

PACS-CSにおける物性物理学研究

ExaFLOPS 201x -

K 2011 -

T2K 2008 -

PACS-CS 2005 - 2007

CP-PACS 1996 -

QCDPAX 1989

SX9 2007

SX8 2004

SX6 2001

SX5 1998

(ES)

SX4 1994

SX3 1989

SX2 1983
NEC

2007 -

U Tokyo

1995 - 2007 U Tsukuba

↑ CNT, Si, ...

*Days of vectorization
and computations*

1985 - 1995 NEC

1983 - 1984 IBM
myself



Lattice QCD

PACS-CSにおける物性物理学研究

ExaFLOPS 201x -

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CP-PACS 1996 -

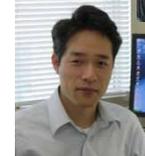
Lattice QCD

QCDPAX 1989

Real-Space
Density-
Functional-
Theory
(RSDFT) Code for
Unprecedentedly
Large-Scale
Calculations

2007 -

U Tokyo



1995 - 2007 U Tsukuba



CNT, Si,...

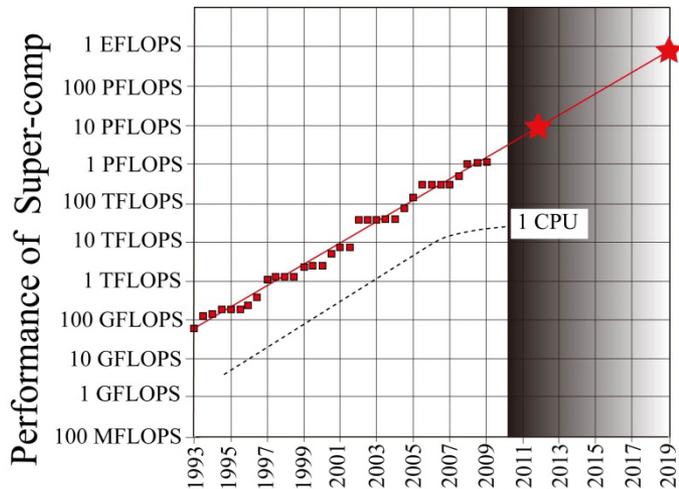
*Days of vectorization
and computations*

1985 - 1995 NEC

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myself

Computics: Fusion of Materials Science and Computer Science

Computers that are changing rapidly



- ✓ Saturation of performance of a single computation processor
Breakdown of Moore's Scaling Law
- ✓ Multi-core massively parallel architecture in the next generation
Kobe-K: 80,000CPU x 8 cores
- ✓ Accelerator introduced in the next next generation
Supercomputer that is a monster

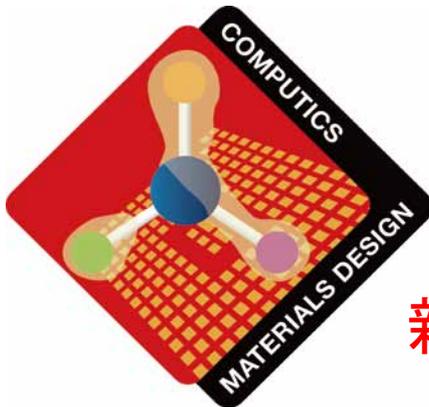
Collaboration between Computational Materials Science and Computer Science is Imperative

- Developments of Mathematical Methodology and Algorithms Suitable to Current Hard wares
- Tuning of High Performance Computation codes based on deep knowledge of hard wares

Computics: Fusion of Materials Science and Computer Science

Computics,

A New Approach to
Materials Science in 21st Century,
in addition to Mathematics
that plays an important role from
Newton's Principia



新学術領域研究: <http://computics-material.jp/>

Development and Status of Computational Materials Science

80s ~ Success of Computational Science Approach

Density Functional Theory (DFT) (W. Kohn: Nobel Prize 1998)

Car-Parrinello Molecular Dynamics (CPMD) (1985)

Various Developments in Quality and Quantity

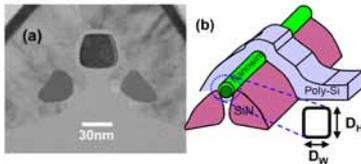
Clarification and Prediction of Material Properties and their Dynamics based on First Principles of Quantum Theory

90s ~ Limitation of DFT (Strongly Correlated System, Excitation Spectrum)

From Now On!

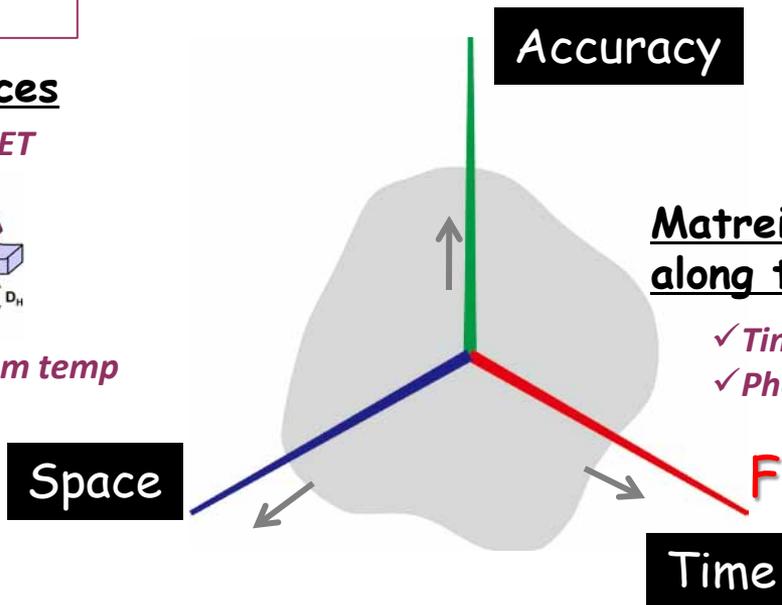
Future in nano devices

Next-Generation NWFET



Quantum effects at room temp

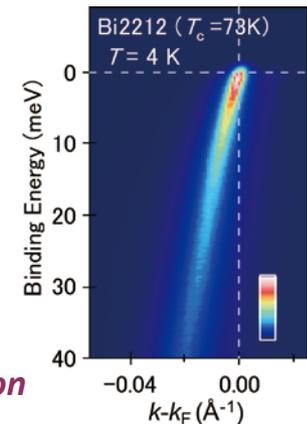
Larger scale:
Nano world =
100,000-atom world



