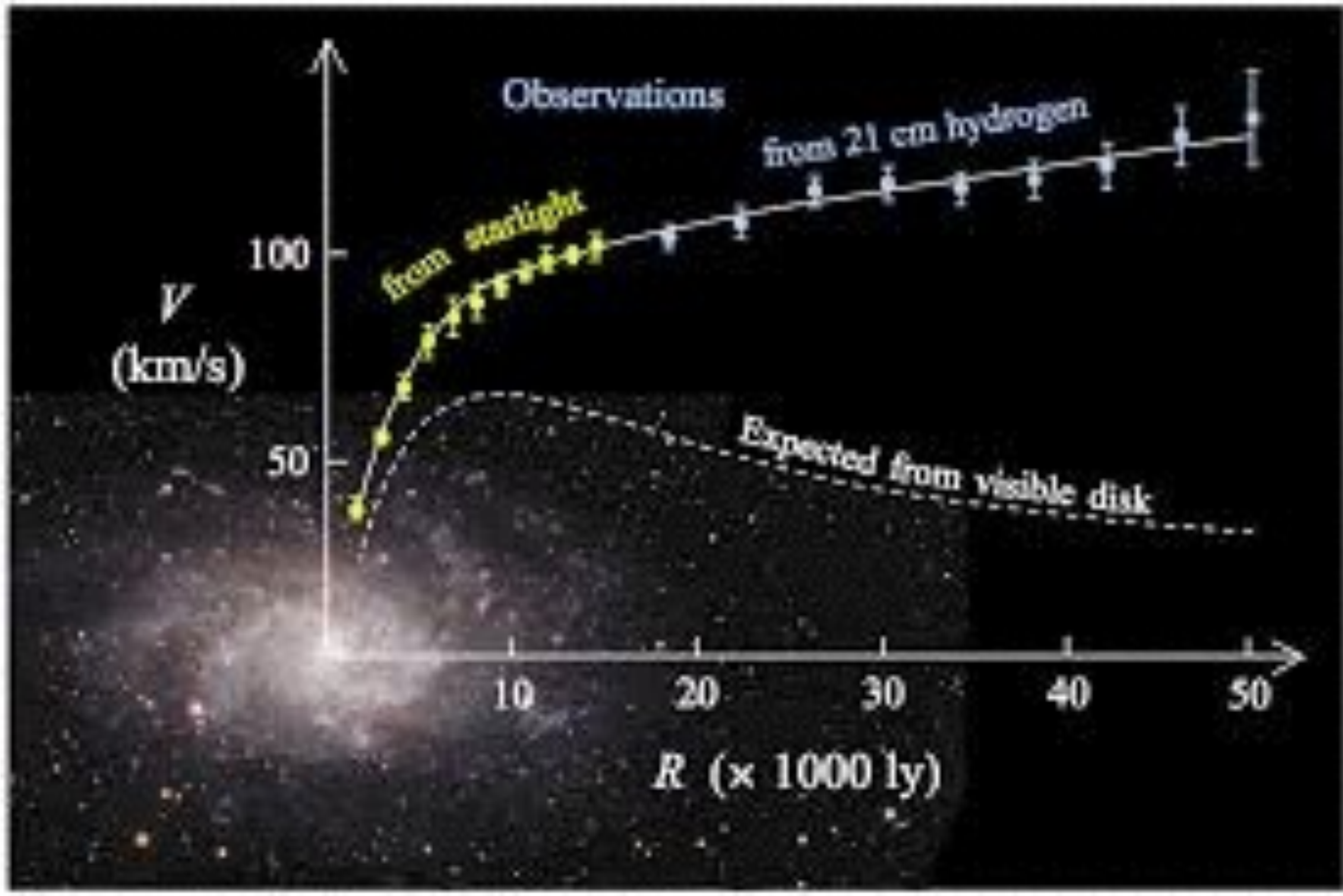
~25% of the mass of the Universe is believed to be Dark Matter

~70% of the mass of the Universe is believed to be Dark Energy

What are important roles for QCD, nuclear physics and current experimental efforts?

- *The assumed existence of Dark Matter (DM) comes from several sources:*

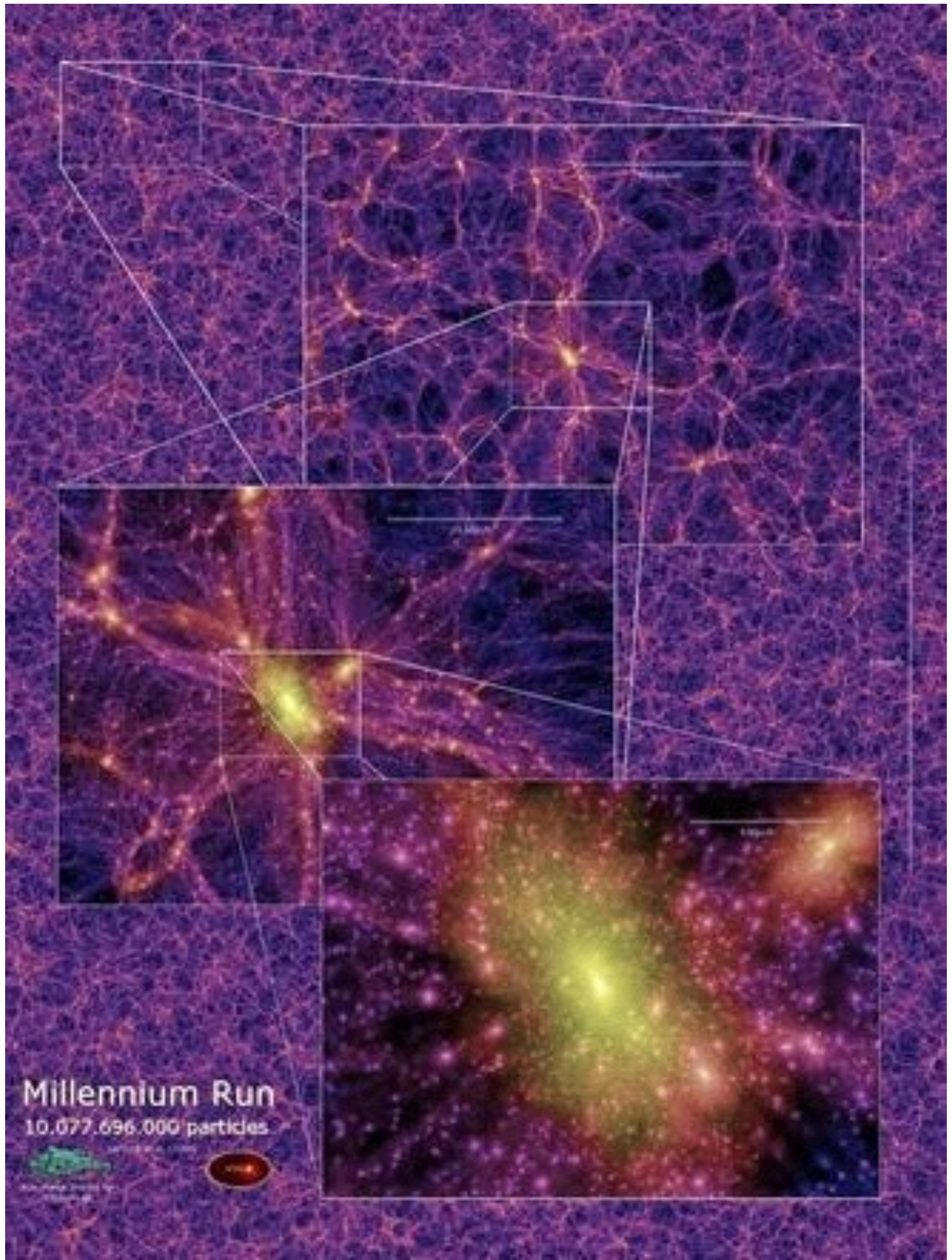
  Velocity curves of rotational galaxies require significantly more gravitating mass than observed



What are important roles for QCD, nuclear physics and current experimental efforts?

- The assumed existence of Dark Matter (DM) comes from several sources:

  N-body simulations of galaxy formation (assuming cold-dark matter) DM gives rise to observed structure of galaxies and galaxy clusters (simulations without DM do not)



What are important roles for QCD, nuclear physics and current experimental efforts?

- The assumed existence of Dark Matter (DM) comes from several sources:

  Bullet-Cluster: two colliding galaxies
  gravitational lensing shows COM moved right through collision and is not observable while visible matter "collided"

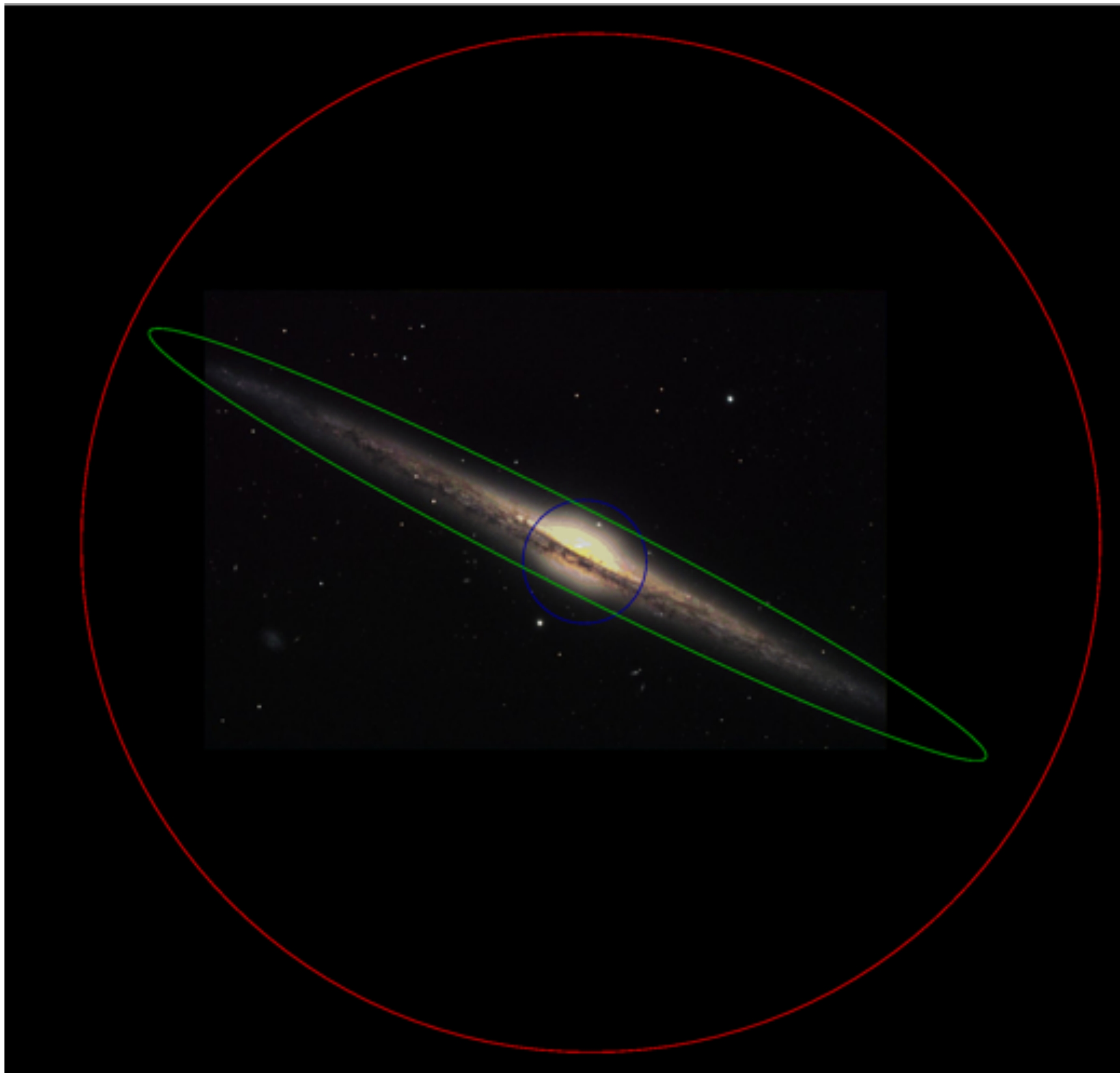
What are important roles for QCD, nuclear physics and current experimental efforts?

- What do we know about Dark Matter?

