



# Numerical Computation

## Triple and Quadruple Precision BLAS Subroutines on GPUs

### Background

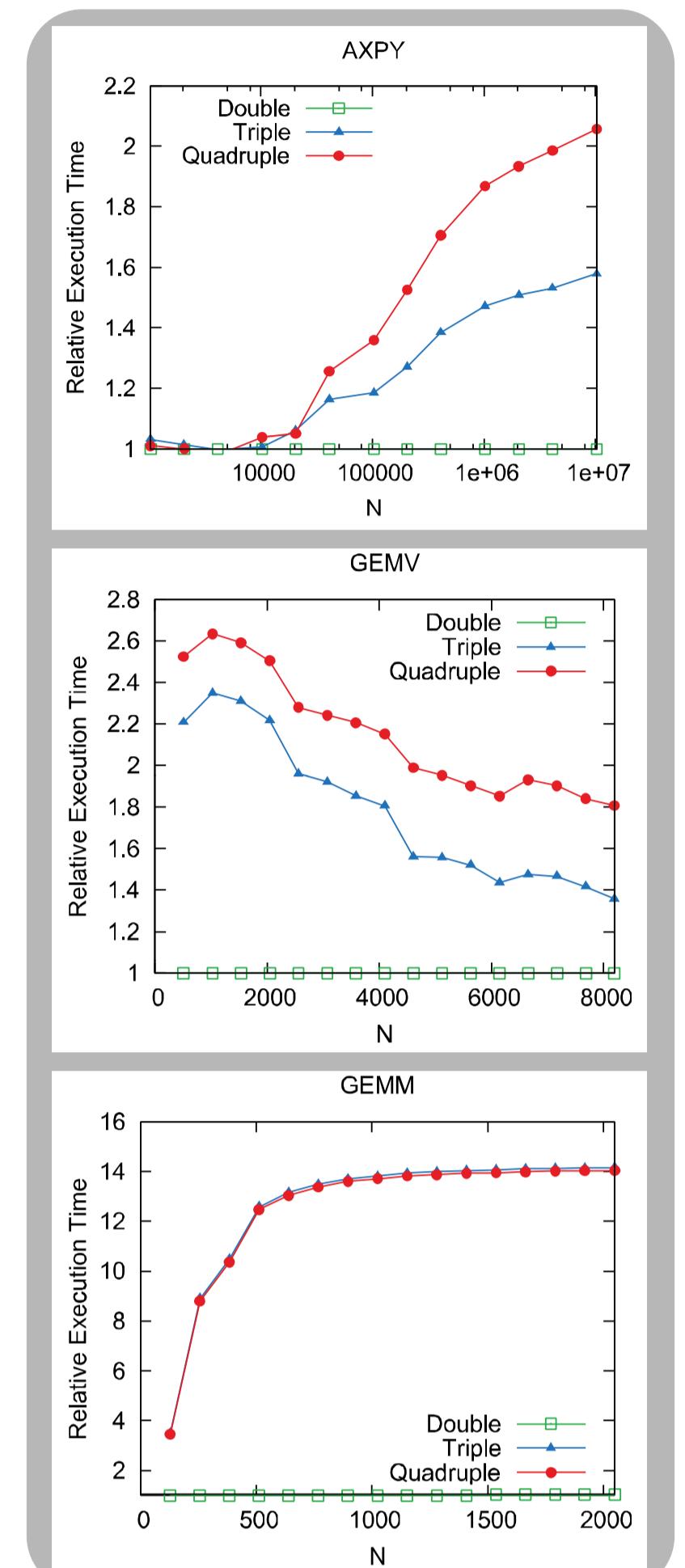
Floating - point operations have round - off errors. These errors may become a critical issue for some applications. Especially in large - scale computing, an accumulation of round – off errors may become a more serious problem. Double precision accuracy may be insufficient in some cases, and there is a demand for higher precision operations.

### Overview

We have implemented triple and quadruple precision Basic Linear Algebra Subprograms (BLAS) subroutines, AXPY ( $y = \alpha x + y$ ), GEMV ( $y = \alpha Ax + \beta y$ ) and GEMM ( $C = \alpha AB + \beta C$ ) on GPUs. For quadruple precision, we used Double – Double (DD) type quadruple precision operations (11bits exponent & 104bits significand). On the other hand, we propose Double+Single (D+S) type triple precision floating – point format (8bites exponent & 75 bits significand) and triple precision operations using DD – operations internally.

