Vlasov-Poisson Simulation of Astrophysical Self-Gravitating Systems

- an alternative to *N*-body simulations -

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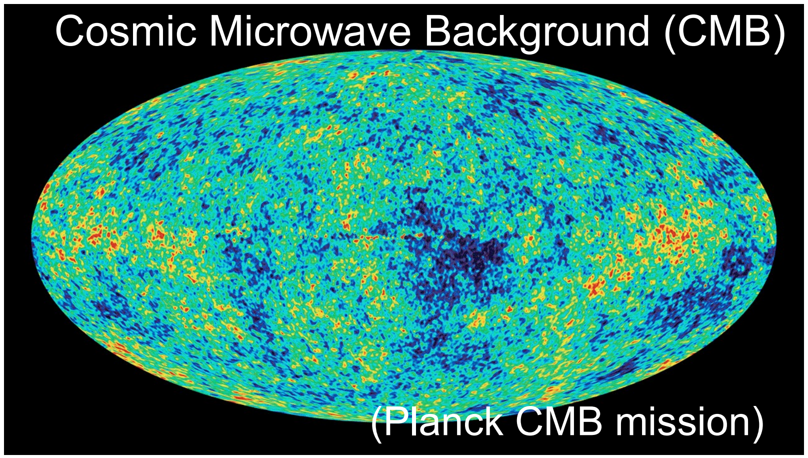
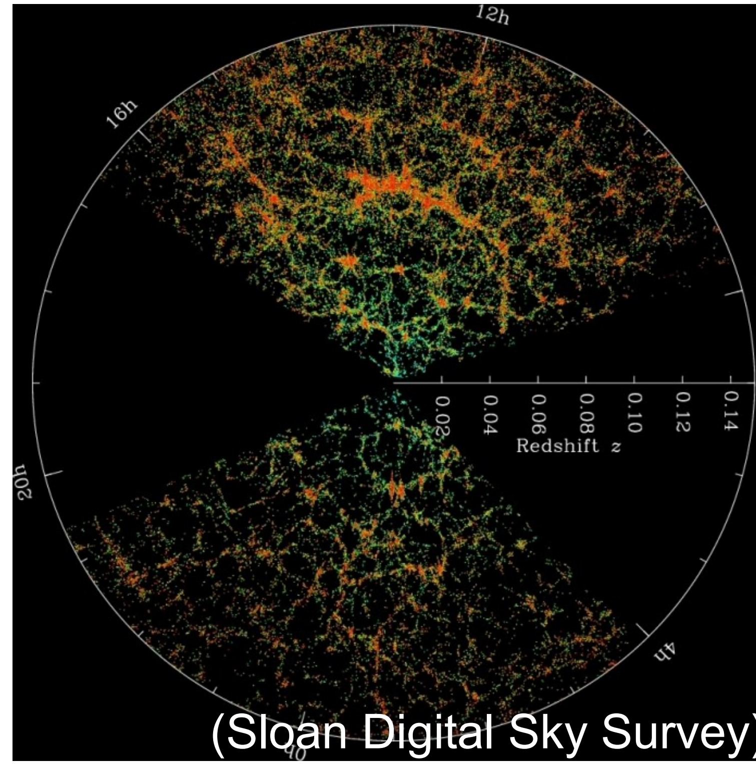
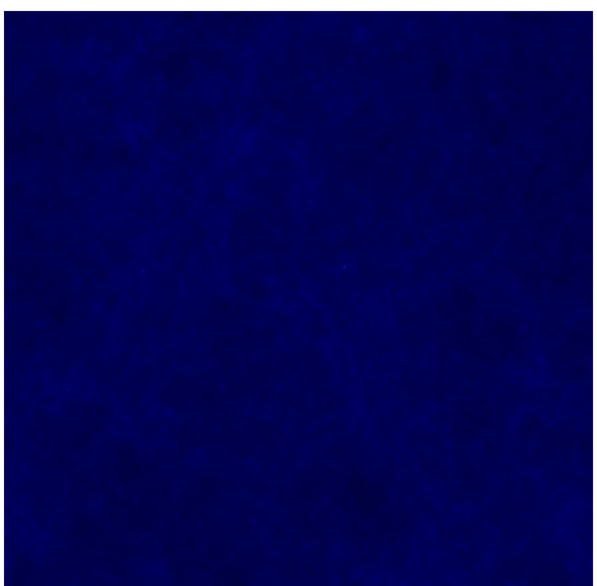
Large-Scale Structure in the Universe





N-body simulation

gravitational interaction



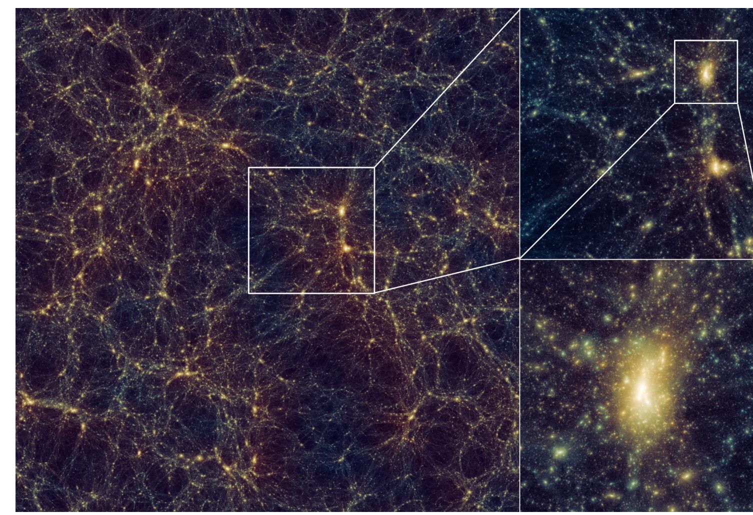
- comparison between the theoretical predictions and the observed universe

→ physical parameters of the universe

N-body Simulation

- a standard method for simulations of self-gravitating systems (galaxies, clusters of galaxies, the LSS) for more than 30 years.
- the mass distribution is sampled by particles in the 6D phase-space volume (**x**, **p**) in a Monte-Carlo manner.