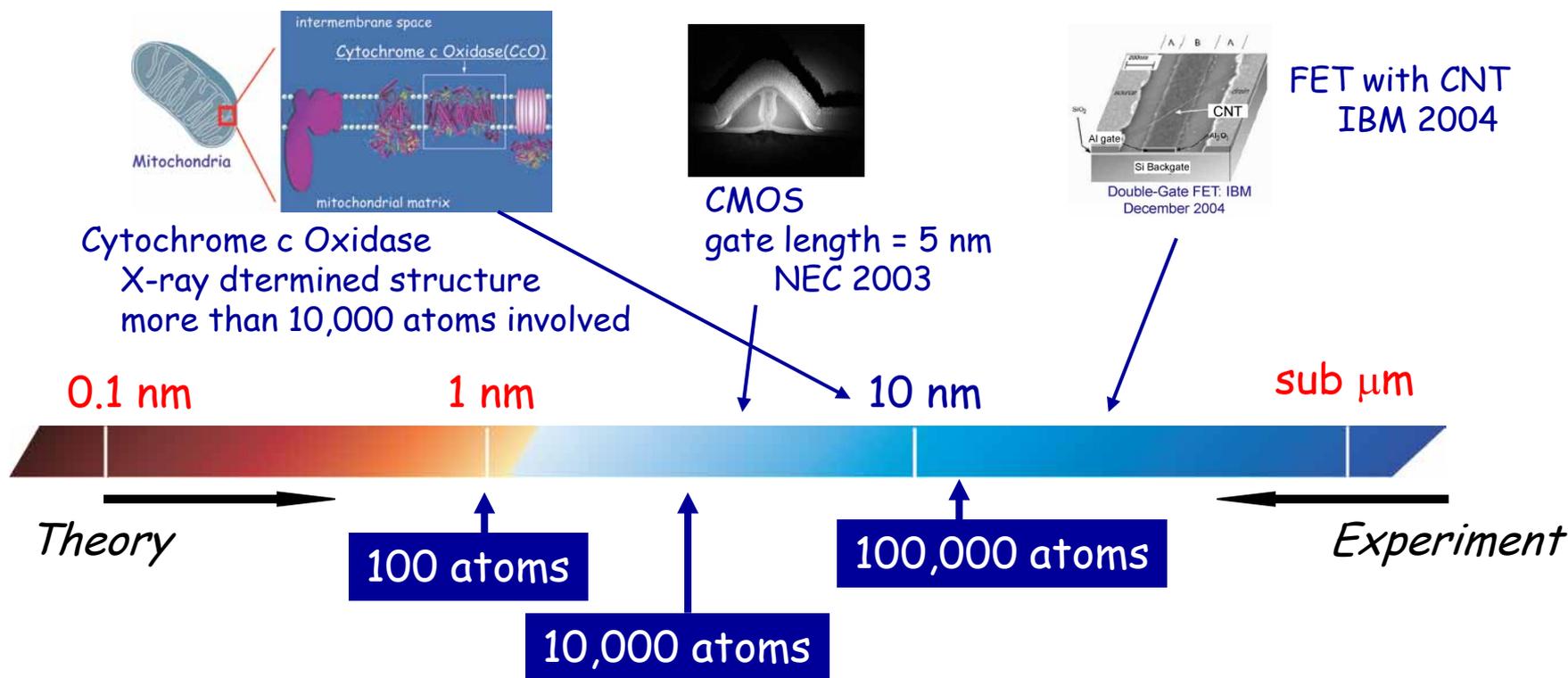
Materials design along time axes

- ✓ *Time-resolved ARPES*
- ✓ *Photo-induced function*

From sub-Femto sec with electrons to Nano sec with ions



Why Large-Scale Density Functional (DFT) Calculations?



Quantum-theory-based large-scale DFT calculations and experiments meet together in Nano World !

Density Functional Theory (DFT)

Total energy of a material is a functional of electron density:

$$\begin{aligned} E[n(\mathbf{r})] &= \langle \Psi | \mathbf{H} | \Psi \rangle = \langle \Psi | \mathbf{T} + V_{\text{nucl}} + V_{\text{ee}} | \Psi \rangle \\ &= T_{\text{S}}[n(\mathbf{r})] + \int v_{\text{nucl}}(\mathbf{r})n(\mathbf{r})d\mathbf{r} + \frac{1}{2} \iint \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}d\mathbf{r}' + E_{\text{XC}}[n(\mathbf{r})] \end{aligned}$$

When the electron density is written as $n(\mathbf{r}) = \sum_{i: \text{occupied}} |\varphi_i(\mathbf{r})|^2$

we obtain Euler equation (Kohn-Sham) equation,

$$\left[-\frac{1}{2}\nabla^2 + v_{\text{eff}}(\mathbf{r}; n(\mathbf{r})) \right] \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r}) \quad v_{\text{eff}}(\mathbf{r}) = v_{\text{nucl}}(\mathbf{r}) + \int \frac{n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}' + \frac{\delta E_{\text{XC}}[n]}{\delta n(\mathbf{r})}$$

Forces acting on nuclei \mathbf{R} are calculated as

$$F_u = -\frac{d}{dR_u} (E[n] + E_{\text{nucl-nucl}}) = \underbrace{\left[\frac{\partial E}{\partial R_u} \right]_{\varphi}}_{\text{Hellmann-Feynman Force}} + \sum_i \int \underbrace{\left(\frac{\delta E}{\delta \varphi_i} \frac{d\varphi_i}{dR_u} + \frac{\delta E}{\delta \varphi_i^*} \frac{d\varphi_i^*}{dR_u} \right)}_{\text{Pulay Correction}} d\mathbf{r}$$

Solving Kohn-Sham Equation

Introducing complete basis set $\chi_n(\mathbf{r})$ to expand Kohn-Sham wave function:

$$\varphi_j(\vec{r}) = \sum_n c_{jn} \chi_n(\vec{r})$$

to convert the differential equation to a matrix equation, and then solve it by iterative techniques. Most convenient basis set has been plane wave basis set:

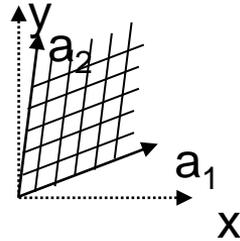
$$\chi_n(\vec{r}) = e^{i(\vec{k} + \vec{G}_n) \cdot \vec{r}}$$

In the plane-wave scheme, several quantities such as $n(\mathbf{r})$, $v(\mathbf{r})$ etc is Fourier-transformed and Inverse-Fourier-transformed frequently.