DM interacts very weakly, if at all, with the SM except through gravity

DM is weakly self-interacting: DM exists in halos rather than disks (matter accumulates to a disk through collisions)

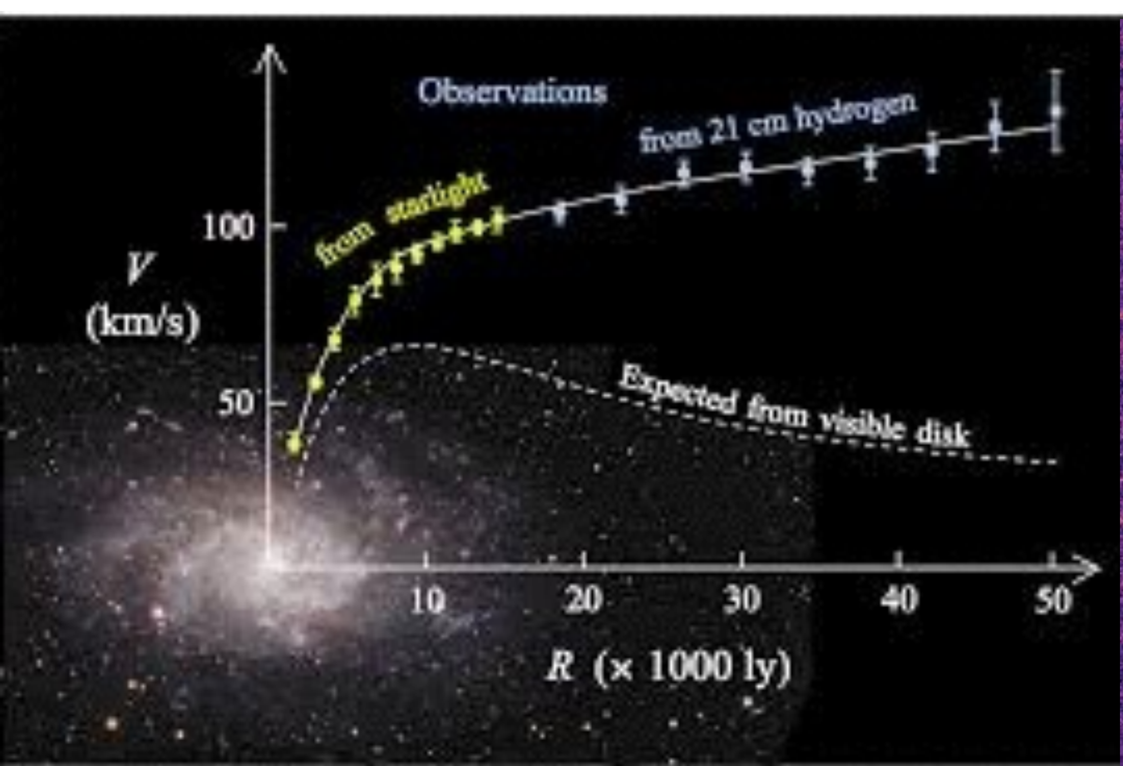
DM is cold (non-relativistic) since it clumps

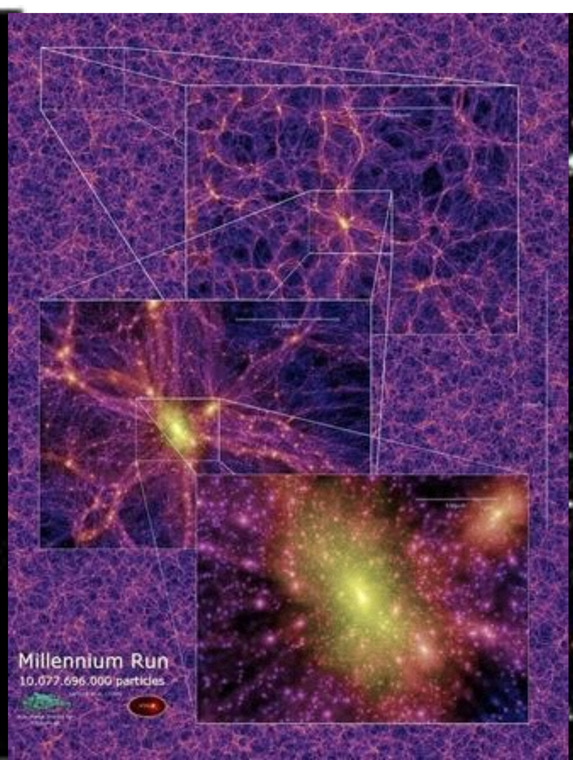


What are important roles for QCD, nuclear physics and current experimental efforts?

- *The assumed existence of Dark Matter (DM) comes from several sources:*

  These three observations, in particular the bullet cluster, are very difficult to explain with modified gravity. Cold Dark Matter is the simplest explanation consistent with all observations



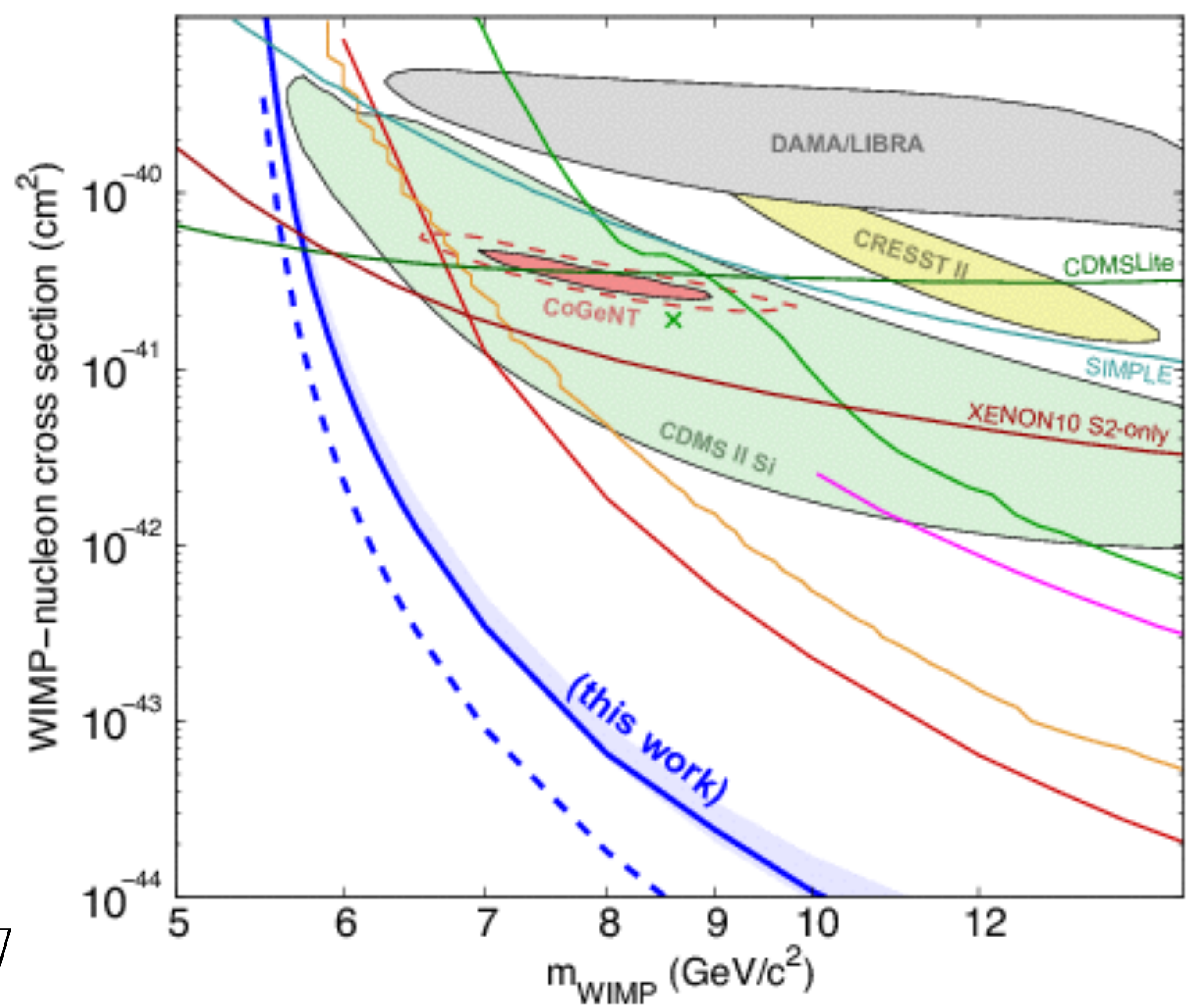


What are important roles for QCD, nuclear physics and current experimental efforts?

- There are several significant experimental efforts underway to try and directly detect Dark Matter - through elastic collisions with matter

These detectors all use nuclei (e.g. Xenon-130) to search for elastic recoil

to interpret constraints/ observations we must understand QCD and possible interactions with DM!



LUX exclusion plot [arXiv:1405.5906]

Testing the Standard Model at low-energy

- To the best of our knowledge, the SM matter in the Universe is comprised entirely of matter and not anti-matter

A measure of the excess matter in the Universe is given by the primordial ratio of the number of baryons to photons - from the CMB, we know this number to be

$$\eta \equiv \frac{X_N}{X_\gamma} \simeq 6.2 \times 10^{-10}$$

However, the SM is nearly symmetric in matter and anti-matter. While this observed asymmetry is small, it is orders of magnitude larger than allowable by the SM

Testing the Standard Model at low-energy

- To produce a matter/anti-matter asymmetry, we need the three Sakharov conditions:
  - baryon number violation
  - C-symmetry and CP-symmetry violation
  - interactions out of thermal equilibrium

C = charge conjugation: $e^- \leftrightarrow e^+$ particle to anti-particle symmetry

P = Parity: mirror symmetry (physics in the mirror should be the same)

T = Time-reversal symmetry: laws of physics going forward and backwards in time is the same

These discrete symmetries are almost perfectly respected by the SM with small violations.

Quantum Mechanics + special relativity → C*P*T conserved

Testing the Standard Model at low-energy

- To produce a matter/anti-matter asymmetry, we need the three Sakharov conditions:
  - baryon number violation
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  - interactions out of thermal equilibrium

CP violation implies T violation which implies there will be permanent electric dipole moments (EDMs) for SM fermions. There are significant experimental efforts to search for permanent electric dipole moments in electrons, protons, neutrons, deuterium, … ^{199}Hg, ^{225}Ra , …

In order to relate constraints/measurements on permanent EDMs in nucleons/nuclei to BSM physics,

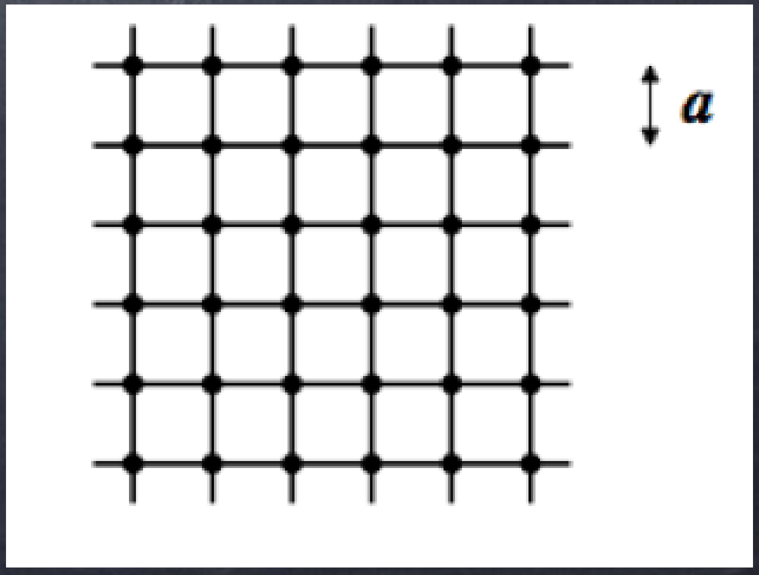
we must be able to solve QCD!

Feynman Path Integrals

$$\mathcal{Z} = \int DA_\mu D\psi D\overline{\psi} e^{iS_{QCD}} \qquad S_{QCD} = \int d^4x \mathcal{L}_{QCD}$$

$$\langle\Omega|\hat{\mathcal{O}}(y)\hat{\mathcal{O}}^\dagger(x)|\Omega\rangle = \frac{1}{\mathcal{Z}} \int DA_\mu D\psi D\overline{\psi} e^{iS_{QCD}} \mathcal{O}(y)\mathcal{O}^\dagger(x)$$

- The path-integral gives us a relation between matrix elements of operators and a high dimensional integral over field configurations.
- We know how to do the integral on the right (in principle at least). The beginning of lattice QFT is to discretize the universe so that we can compute the path-integral representation directly with a computer.
- Suppose we chop the universe into size $32 \times 32 \times 32 \times 64 = 2^{21}$
- our path integral goes over all field configurations on all sites, $n^{2^{21}}$ terms!