  ⇨ need for a very large number of particles

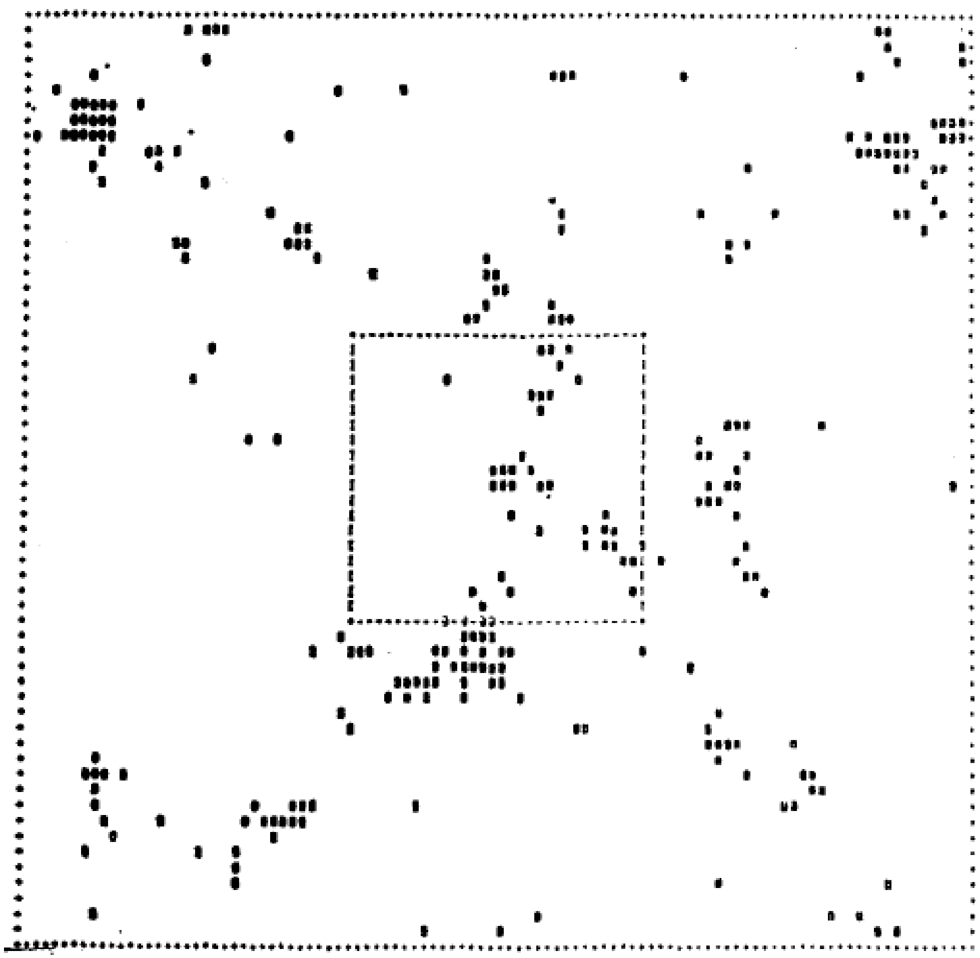


- sophisticated algorithms to treat large number of particles such as Tree and TreePM methods developed
- special- / general-purposed hardware such as GRAPE-family and GPUs to accelerate the computation

Past and Present of Simulations of Large-Scale Structure in the Universe

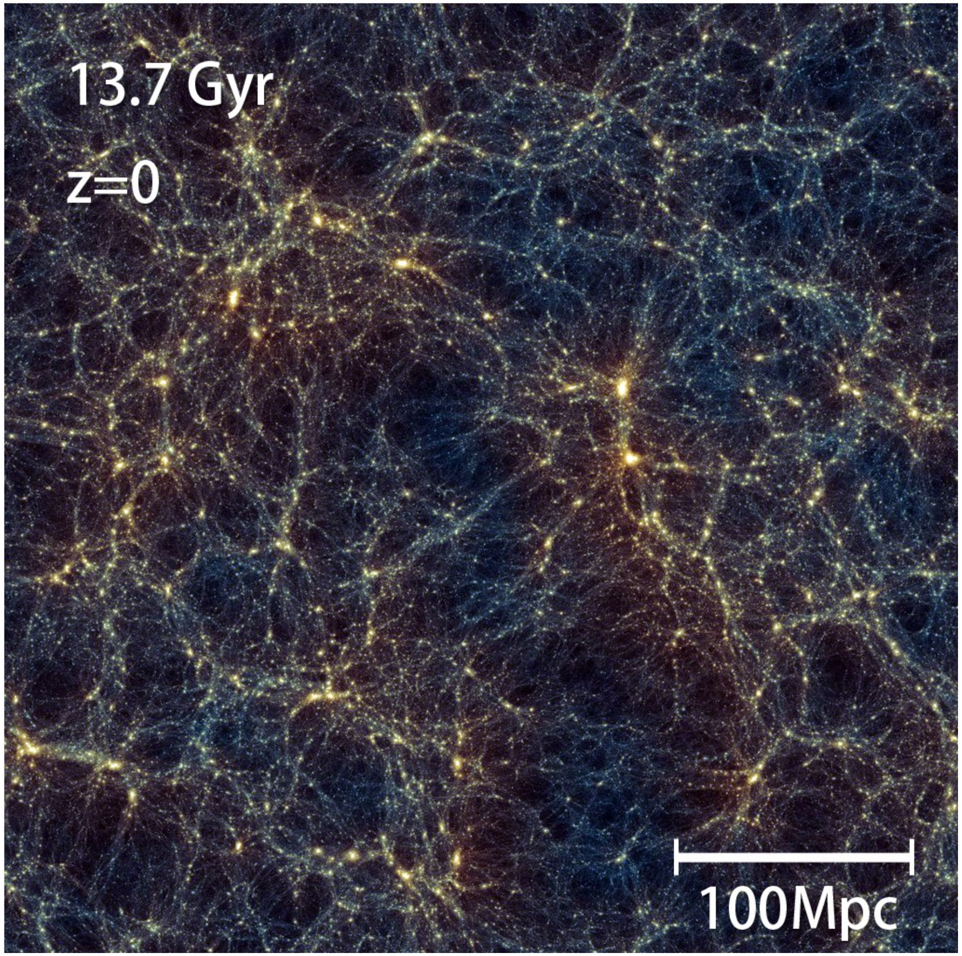
Miyoshi & Kihara (1975) PASJ, 27, 333

N=400 on HITAC 8500



Trillion-body simulation by Ishiyama (2012)

$N=10^{12}$ on K-computer



Drawbacks of N-body Simulations

- intrinsic contamination of shot noise in physical quantities

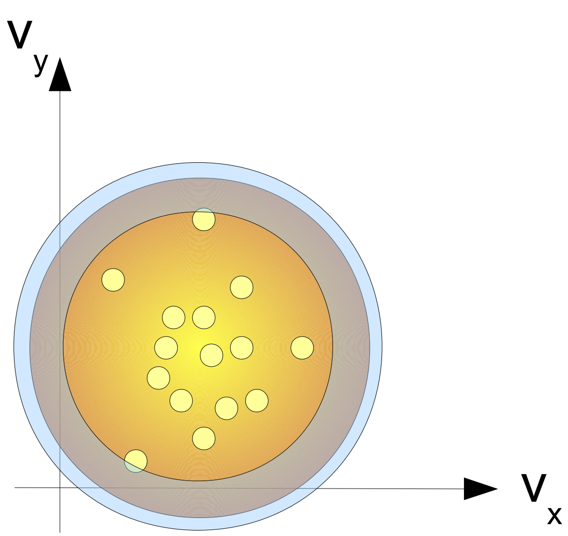
  shot noise term is only proportional to $N^{-1/2}$

- artificial two-body relaxation due to the super-particle approximation

  ⇨ introduces undesired effects in a long-term evolution

- not good at simulating the collisionless damping (Landau damping)

  the collisionless damping is driven by the high velocity component, which is not fairly sampled in N-body simulations



Vlasov-Poisson Equations

$$\begin{cases} \dfrac{\partial f}{\partial t} + \vec{v} \cdot \dfrac{\partial f}{\partial \vec{x}} - \nabla \phi \cdot \dfrac{\partial f}{\partial \vec{v}} = 0 \\ \nabla^2 \phi = 4\pi G \rho = 4\pi G \displaystyle\int f d^3\vec{v} \end{cases}$$

$f(\vec{x}, \vec{v})$: matter density in 6D phase space (distribution function)

- combination of collisionless Boltzmann equation (aka Vlasov equation) and Poisson equation.
- treats the matter as continuum fluid in the phase space instead of statistically sampled particles

  ⇒ free from shot noise contamination seen in the N-body approach
- so far limited to 1D or 2D simulations due to the large amount of required memory space and huge computational costs.

We present the first 3D Vlasov-Poisson simulation in the 6D phase space volume.

Numerical Methods

- Both of physical and velocity spaces are discretized with 3D regular mesh grids.

- Vlasov equation is solved using directional splitting scheme, in which following 1D advection equations are sequentially integrated.