FFT: a heavy communication burden on parallel-architecture computers

Solving Kohn-Sham Equation Using Real-Space Finite-Difference Scheme

Introducing mesh points with spacing H in real space,

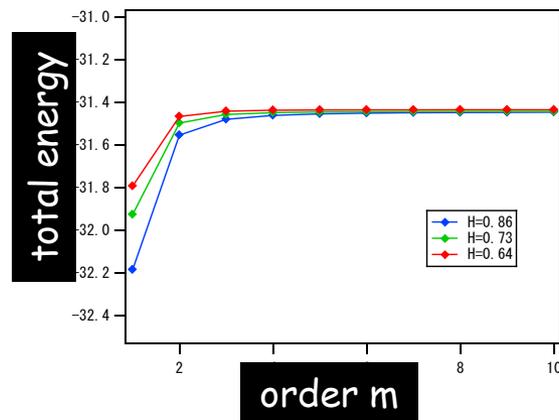


Kohn-Sham differential equation is converted to a **m-th** order finite-difference equation:

$$\frac{\partial^2}{\partial x^2} \varphi_j(x, y, z) = \sum_{m=-M}^M c_m \varphi_j(x + mH, y, z)$$

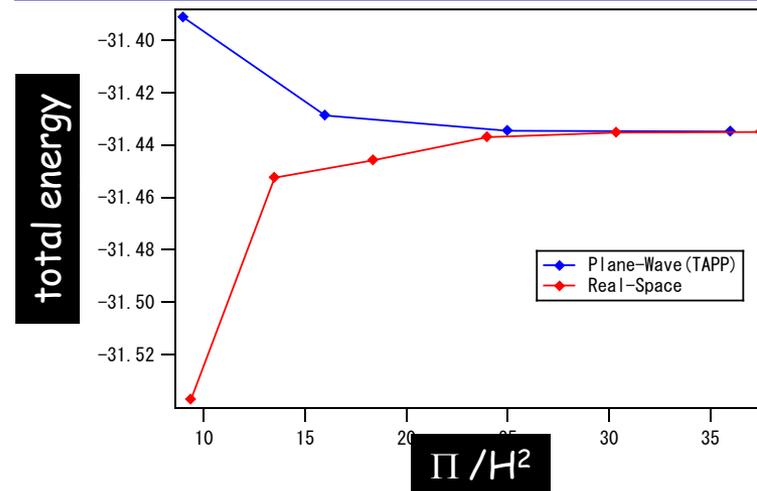
All the quantities computed at each mesh point, KS Hamiltonian expressed as a matrix.

Check of finite difference



Check of mesh spacing

⇒ systematic accuracy improvement

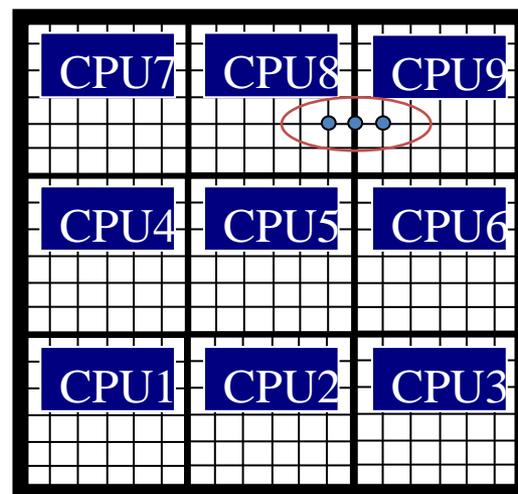


Advantages of Real-Space DFT (RSDFT) in Parallel Computing

- Almost free from FFT, reducing communication burden
 - ⇒ high efficiency
- Flexible boundary condition to wave-functions
 - ⇒ targets expanded including charged objects
- Utilize inherent locality of the system
 - ⇒ prospect to efficient $O(N)$ scheme

On multi-core parallel machines:

- Huge number of 3D mesh points divided into moderate-size cells
- Each cell treated by a single CPU
- Also, electron states are grouped and each group is treated by a single CPU or a core : **Hybrid parallelization**
- **MPI for CPU and OpenMP for core**



Some Details and Their Optimization

- Residual minimization as well as conjugate-gradient minimization to solve KS equation
- Change of algorithm for Gram-Schmidt to utilize BLAS*-level3 matrix-matrix operations
- Divide-and-Conquer algorithm to solve secular equation in subspace

SCF(get-selfconsistent-field) loop

Get-KS wavefunction-for-a-fixed-potential loop $O(N^2)$

conj-grad or residual-min

gram-schmidt orthonormalization $O(N^3)$

diagonalization of a partial matrix $O(N^3)$

(occupied states)

potential (charge) updated

Get-wavefunction loop

conj-grad

or

residual-min

minimize $\langle \phi | H | \phi \rangle$

or

minimize $\| H | \phi \rangle - e | \phi \rangle \|^2$

Hamiltonian

Hamiltonian

operation

operation

gram-schmidt $O(N^3)$

required

residual vector

next guess

& next guess

Order N^3 Calculations, but the N^3 part is highly optimized!

* BLAS = Basic Linear Algebra Subprograms

Gram-Schmidt Orthonormalization: Active use of Level 3 BLAS in $O(N^3)$ Computation

$$\psi'_1 = \psi_1$$

$$\psi'_2 = \psi_2 - \psi'_1 \langle \psi'_1 | \psi_2 \rangle$$

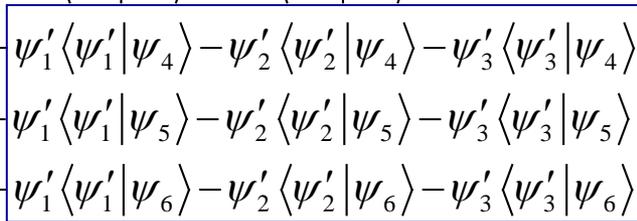
$$\psi'_3 = \psi_3 - \psi'_1 \langle \psi'_1 | \psi_3 \rangle - \psi'_2 \langle \psi'_2 | \psi_3 \rangle$$

$$\psi'_4 = \psi_4 - \psi'_1 \langle \psi'_1 | \psi_4 \rangle - \psi'_2 \langle \psi'_2 | \psi_4 \rangle - \psi'_3 \langle \psi'_3 | \psi_4 \rangle$$

$$\psi'_5 = \psi_5 - \psi'_1 \langle \psi'_1 | \psi_5 \rangle - \psi'_2 \langle \psi'_2 | \psi_5 \rangle - \psi'_3 \langle \psi'_3 | \psi_5 \rangle - \psi'_4 \langle \psi'_4 | \psi_5 \rangle$$

$$\psi'_6 = \psi_6 - \psi'_1 \langle \psi'_1 | \psi_6 \rangle - \psi'_2 \langle \psi'_2 | \psi_6 \rangle - \psi'_3 \langle \psi'_3 | \psi_6 \rangle - \psi'_4 \langle \psi'_4 | \psi_6 \rangle - \psi'_5 \langle \psi'_5 | \psi_6 \rangle$$

Part of the calculations can be performed as Matrix x Matrix operations!