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- How can we actually perform this integral?
- If we Wick-rotate to Euclidean time, $t \to it_E$, then we have

$$\langle\Omega|\hat{\mathcal{O}}(y_E)\hat{\mathcal{O}}^\dagger(x_E)|\Omega\rangle = \frac{1}{\mathcal{Z}} \int DA_\mu D\psi D\overline{\psi} e^{-S^E_{QCD}} \mathcal{O}(y_E)\mathcal{O}^\dagger(x_E)$$

- For zero quark chemical-potential (zero baryon chemical potential)

$$e^{-S^E_{QCD}} \in \mathbb{R}$$

- We can use this factor as a probability measure to importance sample the integral with Monte-Carlo methods for those field configurations that minimize S^E_{QCD}

Feynman Path Integrals

$$\langle \Omega | \hat{\mathcal{O}}(y_E) \hat{\mathcal{O}}^\dagger(x_E) | \Omega \rangle = \frac{1}{\mathcal{Z}} \int DA_\mu D\psi D\overline{\psi} e^{-S^E_{QCD}} \mathcal{O}(y_E) \mathcal{O}^\dagger(x_E)$$

- We can make N_{cfg} different samples of the field configurations and then our correlation functions are approximated with finite statistics

$$\langle \Omega | \hat{\mathcal{O}}(y_E) \hat{\mathcal{O}}^\dagger(x_E) | \Omega \rangle = \lim_{N_{cfg} \to \infty} \frac{1}{N_{cfg}} \sum_{i=1}^{N_{cfg}} \langle \Omega | \hat{\mathcal{O}}(y_E) [A^i_\mu, \psi_i, \overline{\psi}_i] \hat{\mathcal{O}}^\dagger(x_E) [A^i_\mu, \psi_i, \overline{\psi}_i] | \Omega \rangle$$

$[A^i_\mu, \psi_i, \overline{\psi}_i]$ = the i^{th} value of the fields on "configuration" i

- At finite statistics (finite N_{cfg}) we will have an approximation to the correlation functions with some computable statistical uncertainty that can be systematically improved (with more computing time)

Feynman Path Integrals

$$\langle\Omega|\hat{\mathcal{O}}(y_E)\hat{\mathcal{O}}^\dagger(x_E)|\Omega\rangle = \frac{1}{\mathcal{Z}}\int DA_\mu D\psi D\overline{\psi} e^{-S^E_{QCD}}\mathcal{O}(y_E)\mathcal{O}^\dagger(x_E)$$

- What do we expect our Euclidean spacetime correlation functions to look like? Let us take x_E=0 (without loss of generality - translation invariance lets us do this) and $\vec{y}_E = 0$ for simplicity

$$C(t) = \langle\Omega|\hat{\mathcal{O}}(t,\vec{0})\hat{\mathcal{O}}^\dagger(0,\vec{0})|\Omega\rangle$$

- Insert a complete set of states

$$1 = \sum_n |n\rangle\langle n|$$

Lattice QCD results are given by a sum of noisy exponentials - a challenging numerical analysis problem

$$C(t) = \sum_n \langle\Omega|\hat{\mathcal{O}}(t)|n\rangle\langle n|\hat{\mathcal{O}}^\dagger(0)|\Omega\rangle$$

$$= \sum_n \langle\Omega|e^{\hat{H}t}\hat{\mathcal{O}}(0)e^{-\hat{H}t}|n\rangle\langle n|\hat{\mathcal{O}}^\dagger(0)|\Omega\rangle$$

$$= \sum_n Z_n Z_n^\dagger e^{-E_n t}$$

$$Z_n = \langle\Omega|\hat{\mathcal{O}}(0)|n\rangle$$

Integrating the fermion fields

$$\mathcal{Z}_F = \int \mathcal{D}[\psi]\,\mathcal{D}[\overline{\psi}]\,\exp\{-S_W[U,\overline{\psi},\psi]\} \qquad S_W = a^4 \sum_x \overline{\psi}(x)\,[D_W + M]\,\psi(x)$$

This integral is quadratic in the quarks, so we can directly do the integral. We do this because the quarks are Grassmann numbers (anti-commuting)

$$= \det[D_W + M] = \prod_{q=1}^{N_f} \det[D_W + m_q]$$

$$\mathcal{Z}_{QCD} = \int DU_\mu \det[D_W + M]\, e^{-S_G[U_\mu]} \qquad U_\mu = e^{iagA_\mu}$$

How do we evaluate the determinant?

Integrating the fermion fields

How do we evaluate the determinant?

$$\mathcal{Z}_{QCD} = \int DU_\mu \det\left[D_W + M\right] e^{-S_G[U_\mu]}$$

$$\begin{aligned}
\mathcal{Z}_\psi &= \int D[\psi]D[\overline{\psi}]e^{-\int d^4x\overline{\psi}[D_W+M]\psi} \\
&= \det[D_W + M] \\
&= \frac{1}{\det[[D_W + M]^{-1}]} \\
&= \int D[\phi]D[\phi^*]e^{-\int d^4x\phi^*[D_W+M]^{-1}\phi}
\end{aligned}$$

Gaussian integral we can do exactly - for Grassmann variables, we get a determinant. For regular variables, we get the inverse determinant. Introduce pseudo-fermions

We need to invert a large, sparse matrix with small eigenvalues $[D_W + M]^{-1}$

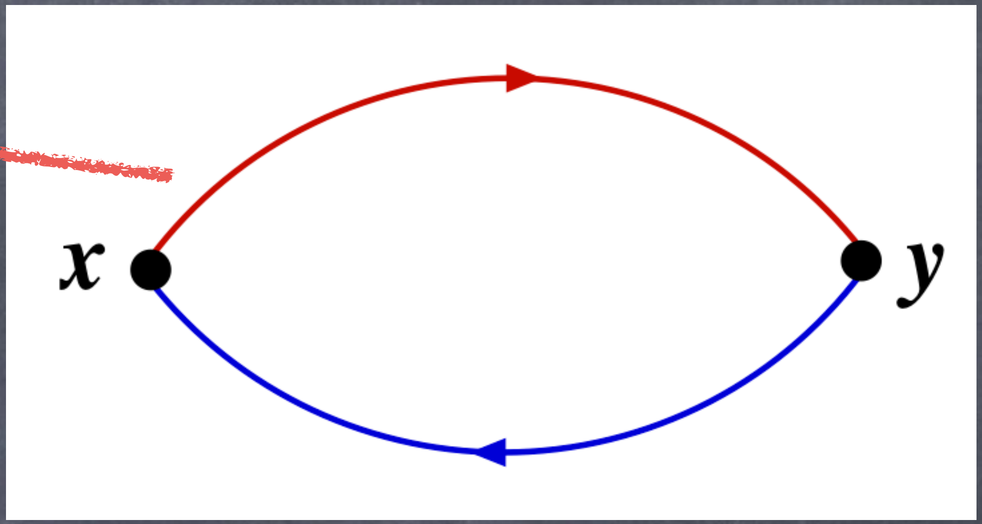
this takes ~90% of CPU cycles for generating ensemble of configurations

Quark contractions: Making protons, pions, ...

$$[D_W + M]\, S(x,y;U) = \frac{1}{a^4}\delta_{xy}$$

Quark propagator

Pion correlation function



To solve for the quark propagator, S, we must invert a large sparse matrix

$$[D_W + M]^{-1}$$

Then - we Wick-contract the quarks together to make states of interest: e.g. the pion

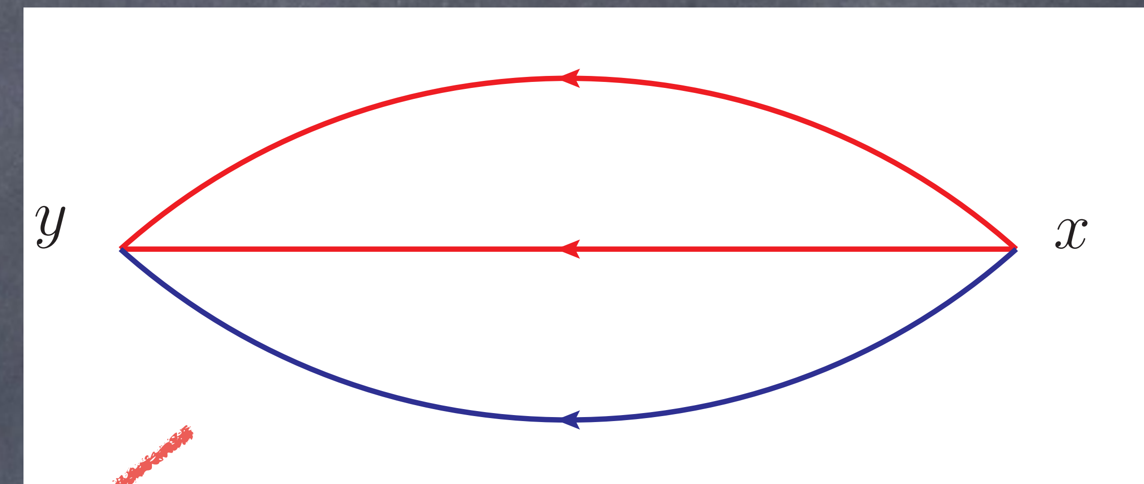
$$\left\langle \overline{u}(x)\gamma_5 d(x)\overline{d}(y)\gamma_5 u(y)\right\rangle_F = -\mathrm{tr}\left\{\gamma_5 S_{dd}(x,y;U)\gamma_5 S_{uu}(y,x;U)\right\}$$

Quark contractions: Making protons, pions, ...

$$[D_W + M]\, S(x,y;U) = \frac{1}{a^4}\delta_{xy}$$

A proton has 2 u-quarks and 1 d-quark. The contractions are slightly more complex - we need to keep track of which u-quark from x goes to which u-quark at y - 2 contractions (Nu! * Nd!)

Proton correlation function



2 protons (proton-proton scattering) has 4! * 2! = 48 contractions
He3 (ppn) has 5! * 4! = 2880 contractions
He4 (ppnn) has 6! * 6! = 518400 contractions!
...

Symmetries can be used to largely reduce this growth
Yamazaki, Kuramashi, Ukawa - Phys.Rev. D81 (2010)
But this Wick-contraction cost can be dominant for multi-nucleon lattice QCD calculations

Signal-to-Noise

Calculations involving nucleons suffer a severe signal-to-noise problem

$$Signal = Ze^{-m_N t}\left[1+\delta Z_n e^{-(E_n-m_N)t}\right]$$

Signal for a proton correlation function

$$\frac{Signal}{Noise} \sim \sqrt{N_{sample}}\; e^{-(m_N-\frac{3}{2}m_\pi)t}$$

Signal-to-noise for a proton correlation function

$$m_N \simeq 939\ \mathrm{MeV}$$
$$m_\pi \simeq 135\ \mathrm{MeV}$$

$$\frac{Signal}{Noise} \sim \sqrt{N_{sample}}\; e^{-A(m_N-\frac{3}{2}m_\pi)t}$$

Signal-to-noise for A nucleons

Solving this problem requires solutions at early Euclidean-time before the Noise becomes large - but this requires sophisticated "wave-functions" for the proton, which compounds the Wick-contraction cost mentioned above

Why exascale?