### Performance

Some level - 1 and 2 BLAS subroutines are memory - bound on the Tesla M2090, not only in single and double, but also triple and quadruple precision: the execution time of triple and quadruple precision subroutines is close to only 1.5x and 2.0x of that of double precision. For memory - bound operations where double precision is insufficient but quadruple precision is not needed, triple precision operations should be used.



## Auto - Tuning of SpMV for CRS format on GPUs

### Background

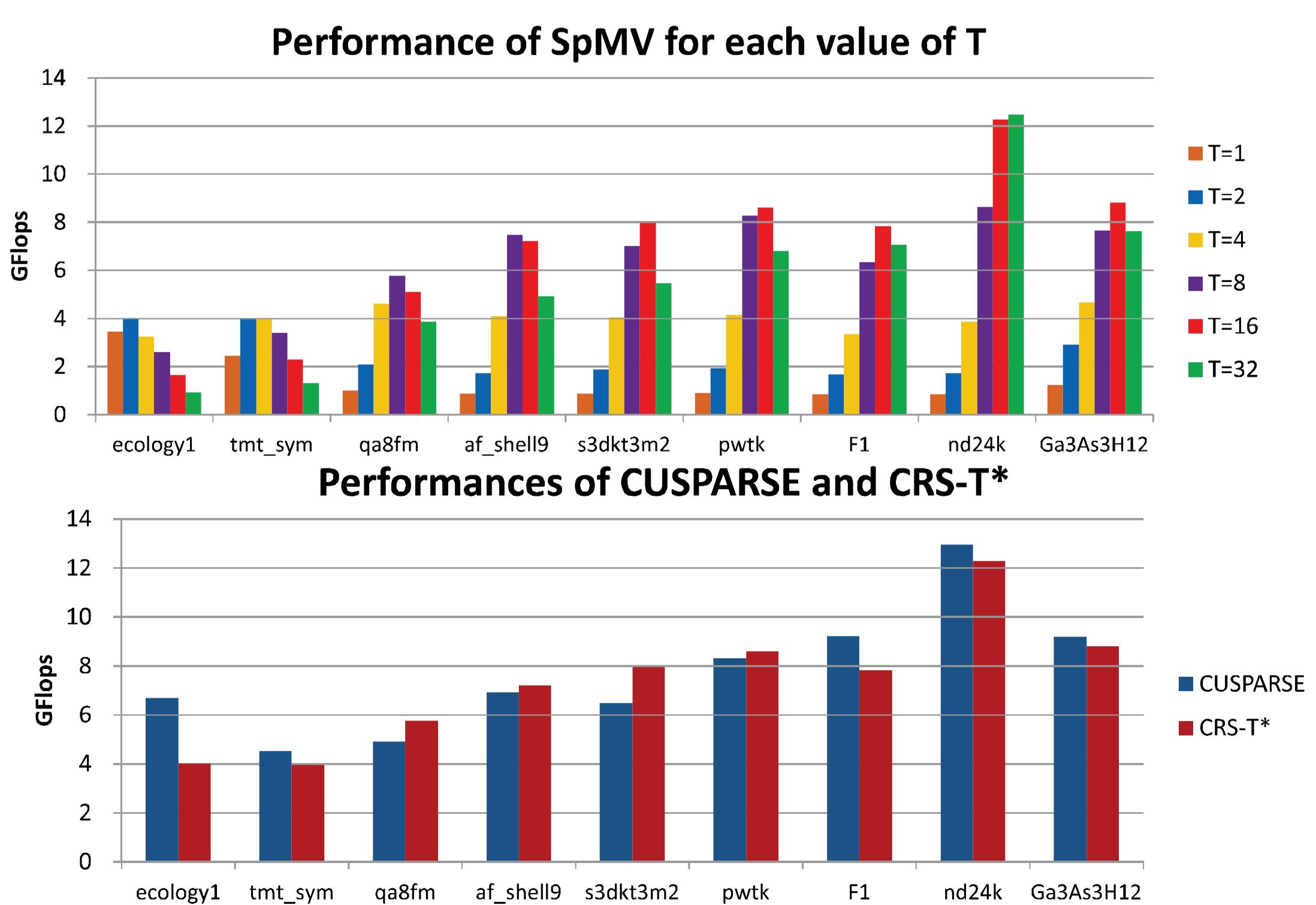
Performance of sparse matrix - vector multiplication (SpMV) on GPUs is highly dependent on the structure of the sparse matrix used in the computation, the computing environment, and the selection of certain parameters.