Performance of Gram-Schmidt Routine
(on 1000 CPUs of Tsukuba PACS-CS)

Theoretical Peak Performance	Our RSDFT (Operation)	Our RSDFT (Operation & Communication)
5.6 GFLOPS/cpu	4.3 GFLOPS/cpu	3.5 GFLOPS/cpu

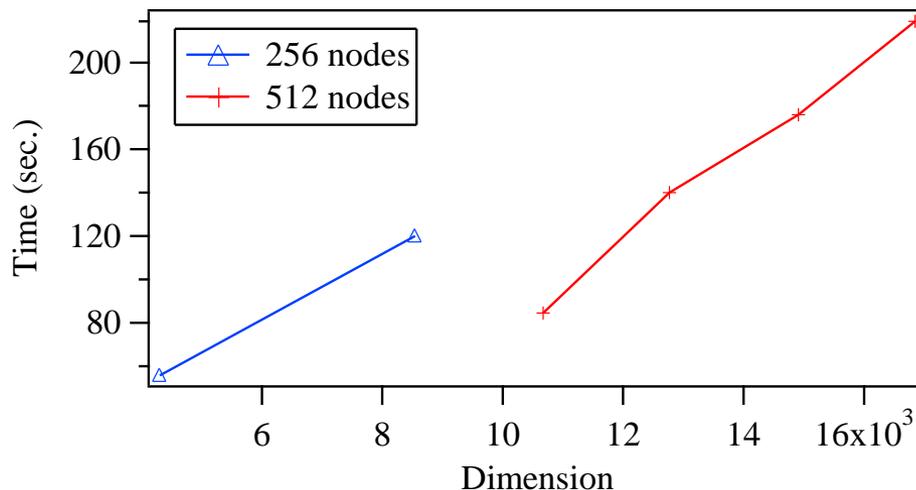
$O(N^3)$ part computed at 80% of the theoretical peak performance!

Divide and Conquer for Subspace Diagonalization

1. Calculation of Matrix Elements

BLAS-3

2. Diagonalization



PDSYEVD
(Solver in SCALAPACK)
using Divide & Conquer

3. Vector (Wavefunction) Rotation

BLAS-3

Now, Imamura Algorithm
In future,
Sakurai-Sugiura Algorithm

RSDFT on K(京) at Kobe

- ✓ 100,000-atom DFT calculations using hundreds of thousands cores with unprecedentedly efficient performance have been done
- ✓ 10,000-atom DFT calculations have become just an overnight task

In collaboration with,

Jun-ichi Iwata (Univ Tokyo)

Yukihiro Hasegawa (Riken)

Miwako Tsushi (Univ Tsukuba)

Daisuke Takahashi (Univ Tsukuba)

Kazuo Minami (Riken)

Taisuke Boku (Univ Tsukuba)

Shinnosuke Furuya (Univ Tokyo)

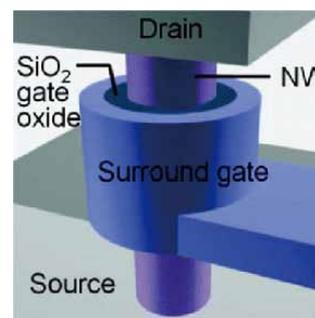
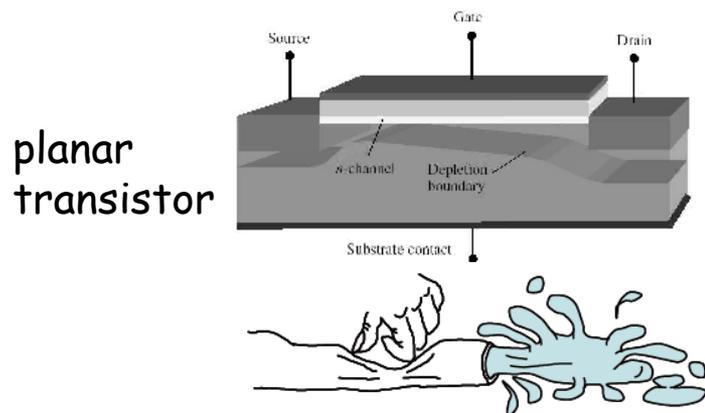
Mitsuo Yokokawa (Riken)

*'''
who are in the fields of
materials science & computer science.*



Prediction of Electron States of Si Nanowires

*Si Nanowire, a Booster
in the next-generation semiconductor technology*
More Moore → More than Moore