QCD is a 4-Dimensional theory. Our current best algorithms scale as

$$t_{MC} \sim \frac{1}{a^6}$$

We require sufficiently small discretization scales, a, such that a continuum limit can be taken. Also, 3 or more discretization scales are needed for a controlled extrapolation

$$L/a = \{64, 128, 256, ...\}$$

The previously mentioned signal-to-noise problem requires millions of stochastically independent samples

With current strategies and algorithms, we can easily occupy an entire exascale-year

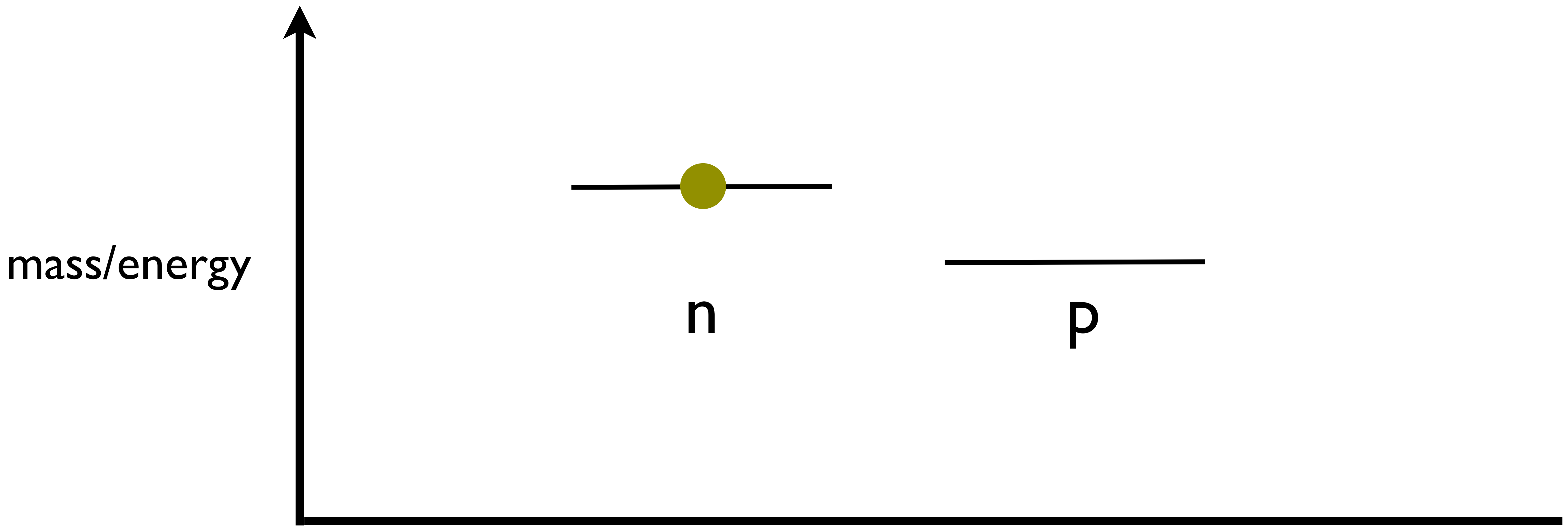
Role of QCD in the Evolution of the Universe

Big Bang Nucleosynthesis

$$T \simeq 1 \text{ trillion K} \rightarrow 1 \text{ billion K}$$

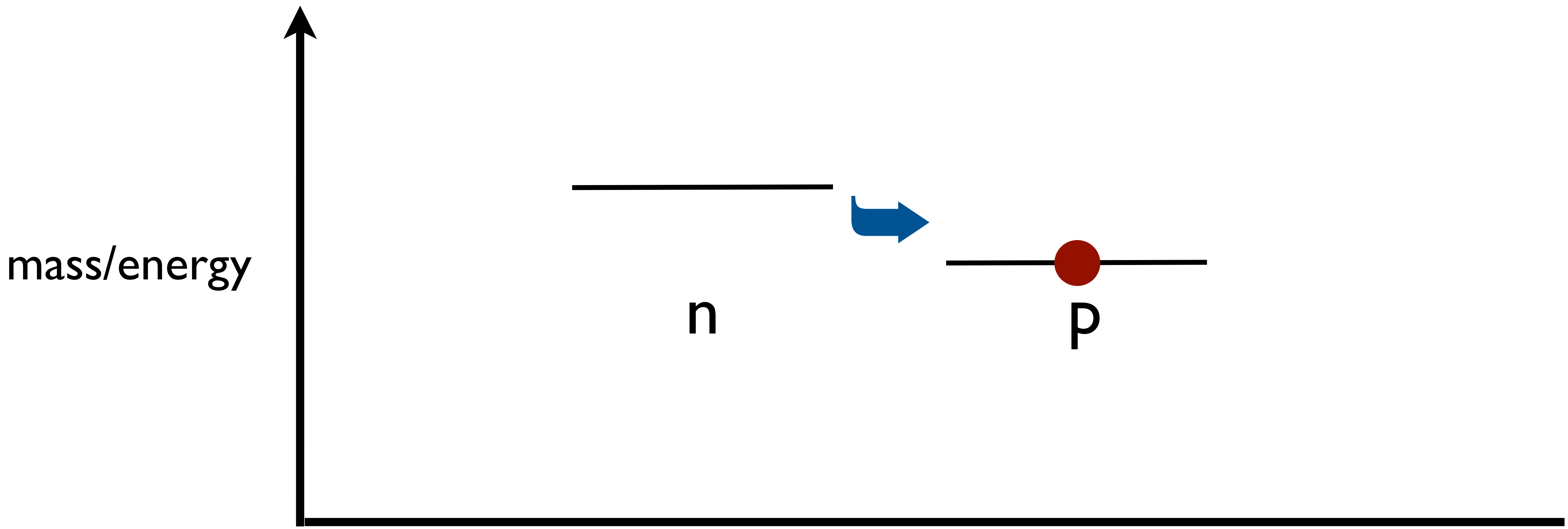
$$t \simeq 3 \times 10^{-5} s \rightarrow 3 \text{ min}$$

when systems cool, they settle into the lowest energy state



$$\tau_n \sim 15 \text{ min}$$

when systems cool, they settle into the lowest energy state

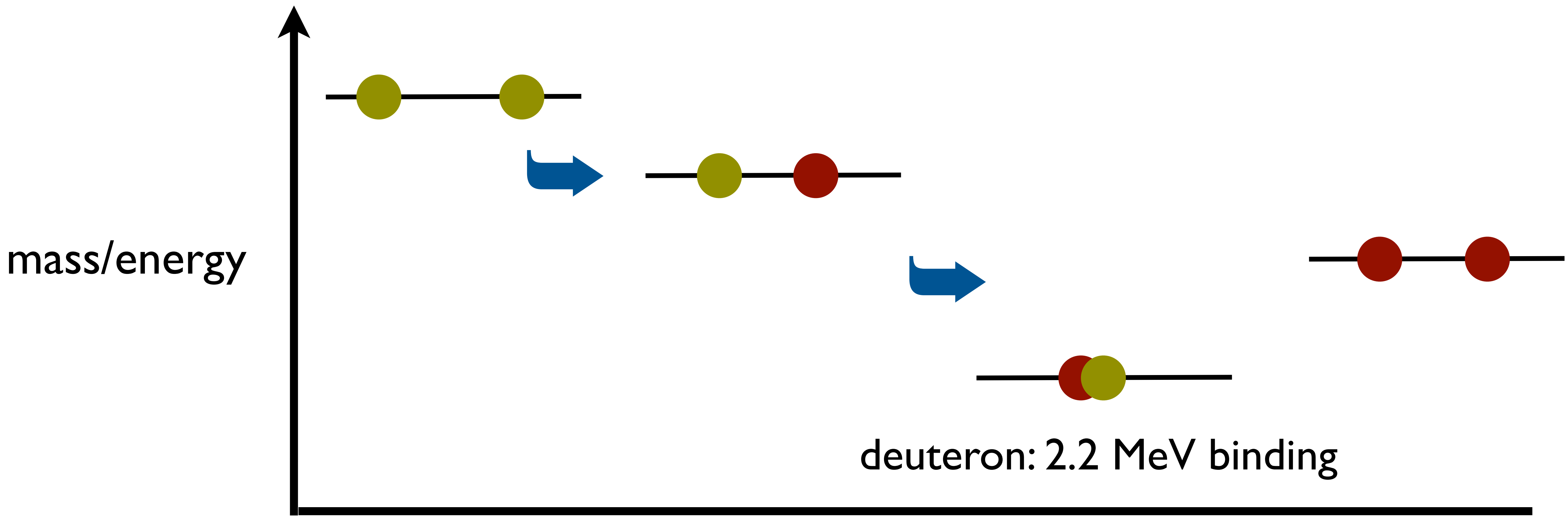


$$\tau_n \sim 15 \text{ min}$$

what prevented this from destroying all the neutrons?

if nothing else were to happen in the next few minutes, our universe would be full of only Hydrogen

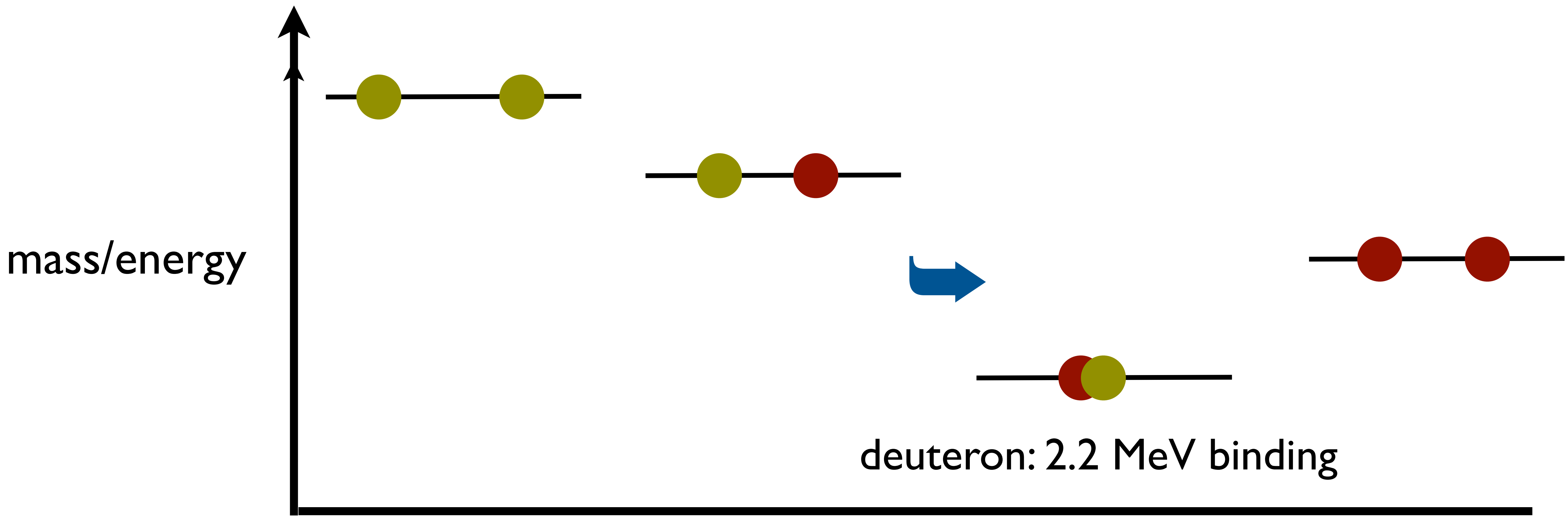
when systems cool, they settle into the lowest energy state

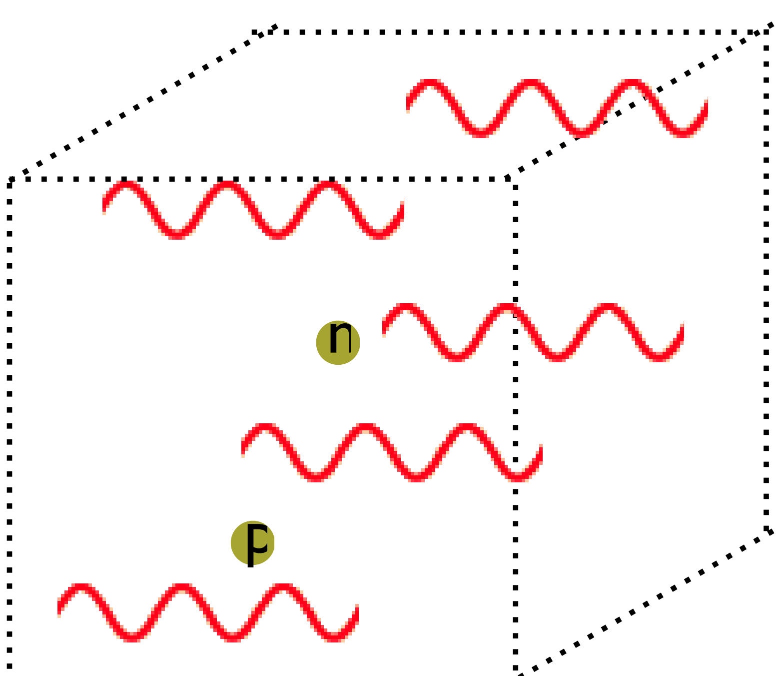


Answer: formation of nuclei

a system with protons and neutrons can collapse to a compact bound state, the deuteron: the attractive binding of a neutron and proton allows neutrons to survive when embedded in nuclei

The deuterium “bottleneck”