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} = 0 \qquad \frac{\partial f}{\partial t} - \frac{\partial \phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0 \quad (i = 1, 2, 3)$$

- Physical requirements for the scheme of 1D advection equations
  - positivity
  - mass conservation → Positive Flux Conservation (PFC) scheme
  - maximum principle

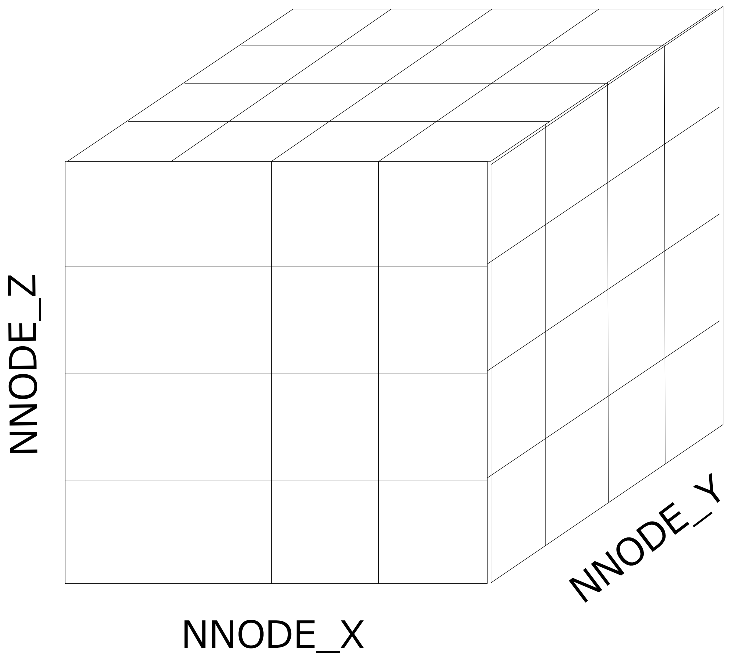
Filbet, Sonnendrucker, Bertrand, J. Comp. Phys. (2001) 172, 166-187

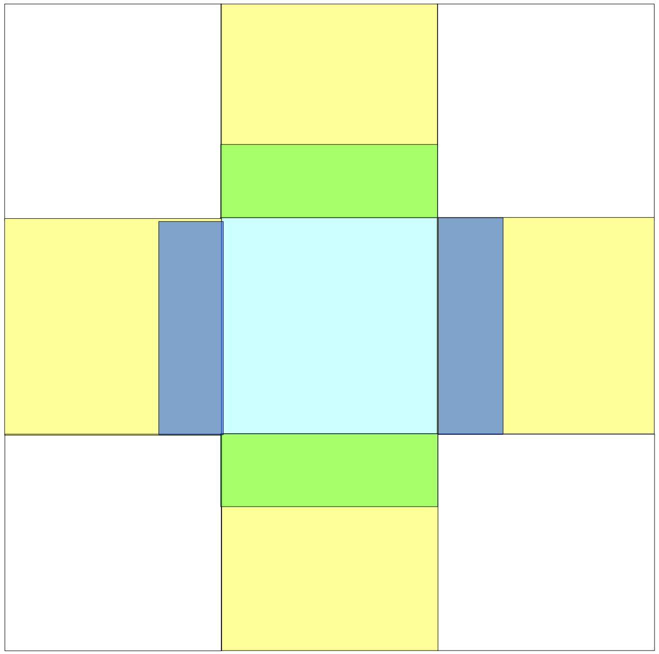
Numerical Methods

- Poisson equation
  - Solved with the convolution method using the Fourier transform
  - Both for the periodic and isolated boundary condition
- Time integration

$$f(\vec{x},\vec{v},t^{n+1}) = T_{v_x}(\Delta t/2)T_{v_y}(\Delta t/2)T_{v_z}(\Delta t/2)$$
$$T_x(\Delta t)T_y(\Delta t)T_z(\Delta t)$$
$$T_{v_x}(\Delta t/2)T_{v_y}(\Delta t/2)T_{v_z}(\Delta t/2) \quad f(\vec{x},\vec{v},t^n)$$

Parallelization





- Only physical (spatial) grids are decomposed among computational nodes.
- Each spatial grids contains the entire velocity (momentum) grids
- Data communication at the boundaries of each decomposed domain.

Test Suite

- Stability test of a stable solution of Vlasov-Poisson equations
- Dynamical collision of two self-gravitating systems
- Gravitational instability and collisionless damping in homogeneous matter

King sphere

- a stable solution of Vlasov – Poisson equations

$$f(E, t=0) = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} \exp[(-E/\sigma^2) - 1] \qquad E < 0$$