Surrounding gate transistor

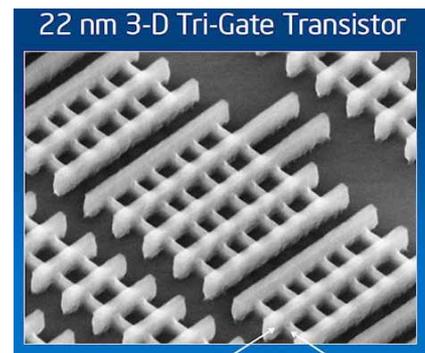


Gate Controllability

→ Suppress short-channel effects

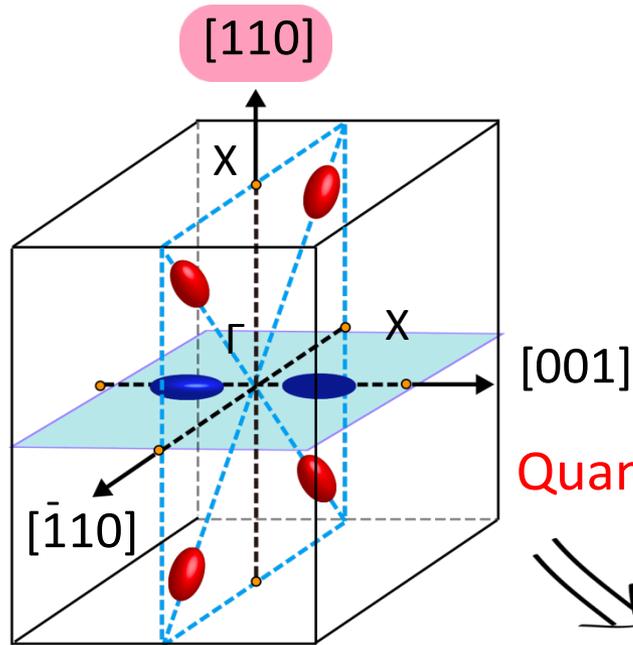
Suppress leaks at off state

→ save energy



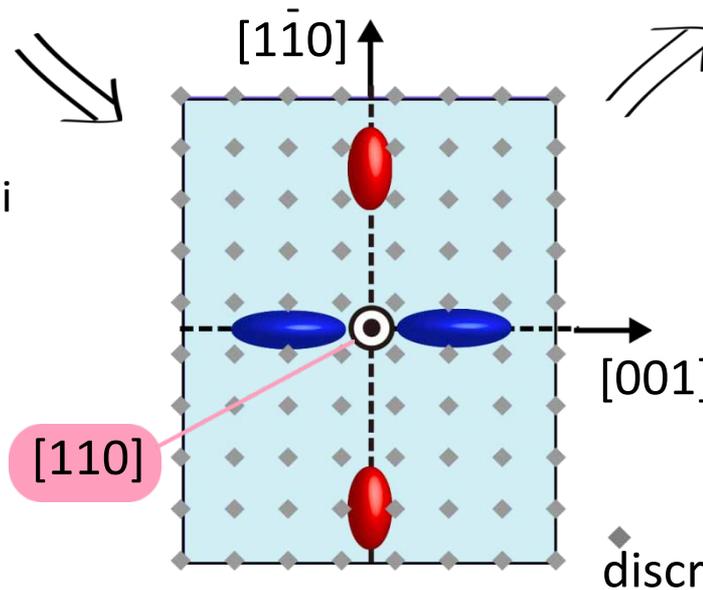
*Actually tri-gate by
Intel in 2011*

Effects of quantum confinement of [110]-SiNWs



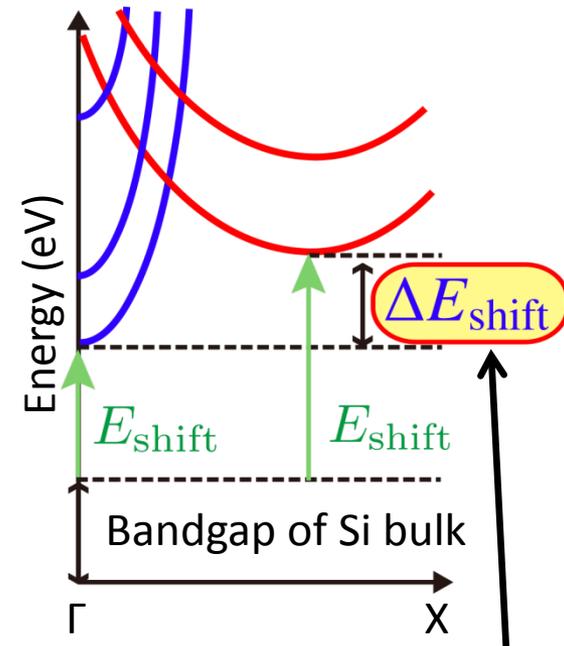
Brillouin Zone (BZ) of bulk Si and six conduction band valleys

Quantum confinement effects



Positions of allowed wave numbers depend on cross-sectional morphology

Conduction band structure of [110]-SiNW

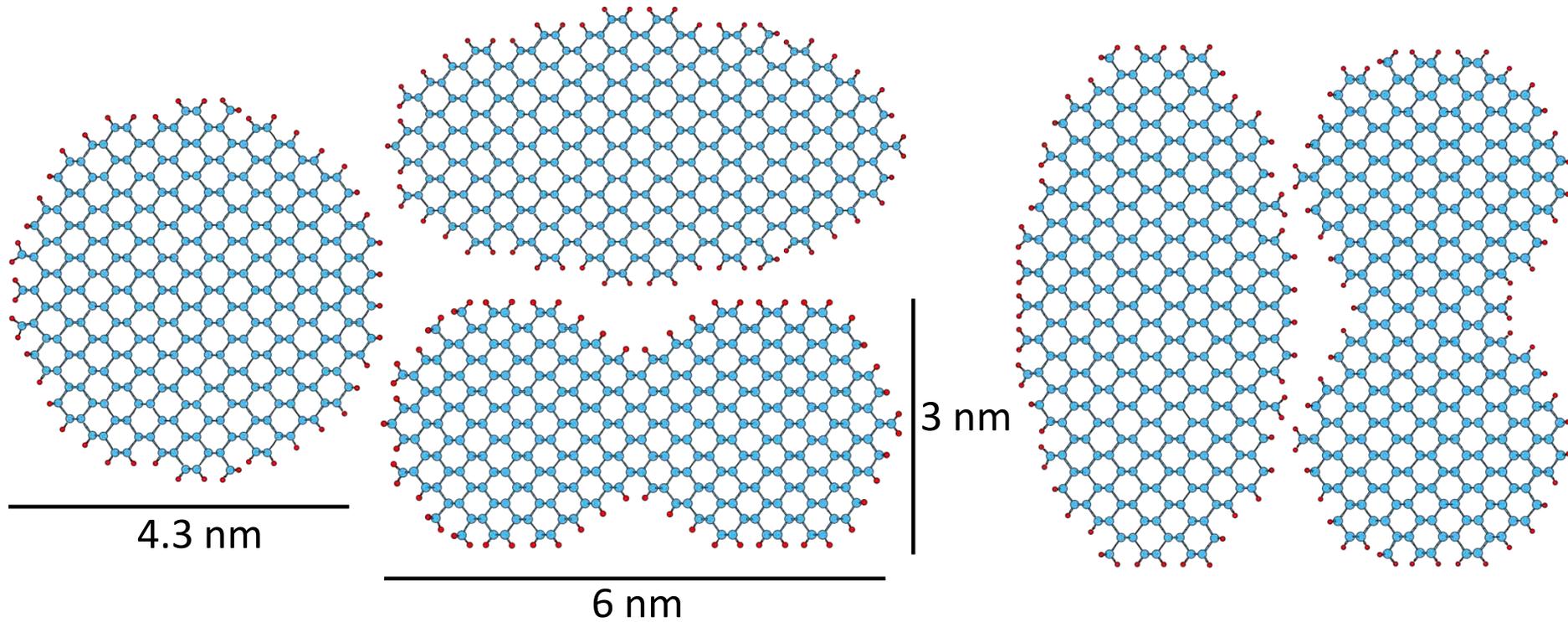


This quantity is decisive in the number of the conduction channels.

Cross Sectional Shapes, Diameters, Directions,,, What is the best Choice?