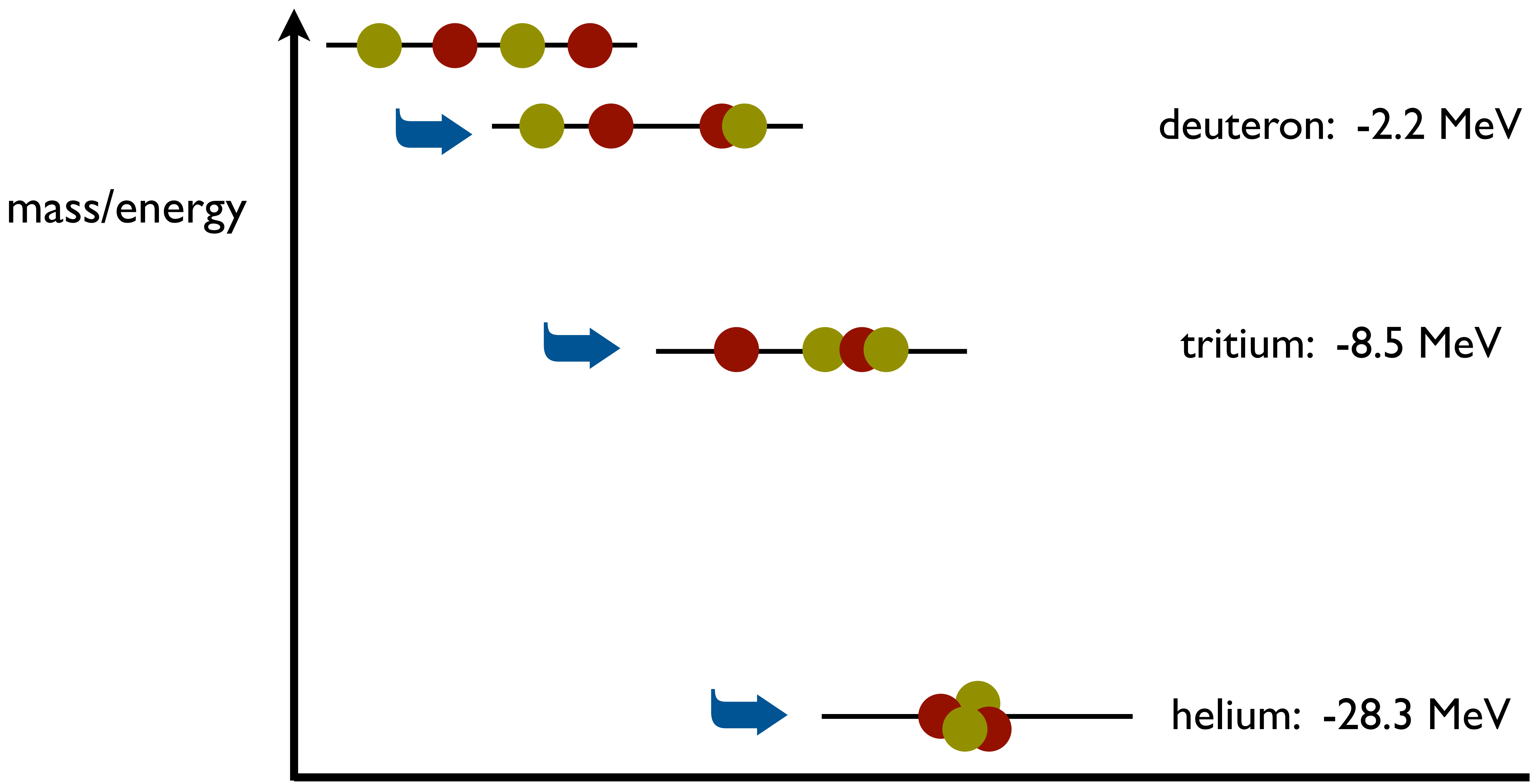


$\eta_{B/\gamma} \sim 10^{-9}$ $\quad p + n \longleftrightarrow d + \gamma$

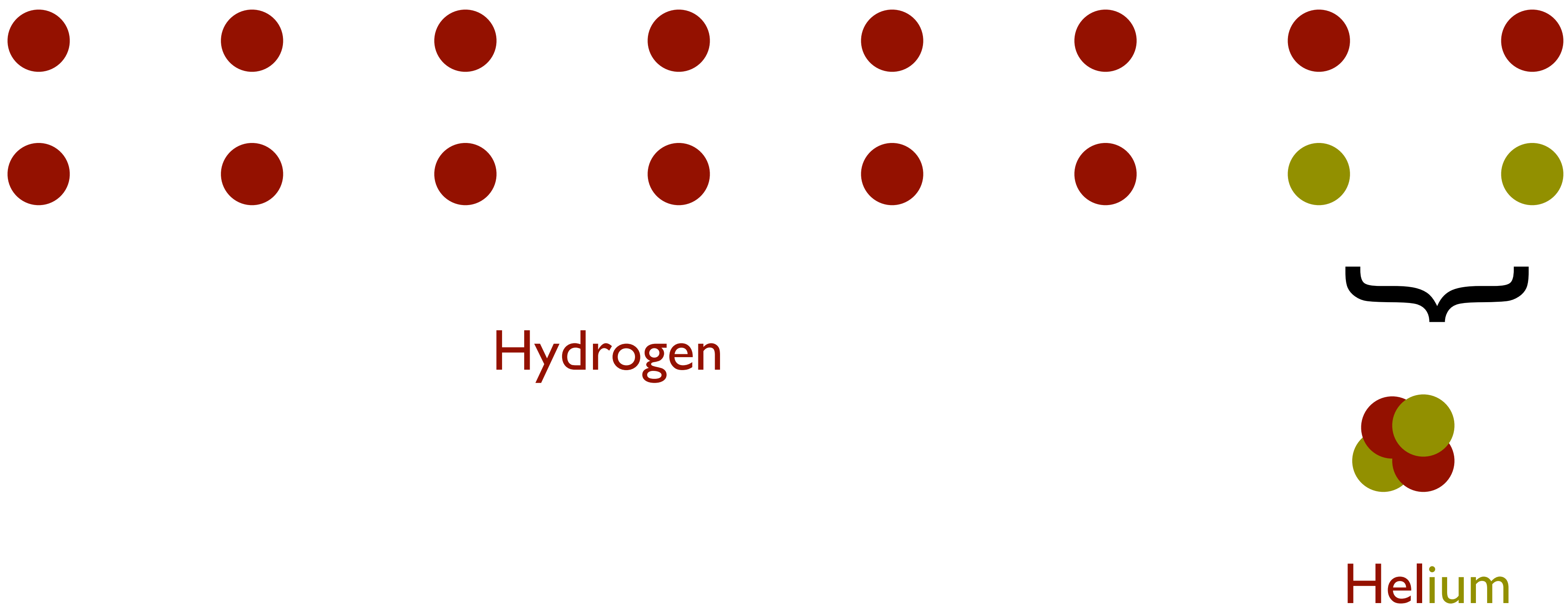
until T ≃ 100 keV (1 billion K),
t ≃ 3 min

$$p + n \longrightarrow d + \gamma$$

The deuterium "bottleneck" is broken, neutrons flow into He



He stability: ↑,↓ protons and ↑,↓ neutrons can be packed together



The early universe contains 75% H and 25% ^{4}He by mass fraction
("all" deuterium converted to ^{4}He)

this picture very sensitive to binding energy of deuterium which is finely tuned (most nuclei have ~8 MeV binding per nucleon)!

$$B_d = 2.22 \ \texttt{MeV}$$

What if

$B_d \ll 2.22$ MeV

more finely tuned

all neutrons decay - no helium

mostly hydrogen stars?

$B_d \gg 2.22$ MeV

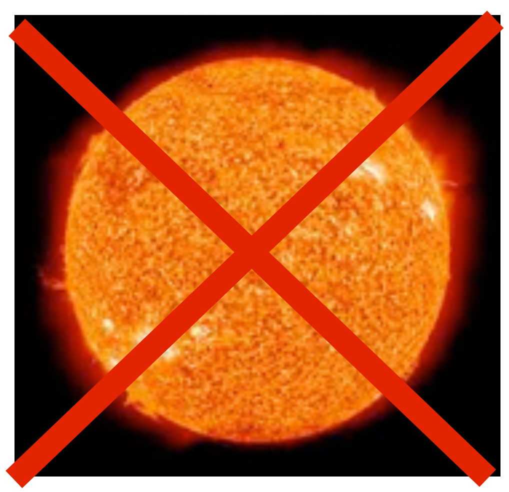
natural scenario

all neutrons captured in deuterium and helium - no hydrogen

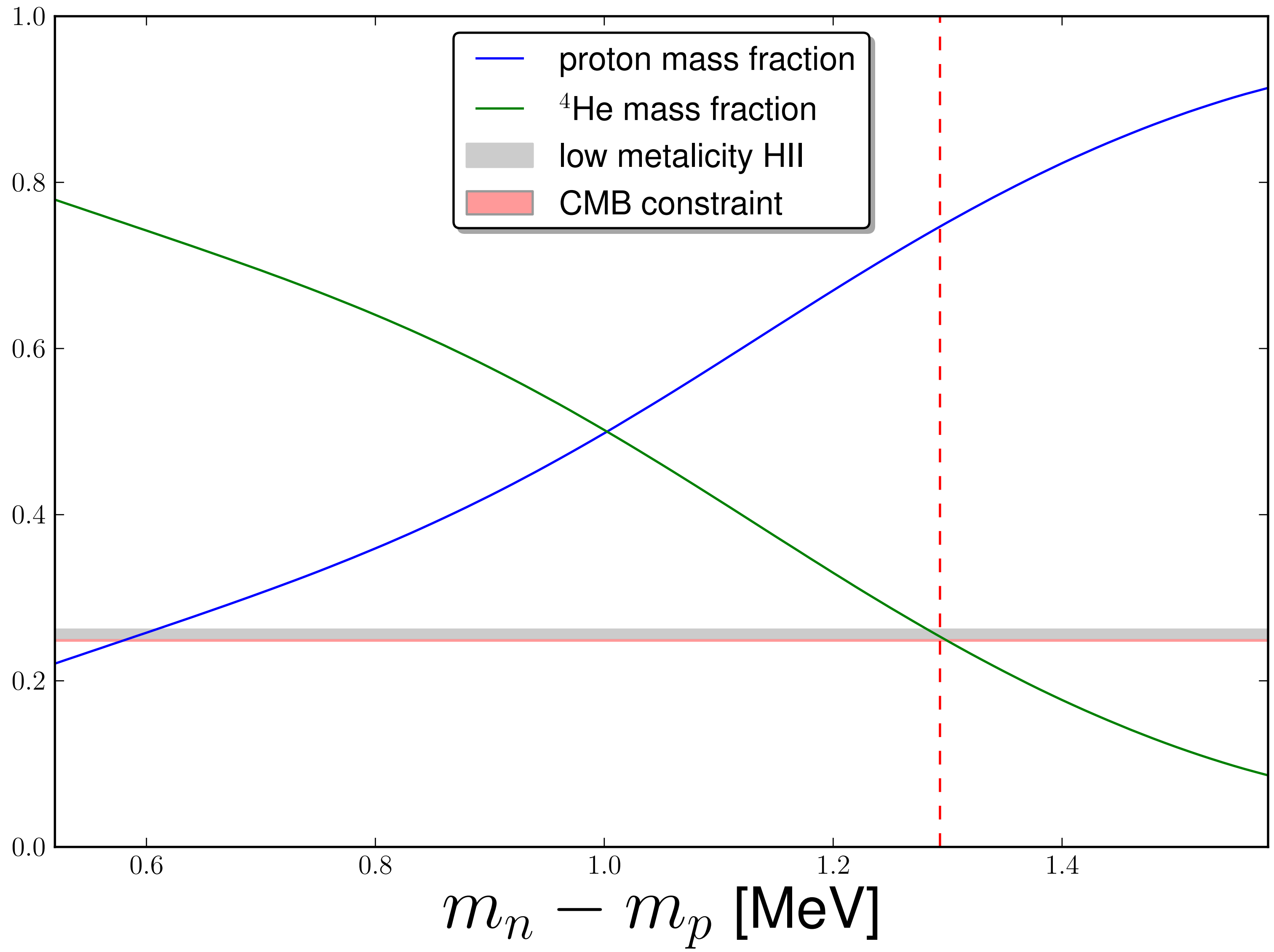
no stars like ours!