$$= 0 \qquad E > 0$$

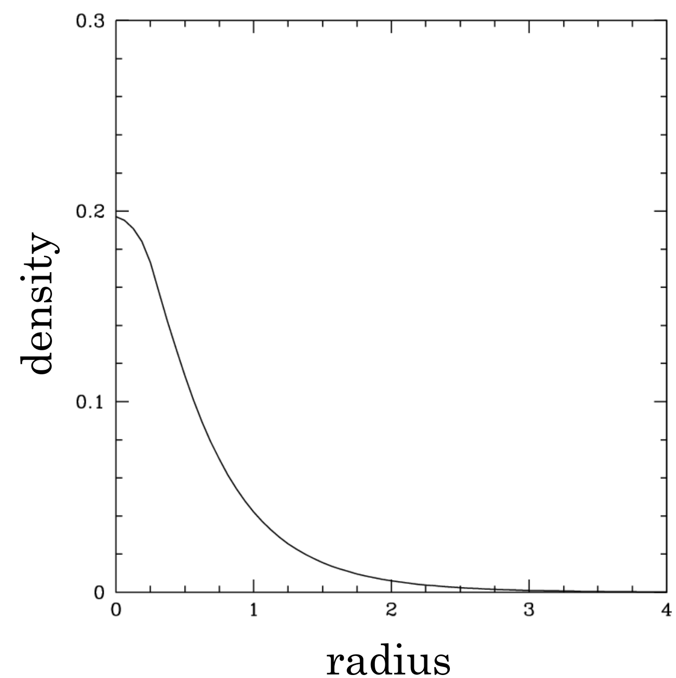
$$E = \frac{1}{2}v^2 + \Phi$$

- number of mesh points

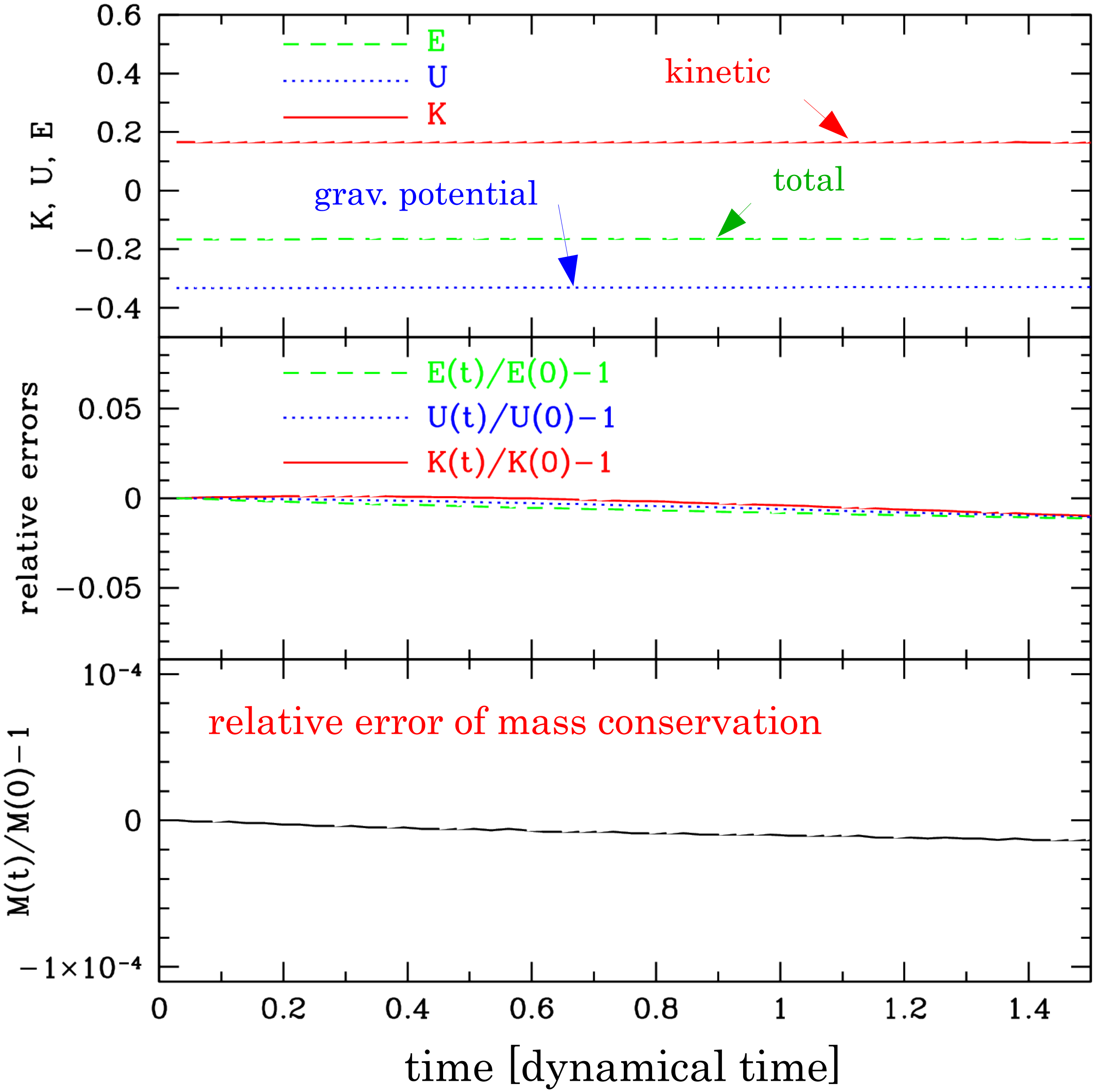
  64^3 for the physical space

  64^3 or 32^3 for the velocity space

- isolated boundary condition

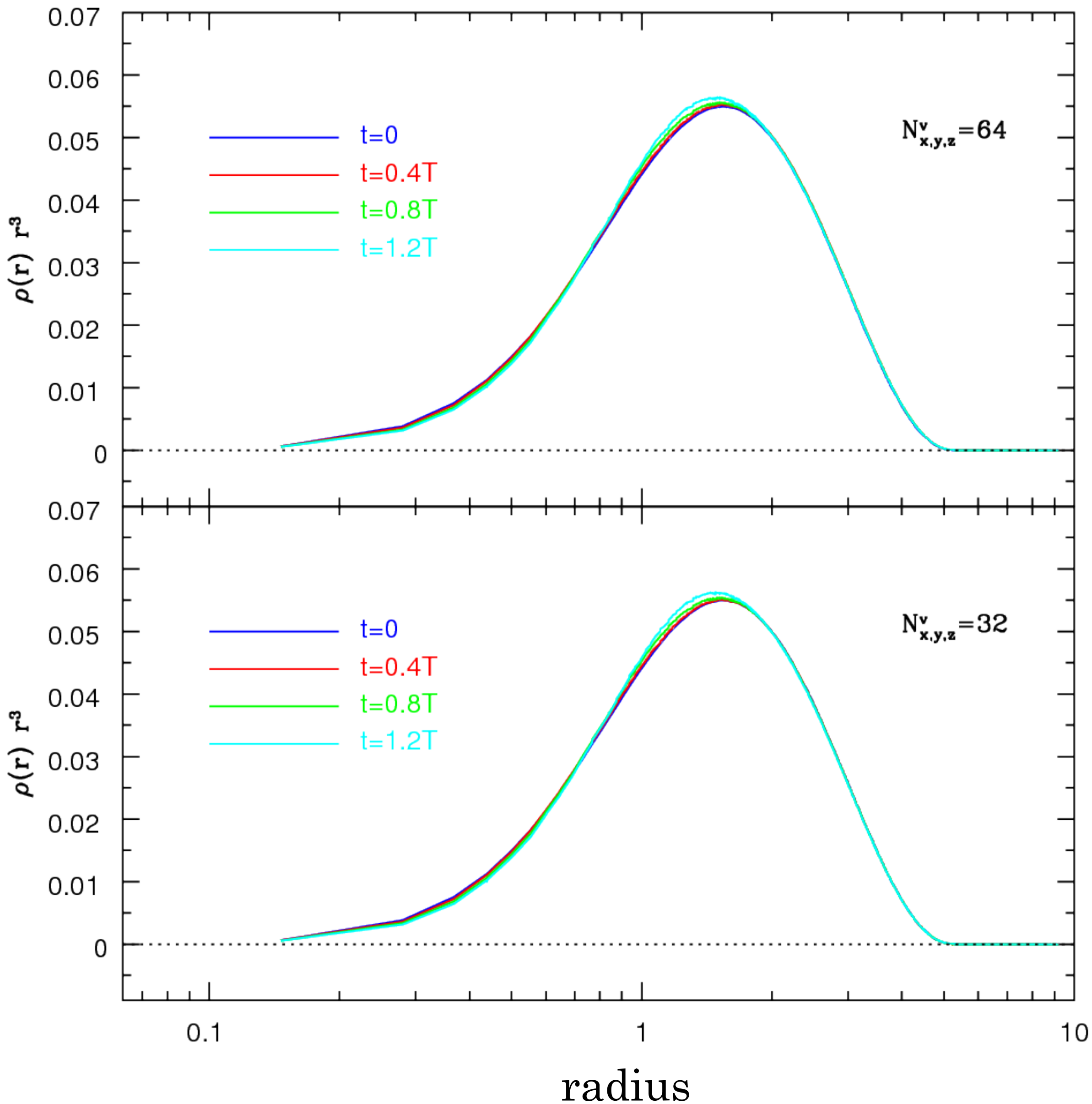


King Sphere



- kinetic and grav. potential energies are almost constant over the dynamical timescale.
- time variation of total, kinetic, and grav. potential energies is sufficiently small (not larger than 1%).
- the total mass is also conserved with sufficiently good accuracy.

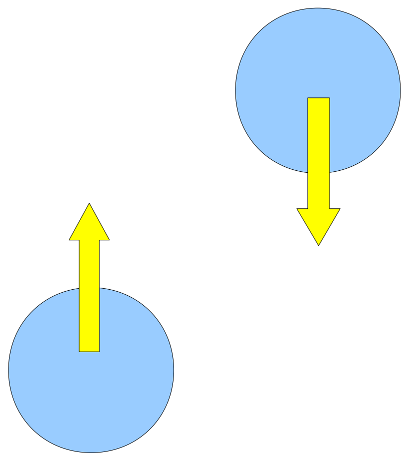
King Sphere



- time evolution of the King sphere
- the profiles almost keep still.
- a little bit of mass transfer from center to outskirts, probably due to poor spatial resolution in the central region.

Merging of Two King Spheres

- initial condition

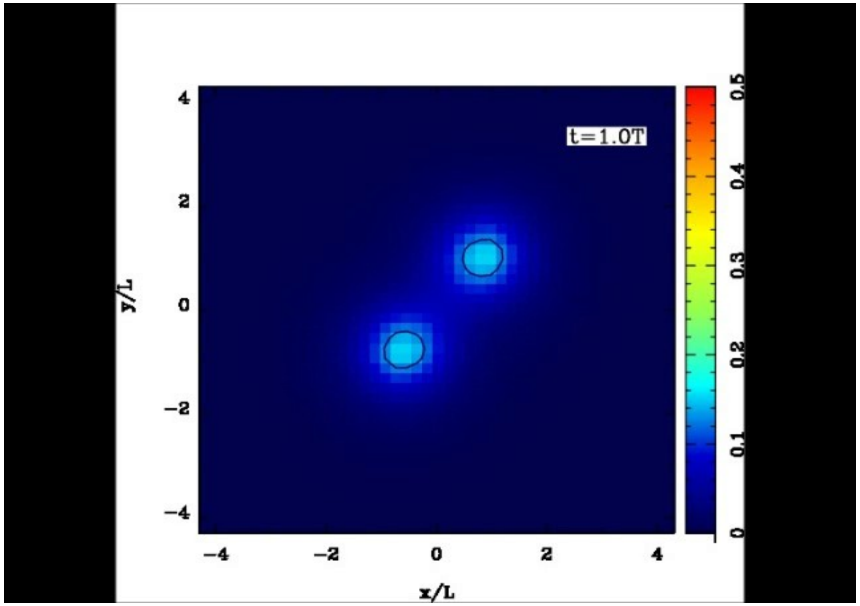


- offset merging of two King spheres
- 64^3 mesh points for both the physical and velocity spaces.