### Overview

We show that the performance achieved using kernel SpMV on GPUs for the compressed row storage (CRS) format depends greatly on optimal selection of a parameter  $T$  that is a number of threads to compute an output vector element, and we propose an efficient algorithm for the automatic selection of the optimal parameter.

### Performance

CRS -  $T^*$  that is kernel SpMV for the CRS format using automatic parameter selection achieves up to approximately 26% improvement over NVIDIA's CUSPARSE library.

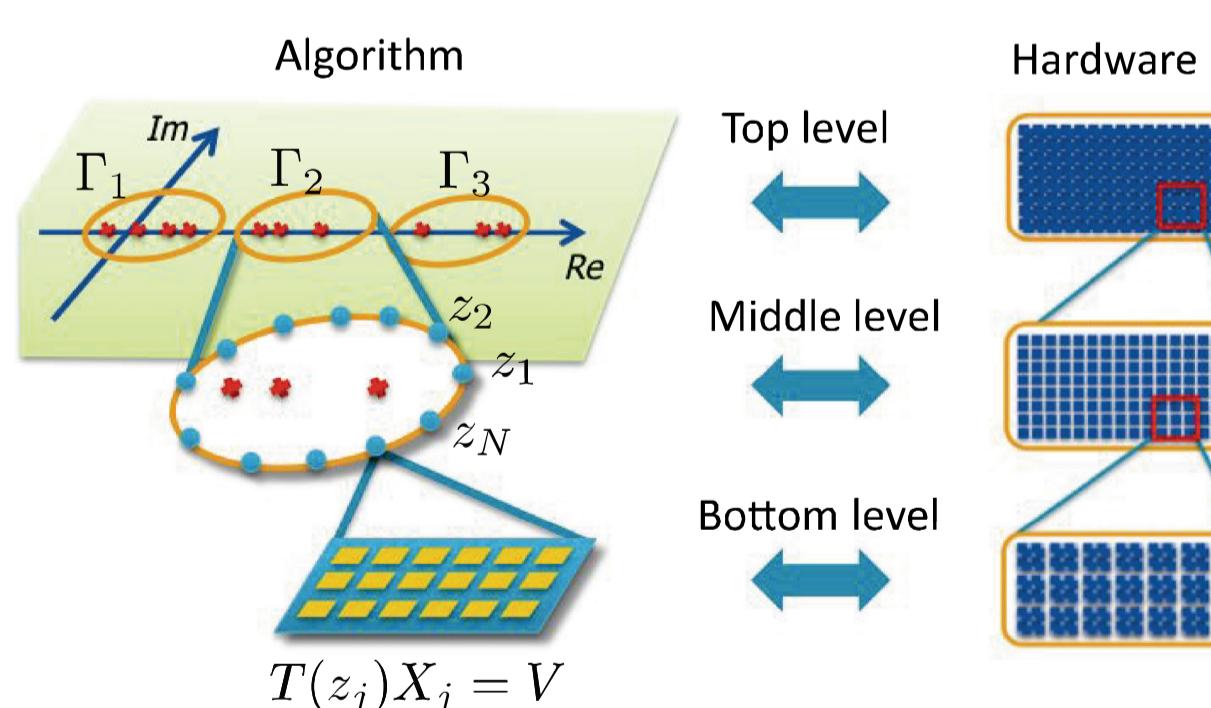


## Development of Parallel Sparse Eigensolver Package: z - Pares

The aim of this research project is to develop numerical software for large-scale eigenvalue problems in post-petascale computing environment. An eigensolver based on contour integral (the SS method) has been proposed by Sakurai and Sugiura [3]. This method has a hierarchical structure and is suitable for massively parallel supercomputers [2]. Moreover, the SS method can be applicable for nonlinear eigenvalue problem [1]. Block Krylov method [4] improve the performance of the method. We are developing software on both Trilinos and PETSc. MATLAB version is available at <http://zparecs.cs.tsukuba.ac.jp/>