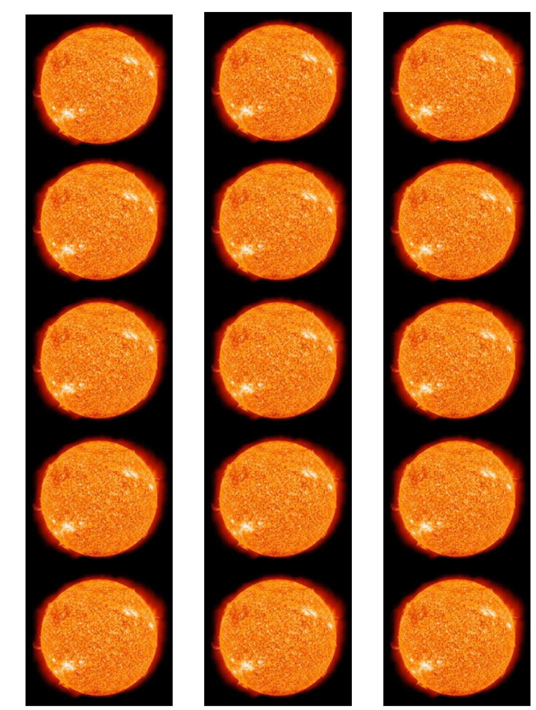
Turns out BBN abundances are also very sensitive to

$$m_n - m_p \propto \begin{cases} m_d - m_u \\ e^2/4\pi \end{cases}$$



No Sun!

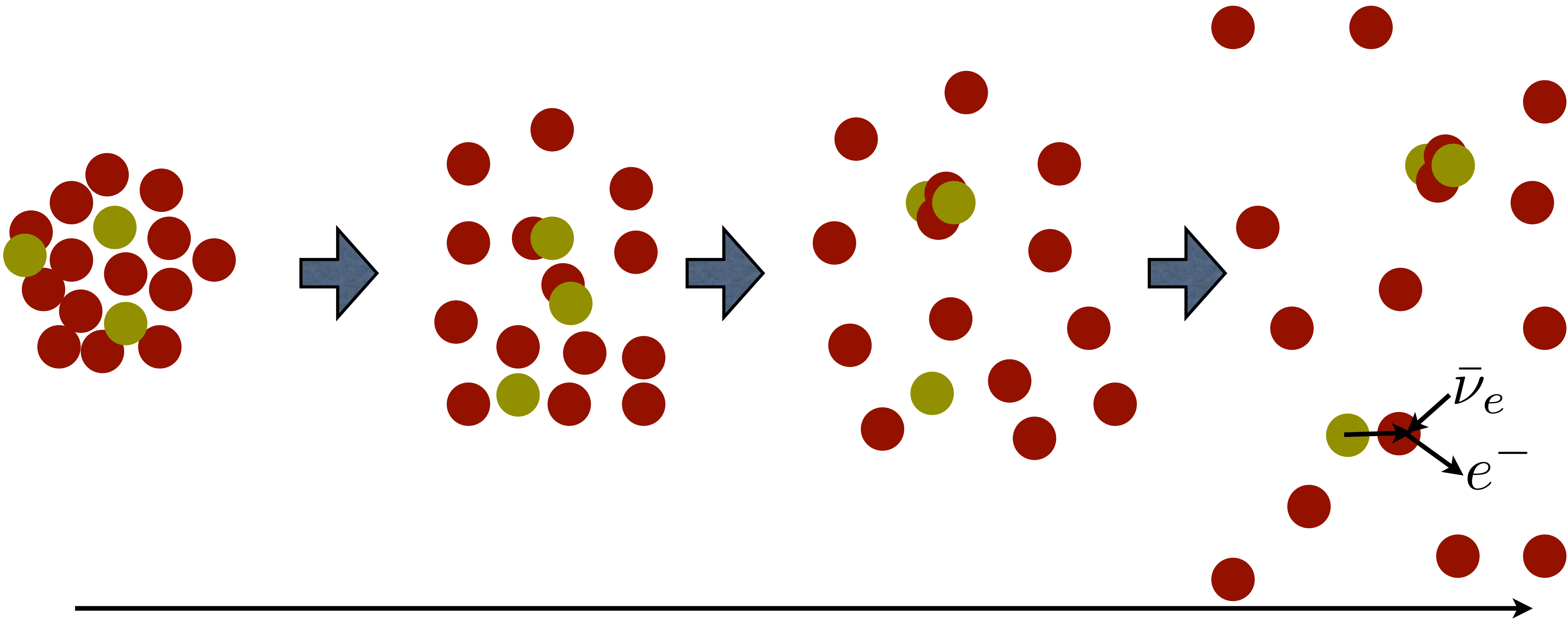




Too many suns?

How does QCD impact light element synthesis in the early Universe? (Will come back to this later)

Big Bang Nucleosynthesis and $m_n - m_p$



$t \sim 1$ sec
$T \sim 1$ MeV

$t \sim 3$ min
$T \sim 0.1$ MeV

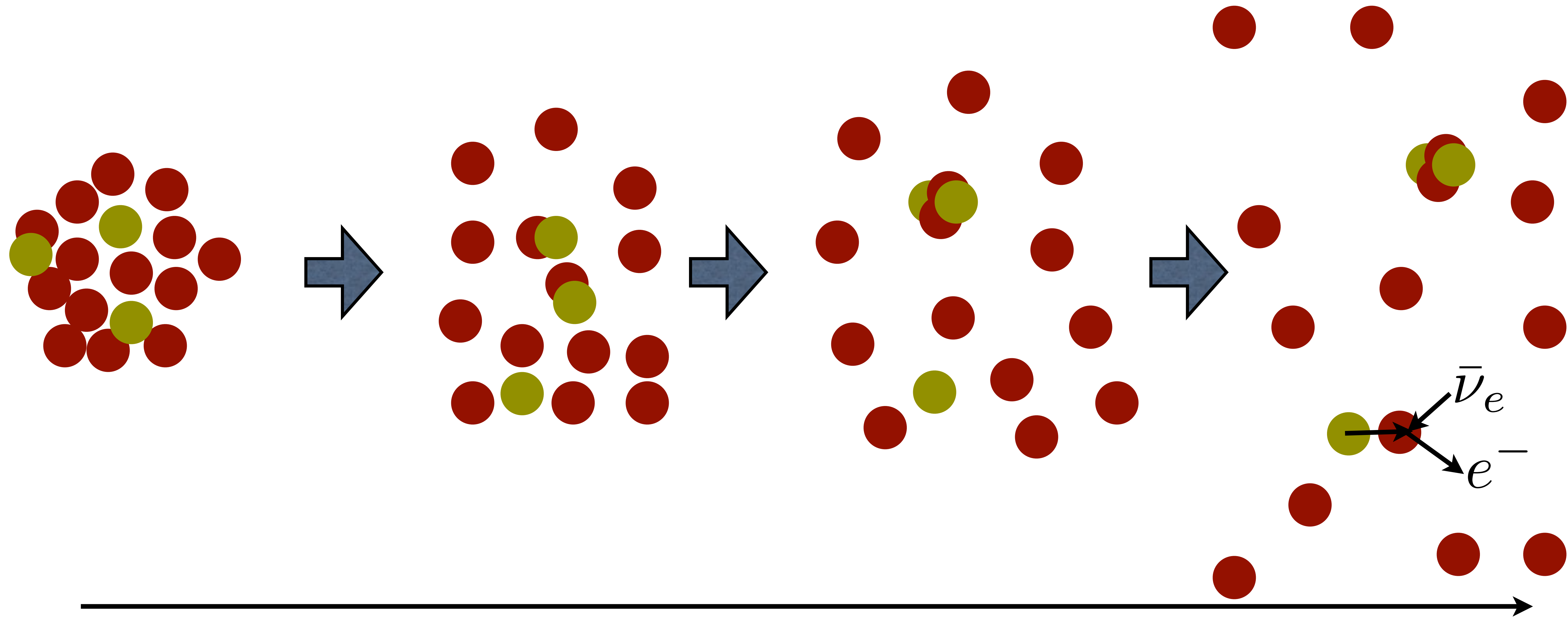
$t \sim 3^+$ min
$T \sim 0.1^-$ MeV

$t \sim 15$min
$T \sim 0.01$ MeV

Light Ion reactions in early universe produce primordial abundances of light nuclei
reactions dominated by radiation
absence of bound A=5,8 nuclei limit synthesis (no ^{12}C)

Alpher, Gamow; Fermi, Turkevich; Hayashi; Alpher; Peebles; Hoyle, Tayler; Wagoner, Fowler, Hoyle; Kawano; Olive; ...

Big Bang Nucleosynthesis and $m_n - m_p$



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$t \sim 15$min
$T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-(m_n - m_p)/T}$$

Initial conditions

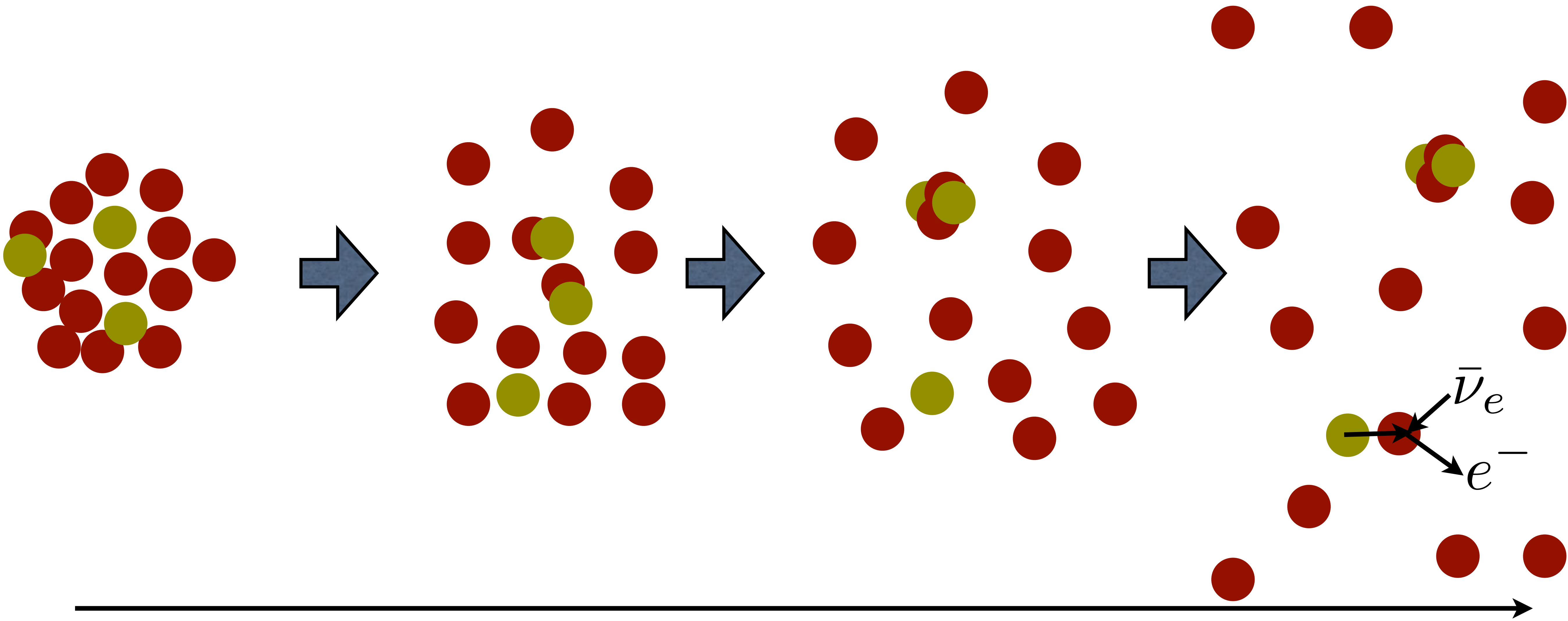
B_d

deuterium
binding energy

τ_n

neutron
lifetime

Big Bang Nucleosynthesis and $m_n - m_p$



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$$\frac{X_n}{X_p} = e^{-(m_n - m_p)/T}$$

Initial conditions

focus on leading isospin breaking

τ_n

neutron lifetime

Big Bang Nucleosynthesis and $m_n - m_p$

initial conditions for ratio of neutron to protons exponentially sensitive to mass splitting

$$\frac{X_n}{X_p} = e^{-(m_n - m_p)/kT}$$

neutron lifetime very sensitive to mass splitting

$$\frac{1}{\tau_n} = \frac{(G_F \cos\theta_C)^2}{2\pi^3} m_e^5 (1 + 3g_A^2)\, f\left(\frac{m_n - m_p}{m_e}\right)$$

$$f(a) \simeq \frac{1}{15}\left(2a^4 - 9a^2 - 8\right)\sqrt{a^2-1} + a \ln\left(a + \sqrt{a^2-1}\right)$$

Griffiths "Introduction to Elementary Particles"