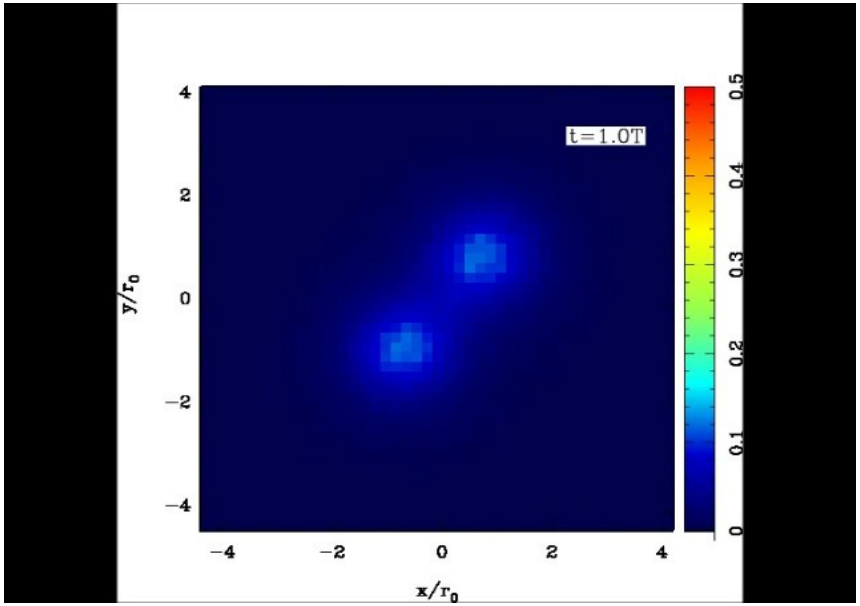
- N-body simulation for the comparison
  - each King sphere is represented with a million particles
  - solved using the Particle – Mesh method with the same spatial resolution as the Vlasov – Poisson simulation.

Merging of Two King Spheres

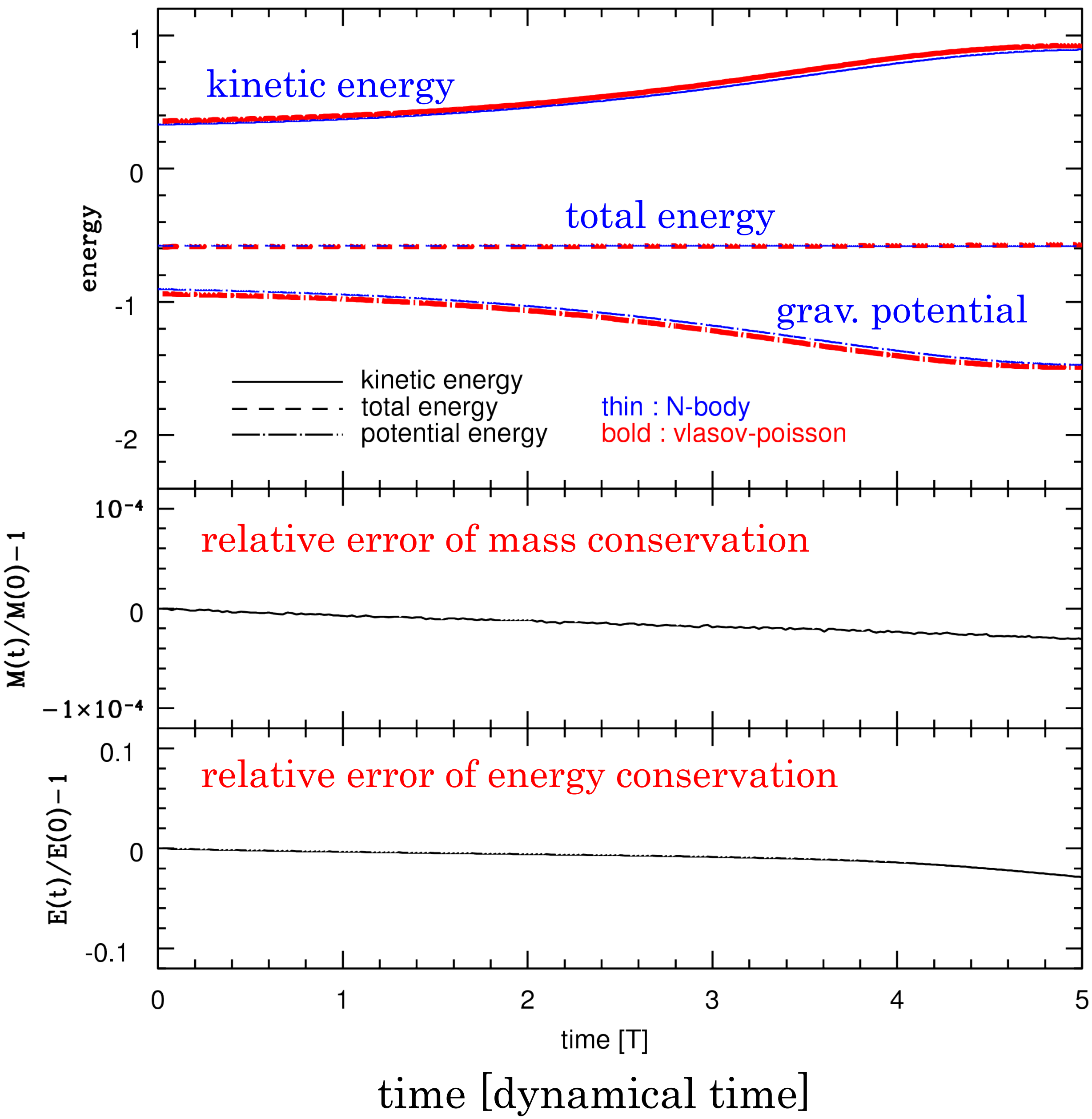
Vlasov simulation



N-body simulation



Merging of Two King Spheres

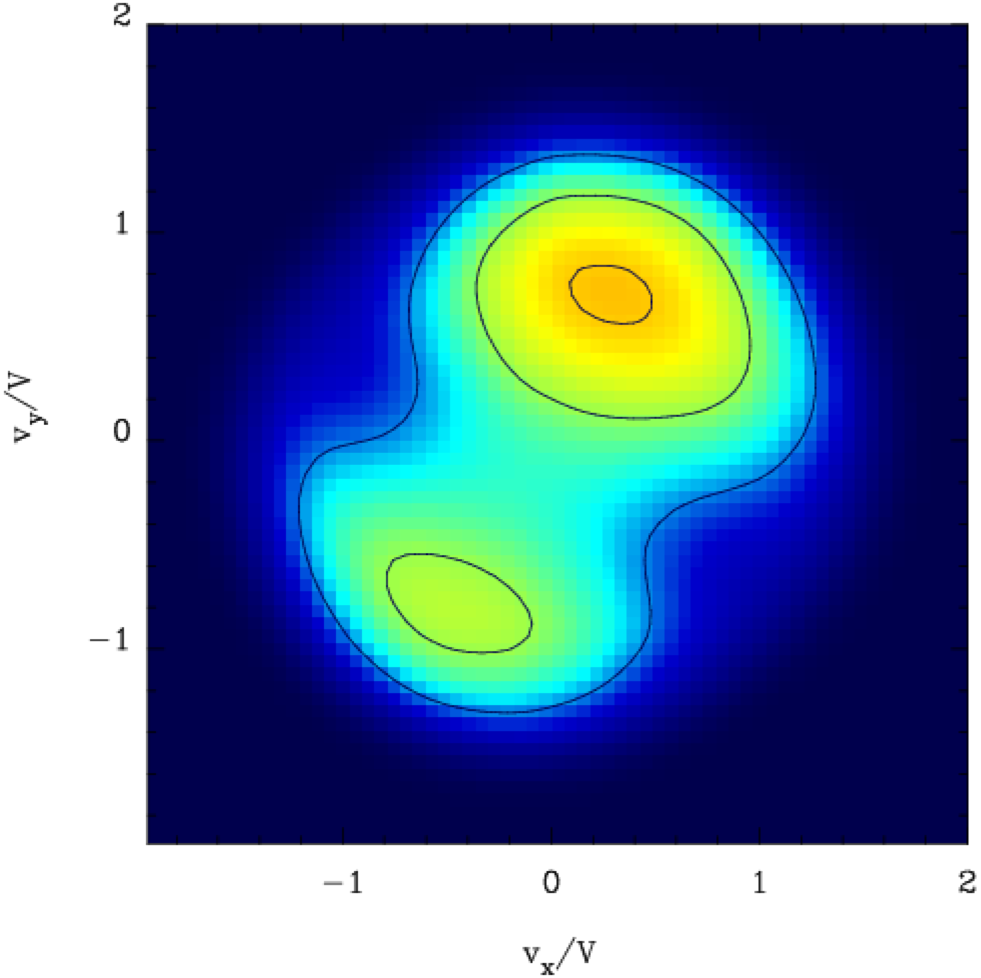


- time evolution of the kinetic, grav. potential and total energy in Vlasov – Poisson and N-body simulations
- good agreement between Vlasov – Poisson simulations and N-body simulations.
- energy conservation is kept within 1% error at t<4.5T.