$[110]$
 $[110]$ $[001]$

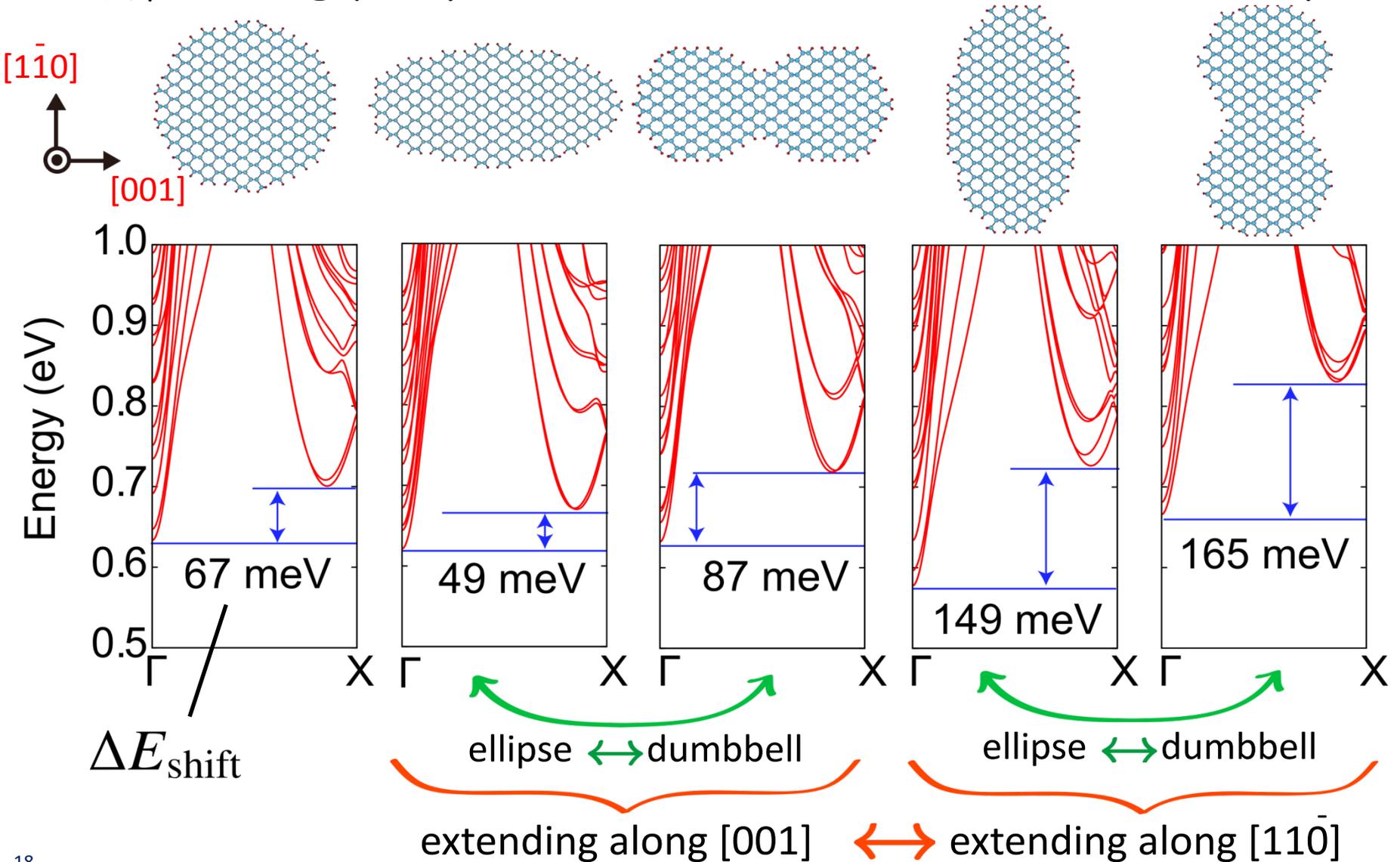
Considering Si (110) Nano Wire,
Directions inequivalent to each other



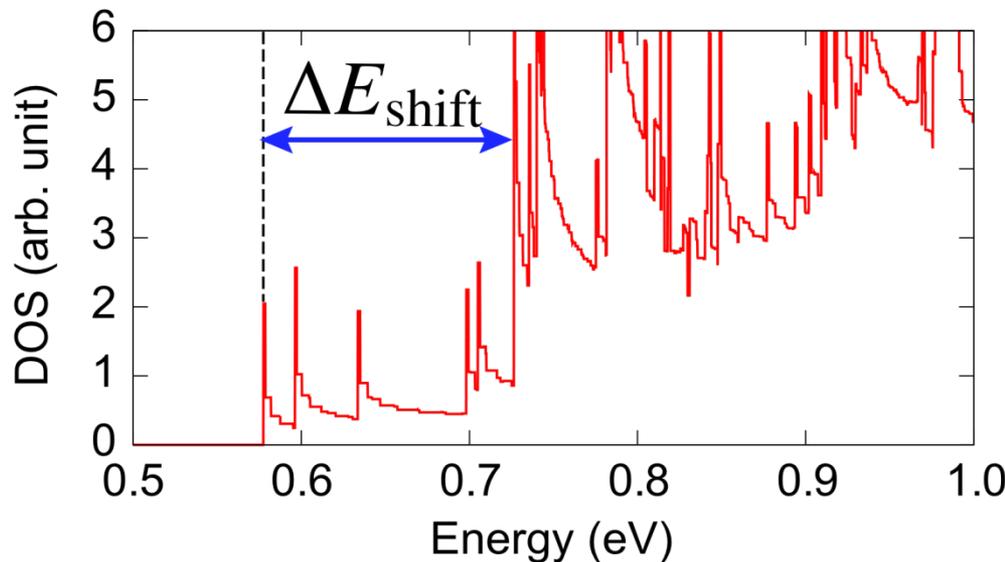
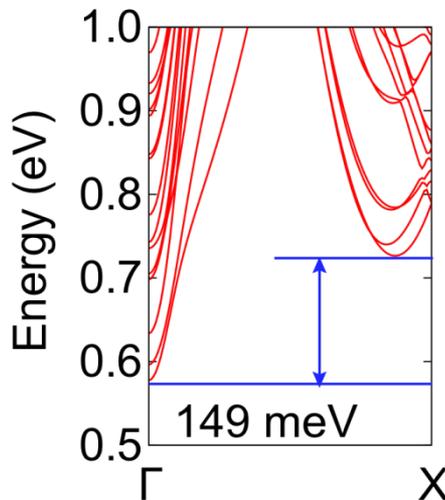
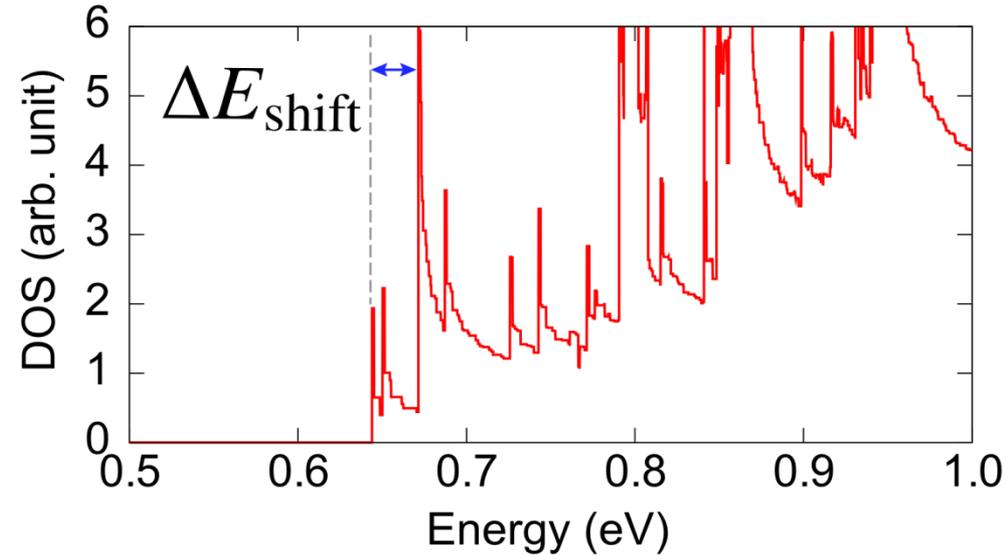
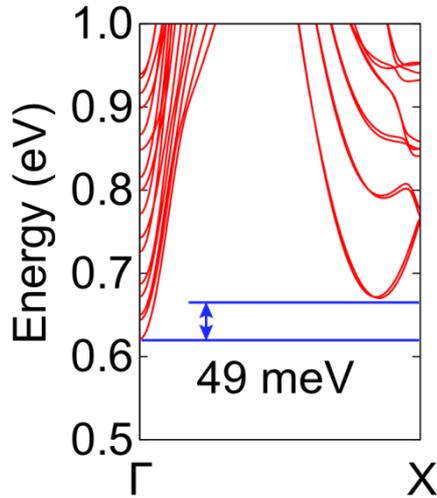
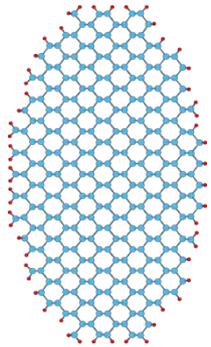
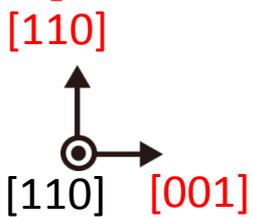
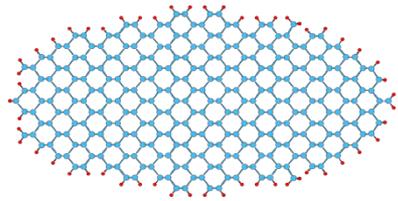
DFT could Give the Answer