10% change in $m_n - m_p$ corresponds to ~100% change neutron lifetime

Big Bang Nucleosynthesis and $m_n - m_p$

$$m_n - m_p = 1.29333217(42)\ \mathrm{MeV}$$

two sources of isospin breaking in the Standard Model

quark mass

$$m_q = \hat{m}\mathbf{1} - \delta\tau_3$$

quark electric charge

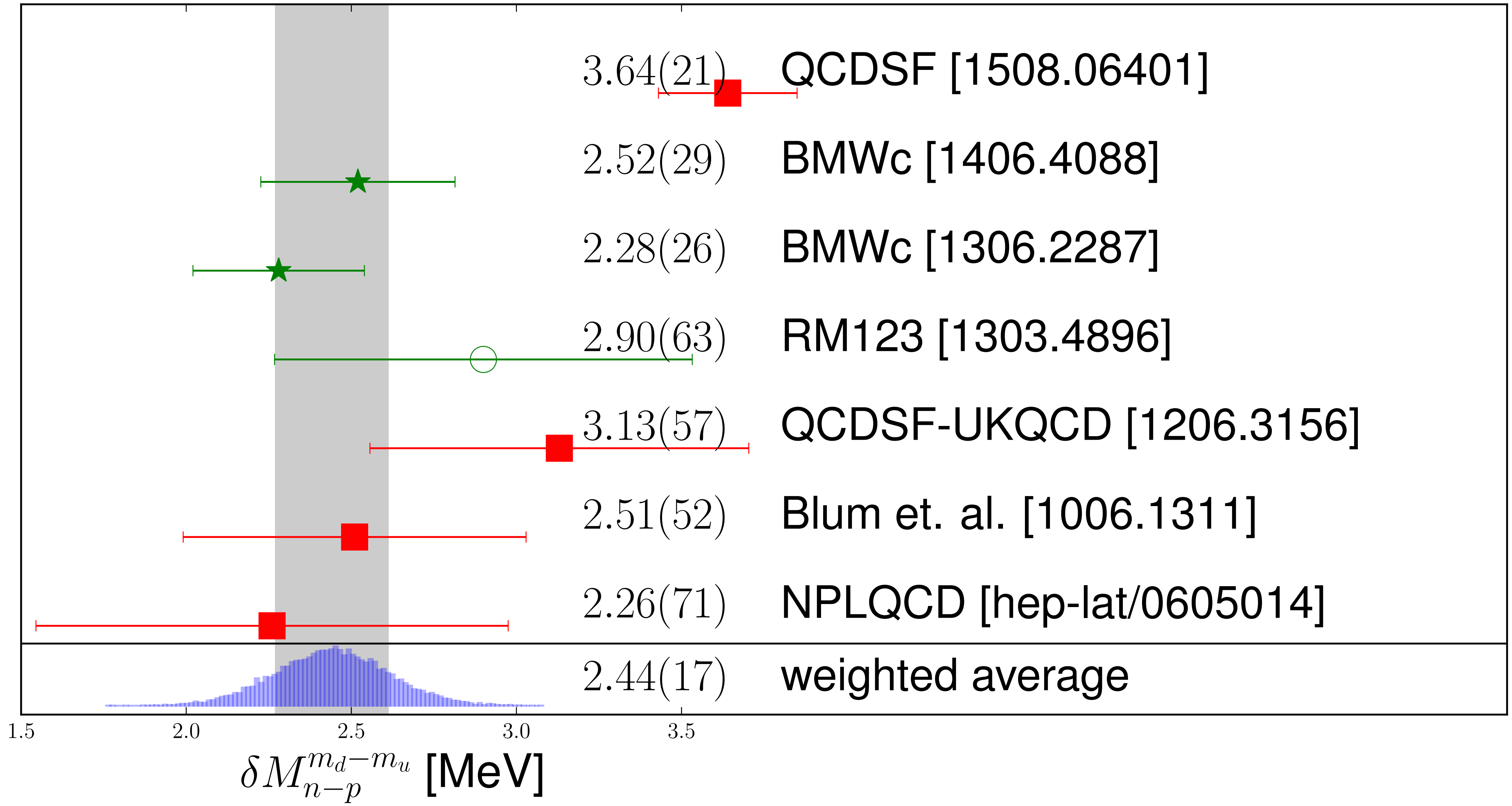
$$Q = \frac{1}{6}\mathbf{1} + \frac{1}{2}\tau_3$$

at leading order in isospin breaking

$$m_n - m_p = \underbrace{\delta M^{\gamma}_{n-p}}_{<0} + \underbrace{\delta M^{m_d - m_u}_{n-p}}_{>0}$$

Big Bang Nucleosynthesis and $m_n - m_p$

- $$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u) + \cdots = 2.44(17)\ \mathrm{MeV}$$



determine electromagnetic contribution through Lattice QCD computation or by comparing with experiment

$$\delta M_{n-p}^{\gamma} = m_n - m_p - \delta M_{n-p}^{m_d - m_u} = -1.15(17)\ \mathrm{MeV}$$

Big Bang Nucleosynthesis and $m_n - m_p$

$$M_n - M_p = \delta M^{\gamma}_{n-p} + \delta M^{m_d - m_u}_{n-p}$$
$$= -178(04)(64)\ \mathrm{MeV} \times \alpha_{f.s.} + 1.08(6)(9) \times (m_d - m_u)$$

(lattice average)

Big Bang Nucleosynthesis highly constrains variation of $M_n - M_p$ and hence variation of fundamental constants

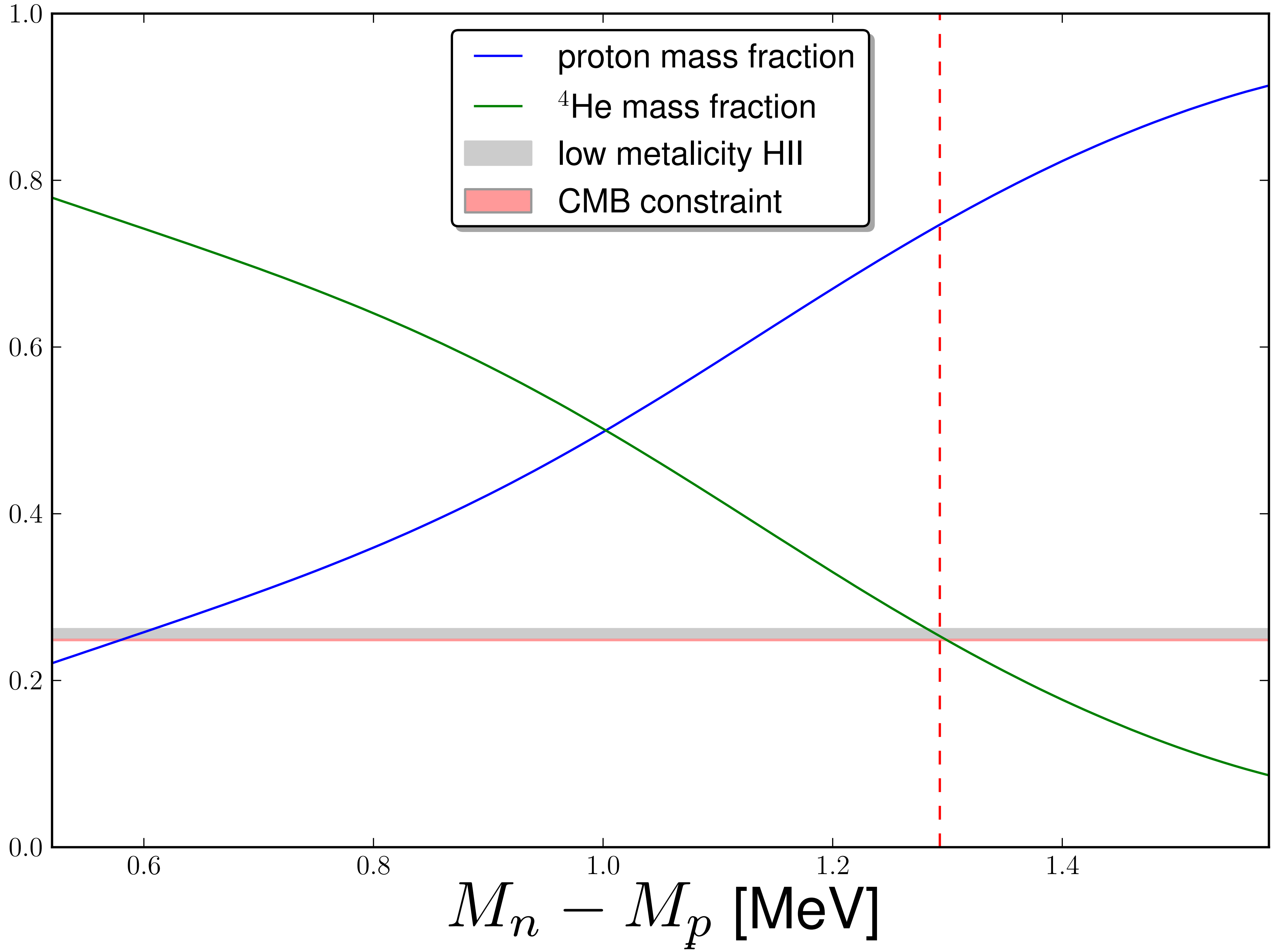
considering $\alpha_{f.s.}$ and $m_d - m_u$ simultaneously relaxes constraints (not yet simultaneously considered)

for now - freeze electromagnetic coupling and just look at effects of quark mass splitting