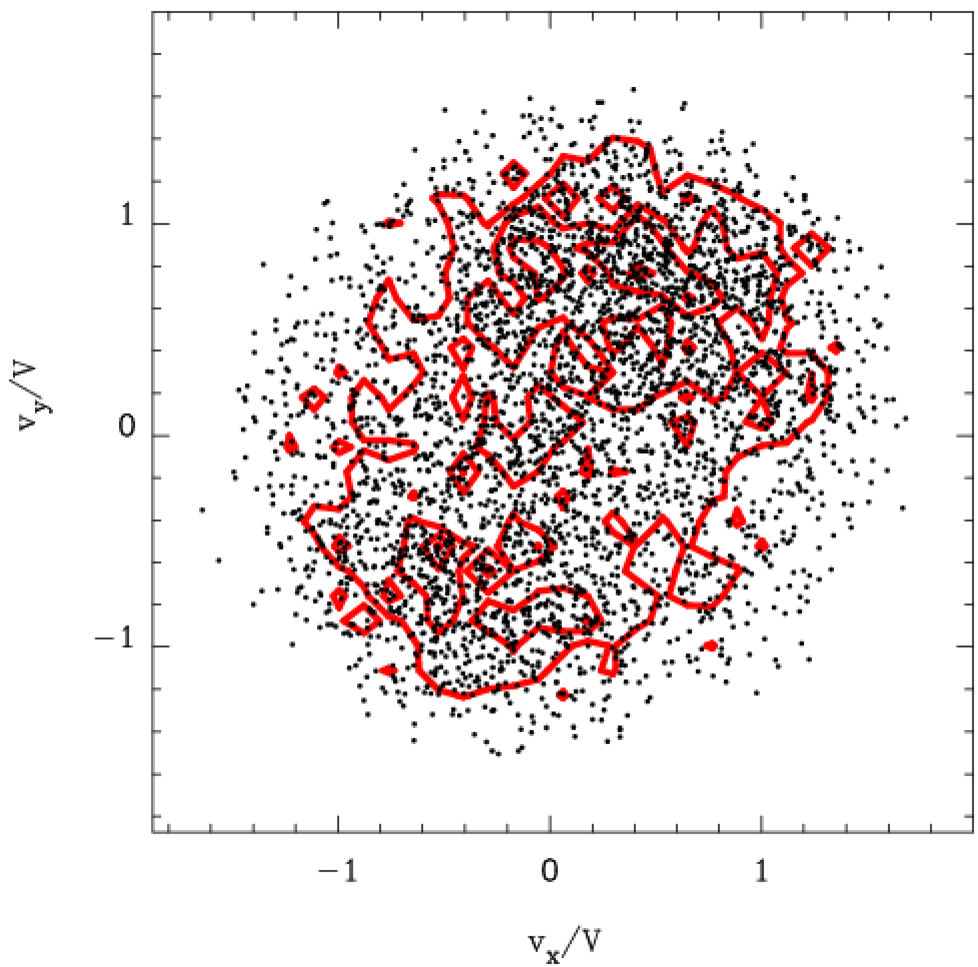
Velocity Distribution

- phase space density in the central region at a time of the closest approach.