Conduction Band Structures of Various Si(110) NWs

ΔE_{shift} strongly depends on the cross-sectional nano-shapes



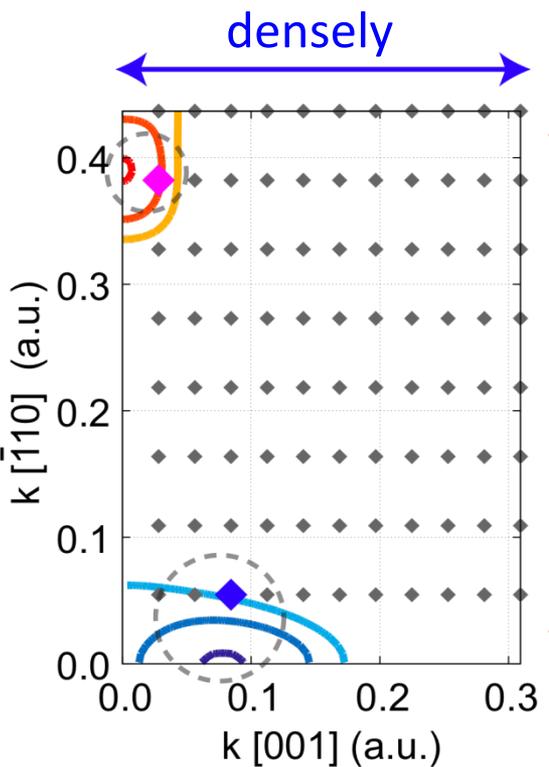
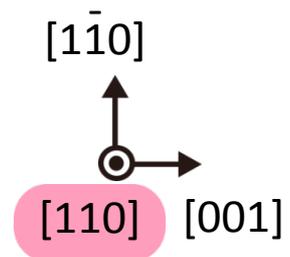
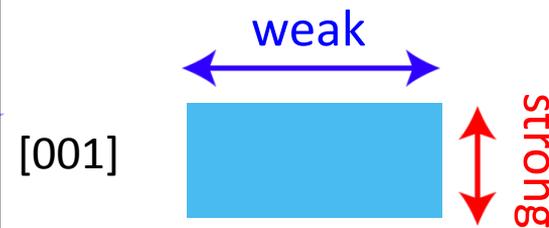
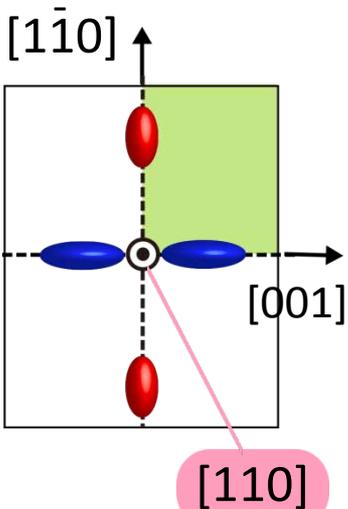
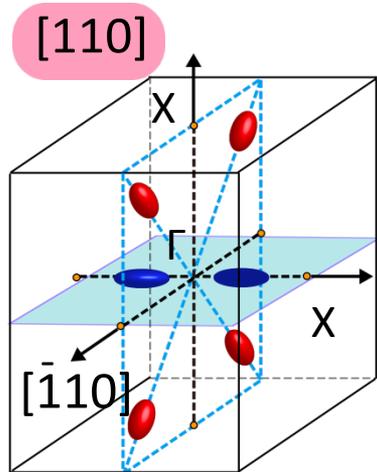
Density of states of SiNWs



The number of channels near the CBM is strongly related to ΔE_{shift}

Interpretation of calculated results

- ◆ discrete k
- ◆ CBM
- ◆ Second minimum



- ◆ CBM
- ◆ Second minimum
- ◆ ΔE_{shift}

Correspond to the results from our first-principle calculations

SiNWs with sidewall roughness

Roughness makes the radius of NWs vary along circumference and wire axis.