Big Bang Nucleosynthesis and $m_n - m_p$

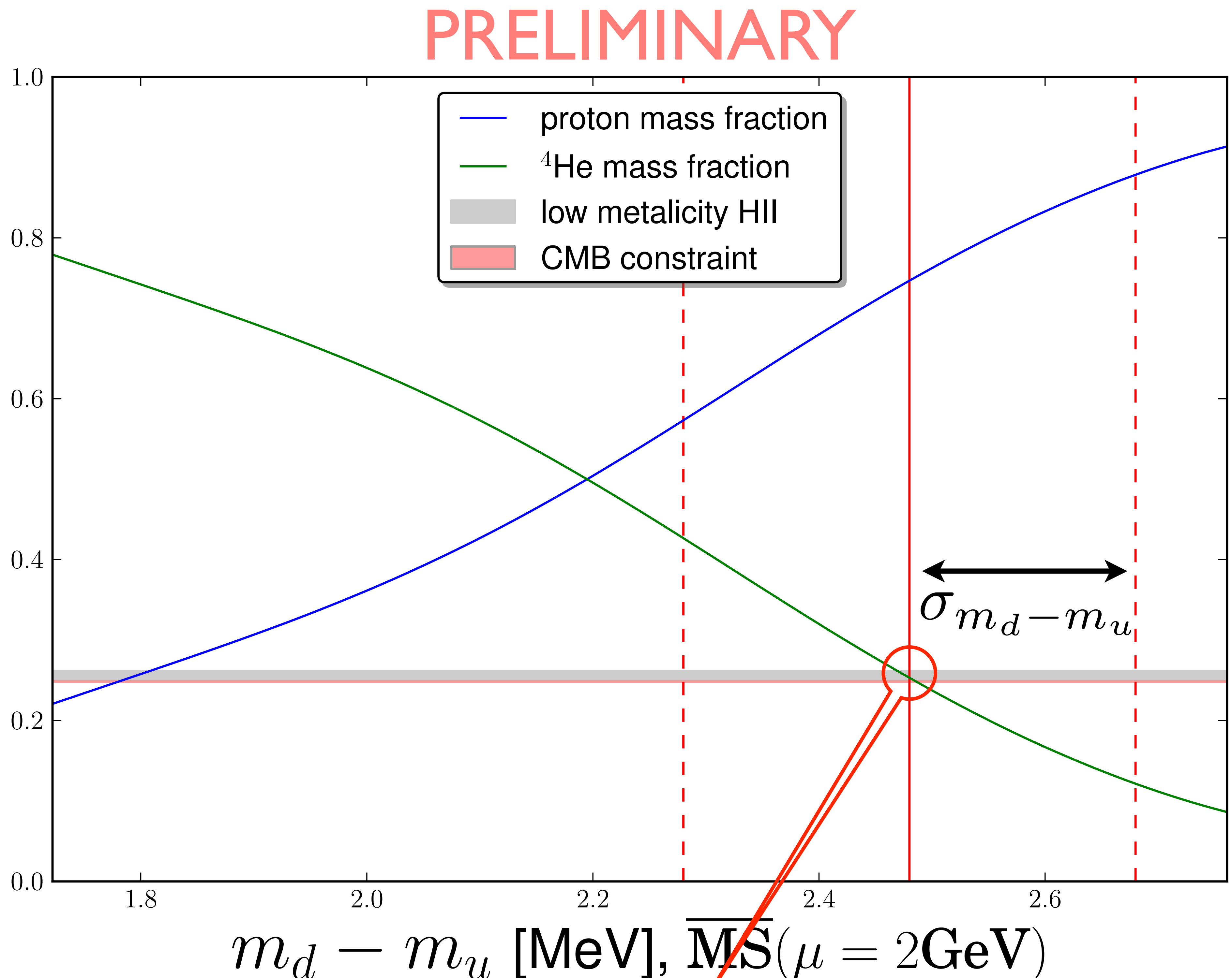
P. Banerjee, M. Heffernan, AWL

PRELIMINARY



Big Bang Nucleosynthesis and $m_n - m_p$

P. Banerjee, M. Heffernan, AWL



Lattice QCD

A precise determination of α + BBN can constrain $m_d - m_u$

$$\delta M_{n-p}^{m_d - m_u} \equiv \alpha(m_d - m_u)$$

Big Bang Nucleosynthesis and $m_n - m_p$

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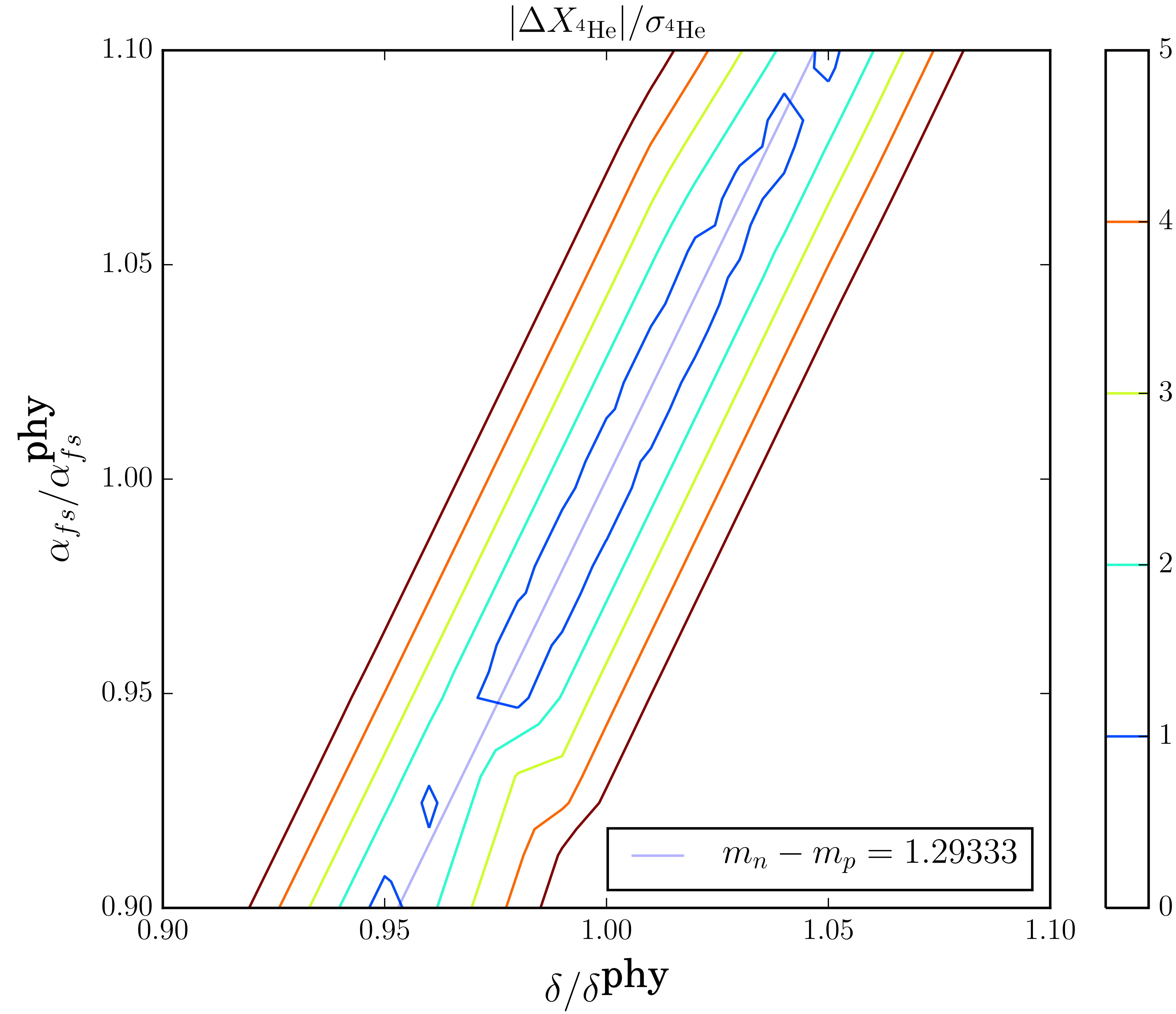
PRELIMINARY



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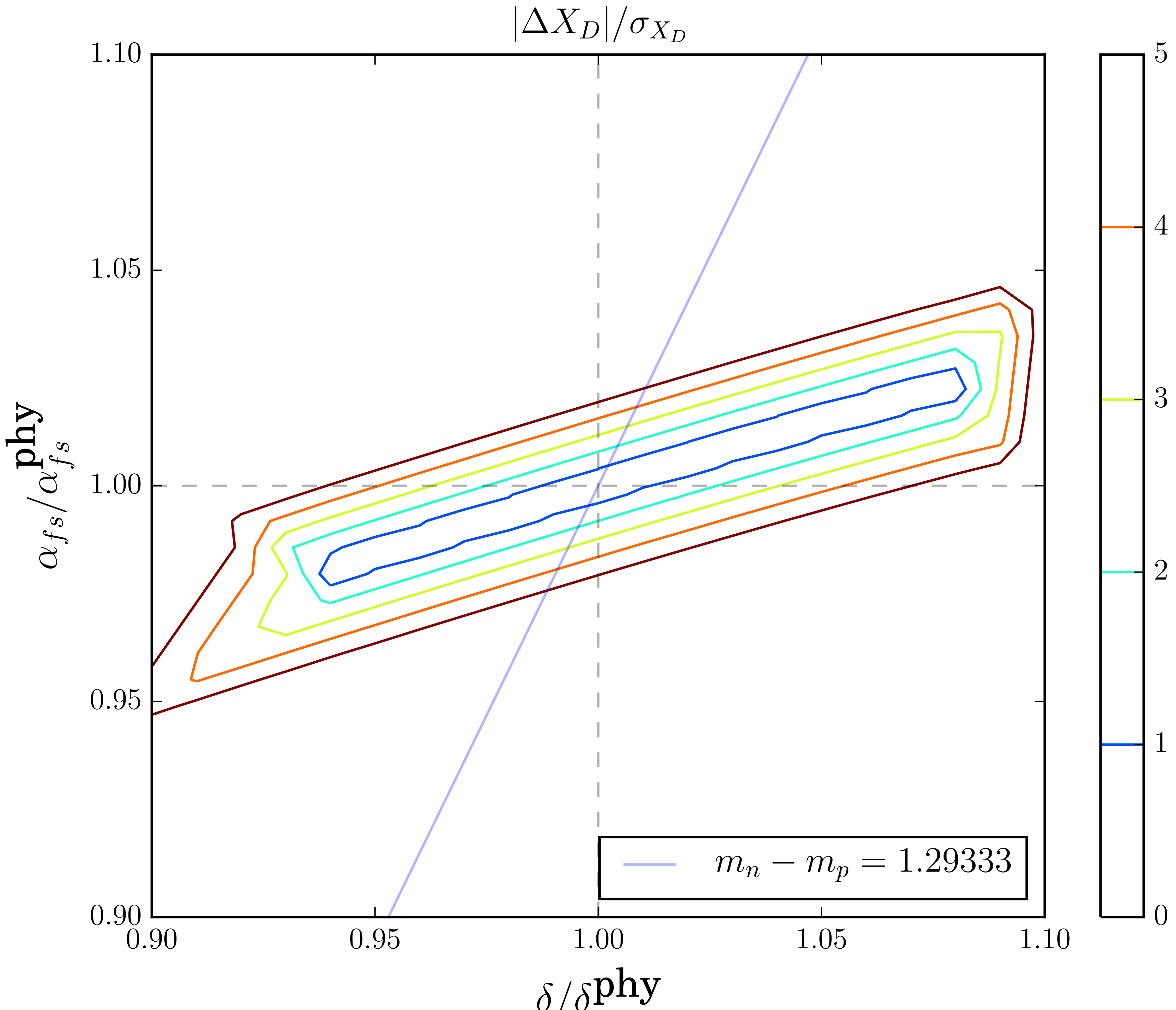

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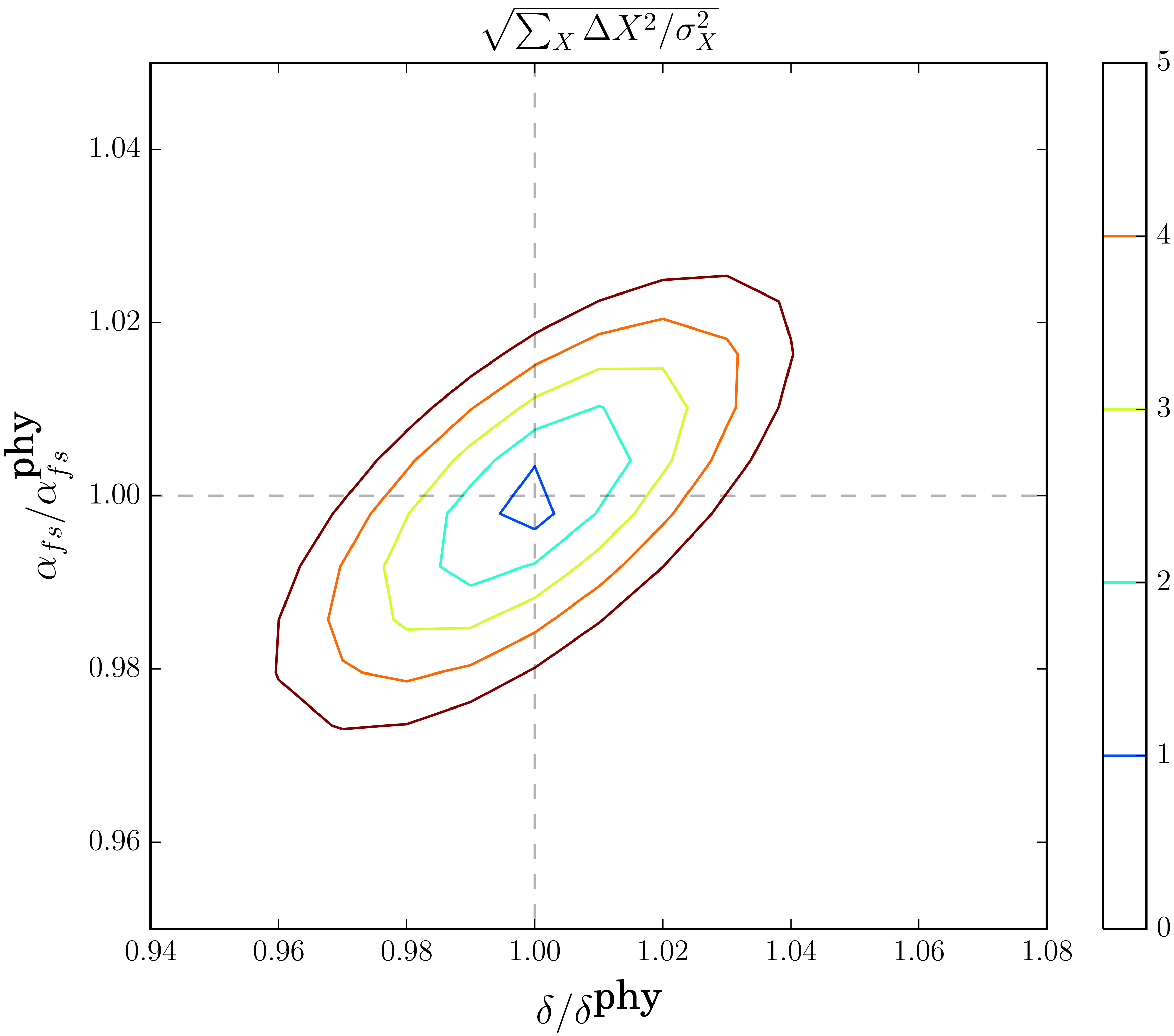

Conclusions

- Understanding nuclear physics from the fundamental theory of strong interactions, QCD, is exciting and important for these and other reasons:
    - Quantitative connection between QCD and the rich nuclear phenomenology
    - Understanding precision low-energy nuclear physics to constrain the SM and searches for BSM physics
- The growth of computing power and algorithms means that TODAY is the beginning of a renaissance in nuclear physics where these exciting things are just becoming possible!

Thank You