Vlasov – Poisson simulation



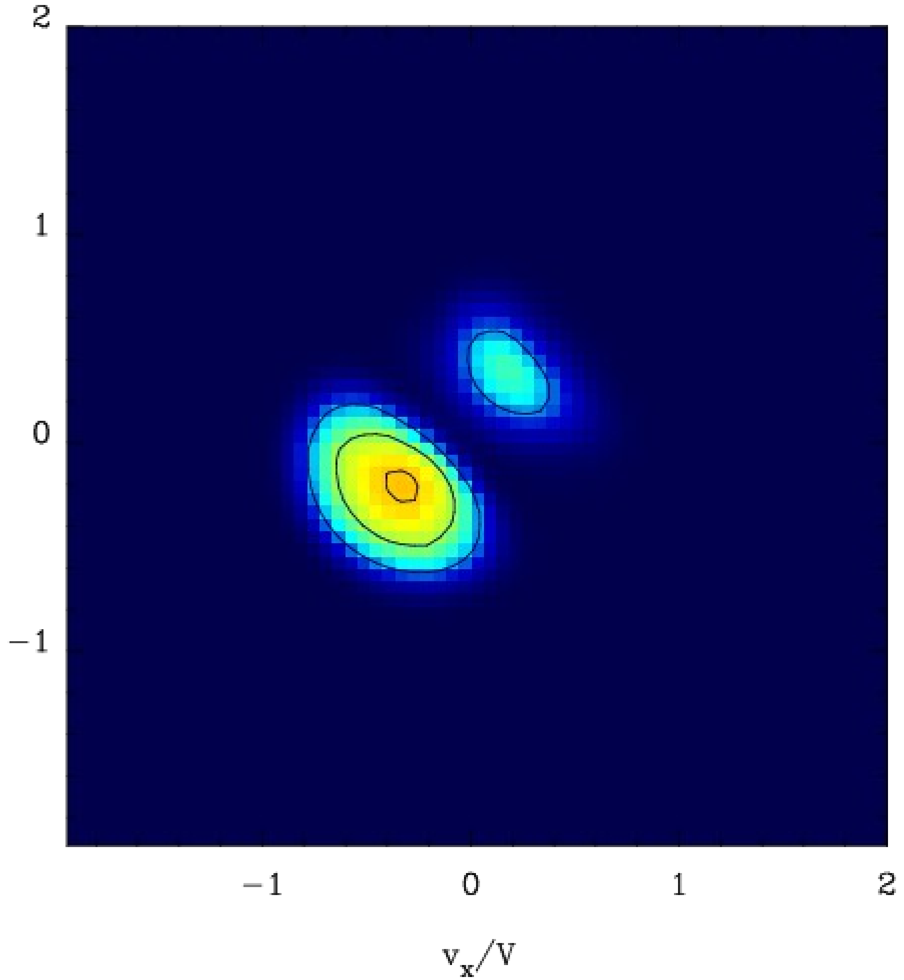
N-body simulation



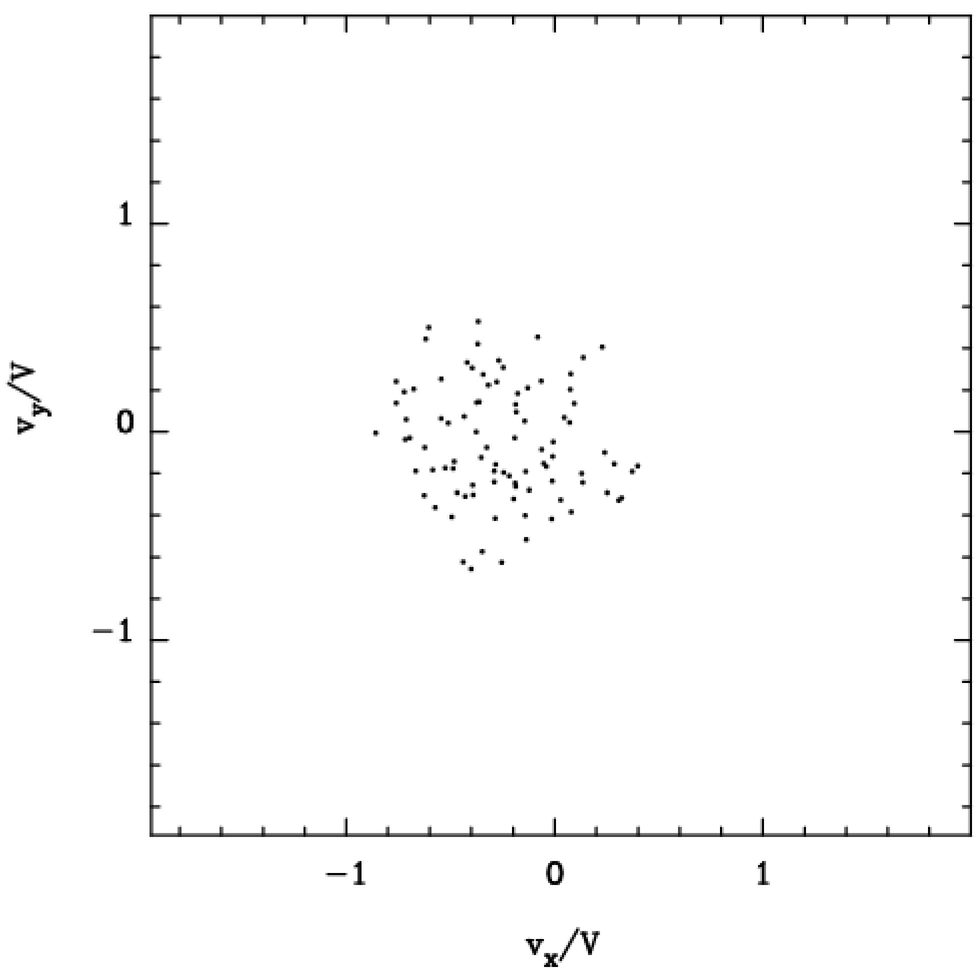
Velocity Distribution

- phase space density in the outskirts at a time of the closest approach.

Vlasov – Poisson simulation



N-body simulation



3D Gravitational Instability and Collisionless Damping

Initial condition

$$\begin{cases} f(\vec{x}, \vec{v}, t=0) = \dfrac{\bar{\rho}(1+\delta(x))}{(2\pi\sigma^2)^{3/2}} \exp\left(-\dfrac{|\vec{v}|}{2\sigma^2}\right) \\ \rho(x, t=0) = \bar{\rho}(1+\delta(x)) \end{cases}$$

- The density fluctuation $\delta(x)$ is given so that it has a white noise power spectrum.
- periodic boundary condition
- Jeans wave number

$$k_{\rm J} = \frac{\sqrt{4\pi G\bar{\rho}}}{\sigma}$$

$k > k_{\rm J}$ → collisionless damping

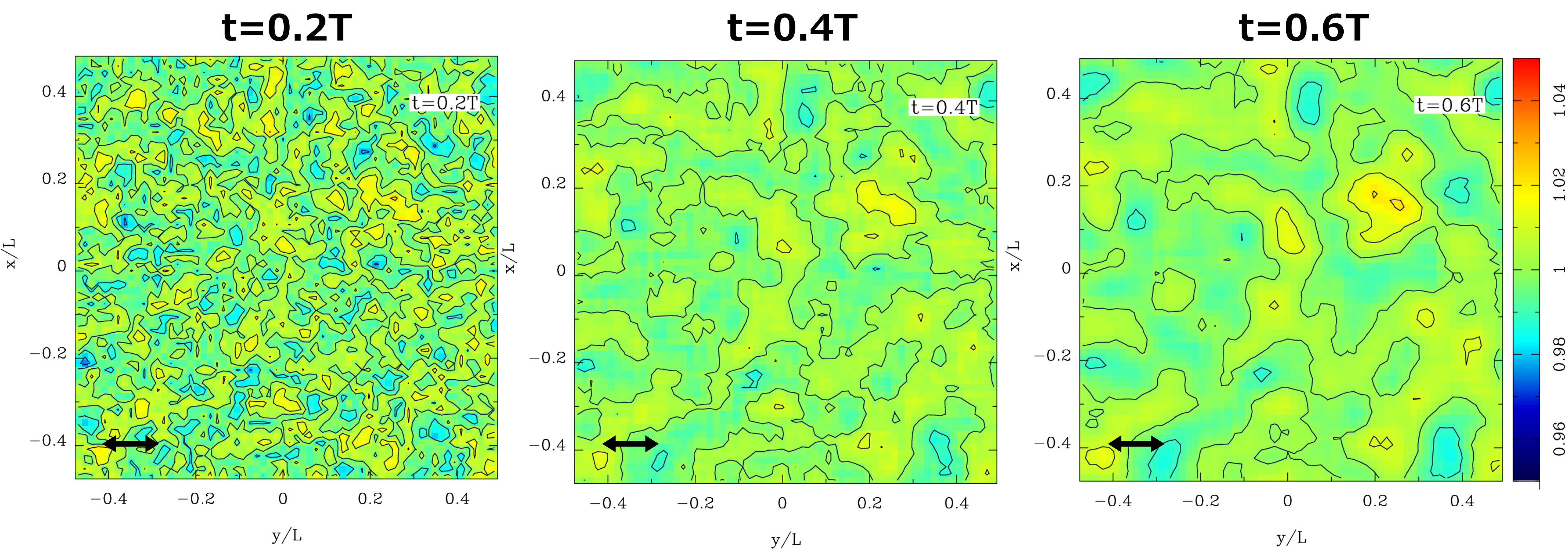
$k < k_{\rm J}$ → gravitational instability