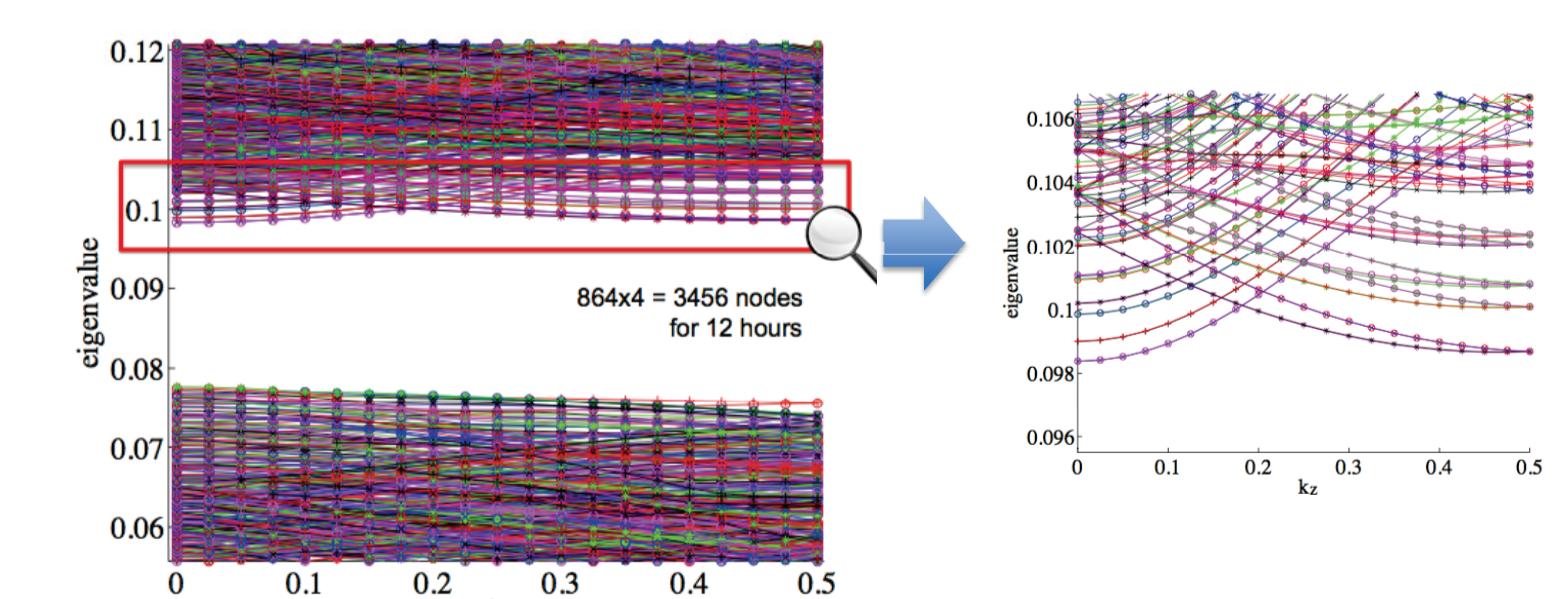
### Hierarchical Parallel Structure

Hardware is grouped according to a hierarchical structure of the algorithm.



### Numerical Example on the K Computer

Application for band calculation with real space density functional theory (RSDFT) [2].



Band structure of silicon nanowire of 9,924 atoms. (matrix size = 8,719,488, Number of cores = 6,144)

\*The results are tentative since they are obtained by early access to the K computer.

### Reference

- [1] J. Asakura, T. Sakurai, H. Tadano, T. Ikegami and K. Kimura, A numerical method for nonlinear eigenvalue problems using contour integrals, JSIAM Letters, 1 (2009) 52-55.
- [2] Y. Futamura, T. Sakurai, S. Furuya and J.-I. Iwata, Efficient algorithm for linear systems arising in solutions of eigenproblems and its application to electronic-structure calculations, Proc. 10th International Meeting on High-Performance Computing for Computational Science (VECPAR 2012) (accepted).
- [3] T. Sakurai and H. Sugiura, A projection method for generalized eigenvalue problems, J. Comput. Appl. Math., 159 (2003) 119-128.
- [4] H. Tadano, T. Sakurai and Y. Kuramashi, Block BiCGGR: A new block Krylov subspace method for computing high accuracy solutions, JSIAM Letters, 1 (2009) 44-47.

### Acknowledgment

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