- number of mesh grids

  64^3 for the physical space

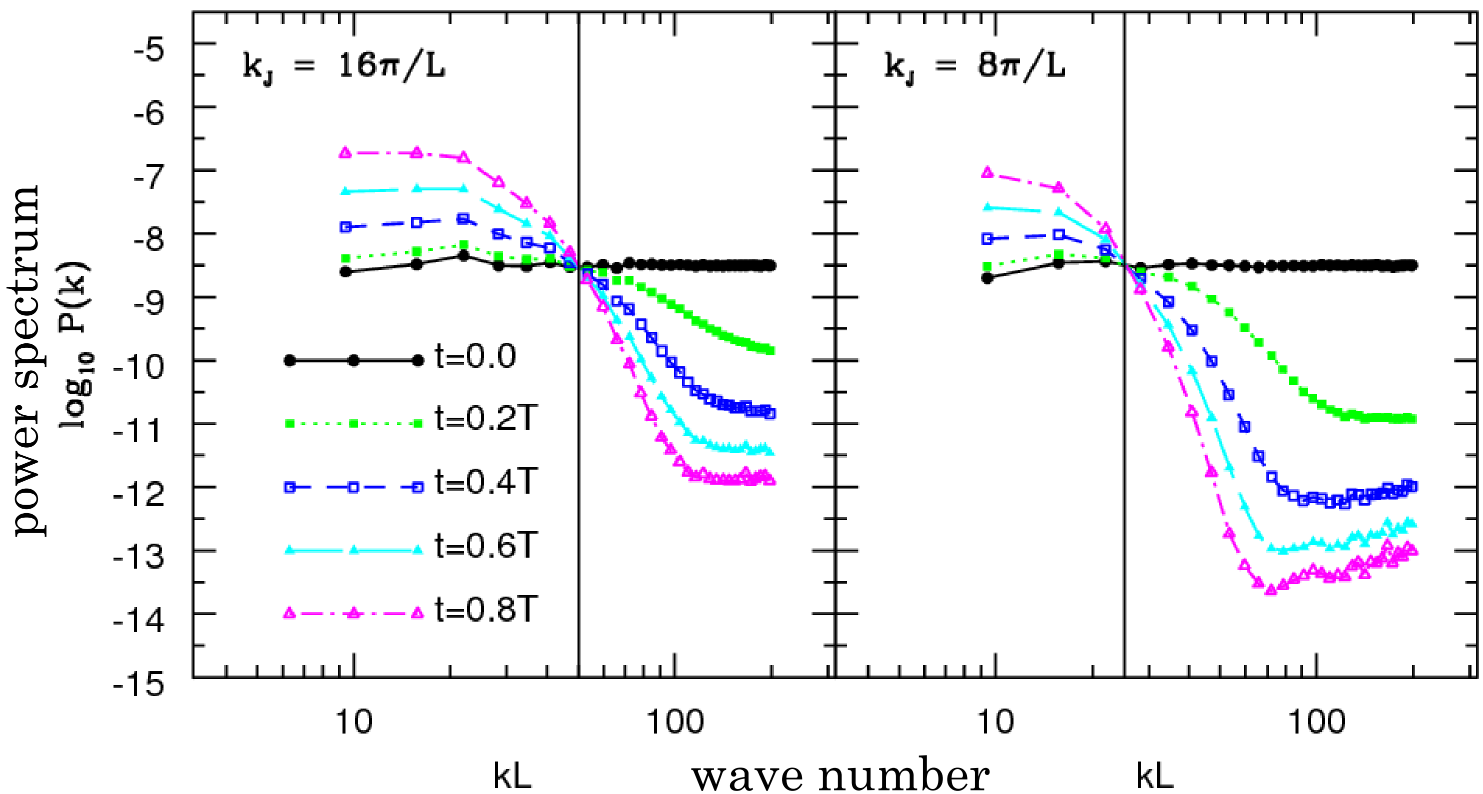
  64^3 or 32^3 for the velocity space

3-D Self-Gravitating System



- time evolution of the density map
- small scale fluctuation at the initial condition damps through the collisionless damping
- density fluctuations with a scale larger than Jeans length grow through the gravitational instability

Power Spectra



- Growth and damping of the density fluctuations switch each other clearly at the Jeans wave number.

Pros and Cons

- The resolution in the velocity space is significantly better than that of N-body simulations

  → Physical processes sensitive to the velocity distribution such as collisionless damping can be simulated acculately.

- It is free from shot noise contamination and artificial effect due to super-particle approximation.

  → suitable to follow the long-term evolution of self-gravitating systems

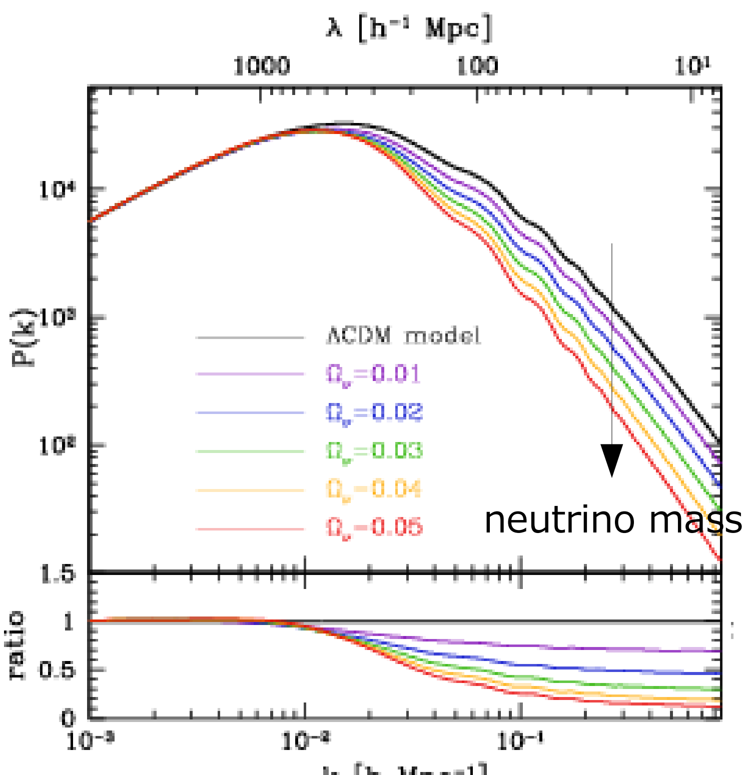
- Current spatial resolution of the Vlasov – Poisson simulation is rather poor compared with the conventional N-body simulations, due to the large amount of computational costs and required memory.

  → needs for hierarchical or adaptive mesh structure

Astrophysical Application of Vlasov-Poisson Simulation

- abundant relic neutrinos with finite mass
- due to low mass of neutrinos, they have very large thermal velocity dispersion

  → collisionless damping of density fluctuation in the large-scale structure in the universe



- scale and depth of the damping are the measures of the neutrino absolute mass

- hybrid of N-body + Vlasov simulations

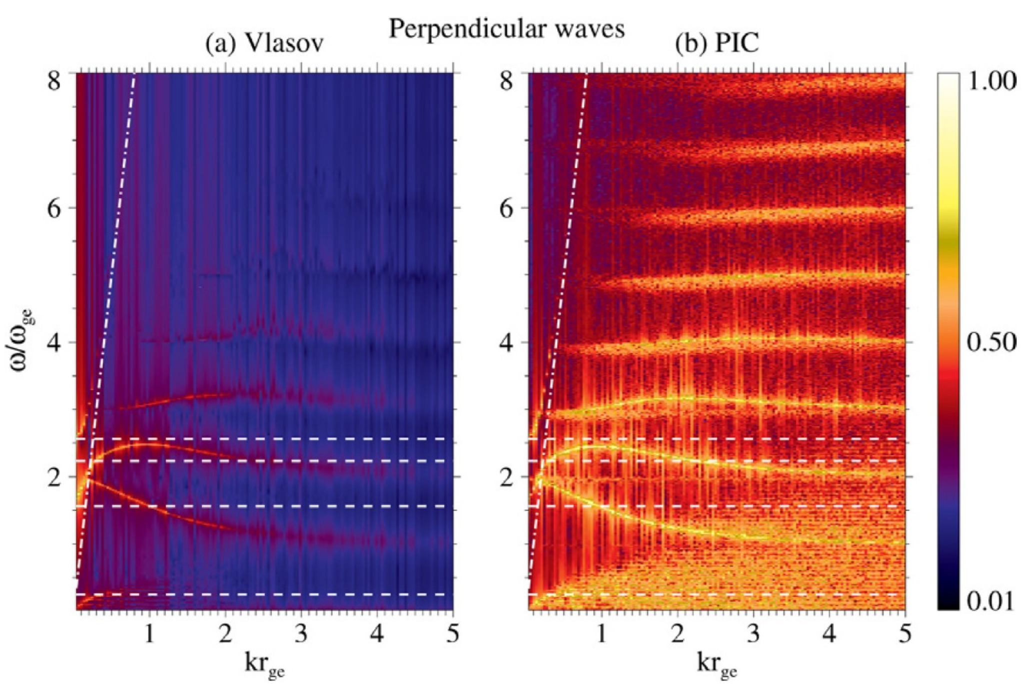
  neutrino : Vlasov-Poisson simulation

  Cold Dark Matter : N-body simulation

Vlasov-Maxwell Simulation

$$\begin{cases} \dfrac{\partial f}{\partial t} + \vec{v} \cdot \dfrac{\partial f}{\partial \vec{x}} + q(\vec{E} + \vec{v} \times \vec{B}) \cdot \dfrac{\partial f}{\partial \vec{p}} = 0 \\ \nabla \cdot \vec{E} = \rho / \epsilon_0 \qquad \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \dfrac{\partial \vec{E}}{\partial t} \right) \end{cases}$$

- counter part of the Particle-In-Cell (PIC) method for plasma simulations



- The electron cyclotron (Bernstein) modes can be clearly seen in Vlasov simulations
- The results in PIC simulations are contaminated by shot noise and fake signals

Minoshima et al., JCP (2012) 230, 6800

Summary

- 3D Vlasov-Poisson simulations of self-gravitating systems in the 6D phase space volume as an alternative to N-body simulations
- Vlasov-Poisson simulations and N-body simulations are complementary with each other.
- Collisionless damping of the large-scale structure by cosmic relic neutrinos is one of the suitable applications of Vlasov-Poisson simulations.
- Vlasov simulations can be applied to plasma simulations as a replacement of PIC simulations