⇒ Lorentzian autocorrelation function

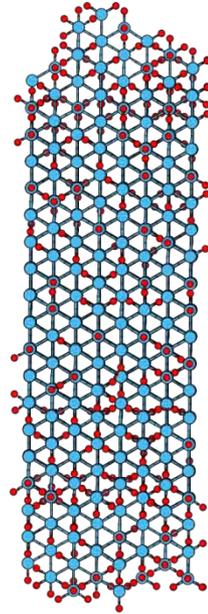
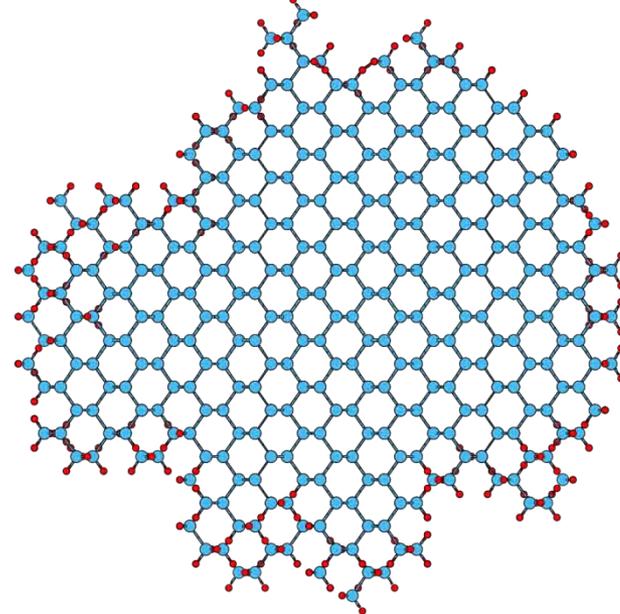
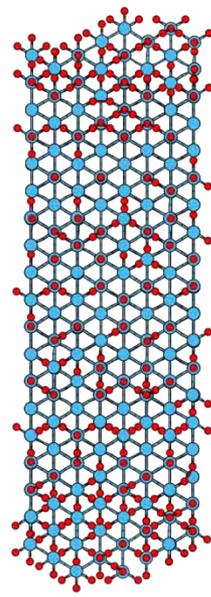
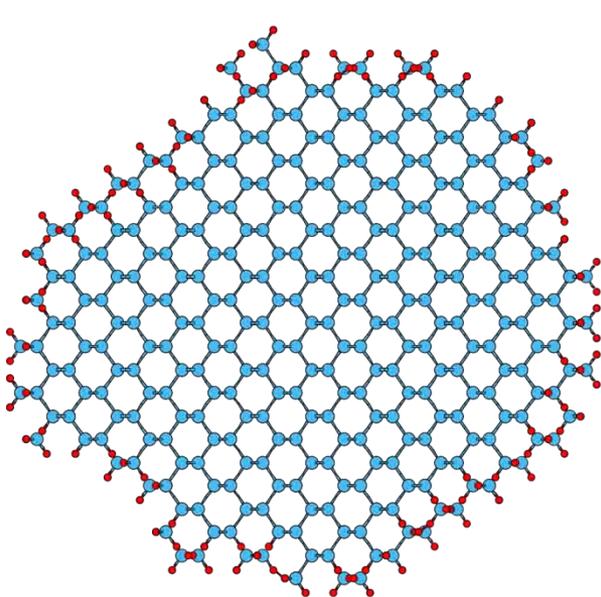
$$\langle \delta R(\mathbf{r}') \delta R(\mathbf{r}' - \mathbf{r}) \rangle = \Delta^2 e^{-\frac{r}{L_r}} \leftarrow \text{correlation length : } L_r = 0.54 \text{ nm}$$

$$(\delta R(\mathbf{r}) = R(\mathbf{r}) - R_0, R_0 = 2.2 \text{ nm})$$

This model is obtained by experimental data of Si(100)-SiO₂ interface.

$$\Delta = 0.2 \text{ nm}$$

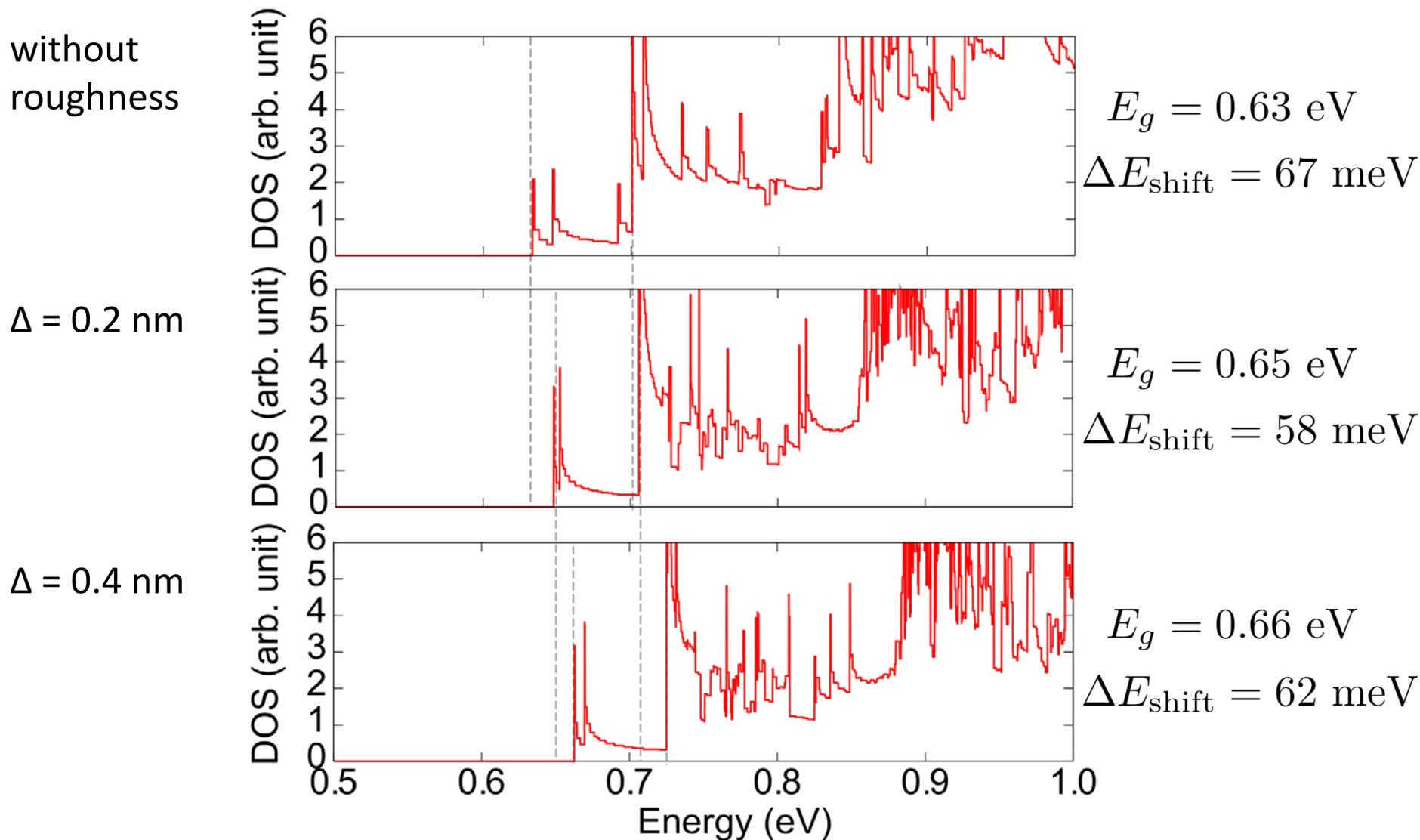
$$\Delta = 0.4 \text{ nm}$$



cross-sectional view

side view

Effects of DOS on surface roughness

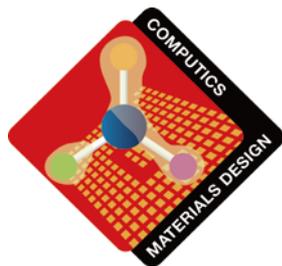
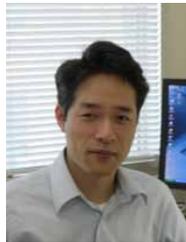


Effects of roughness are **minor** when Δ is smaller than 0.4 nm

PACS-CSにおける物性物理学研究

—RSDFTを中心に—

- ◆ 物質科学と計算機科学の共同での、RSDFTコードの開発：超並列マルチコア・アーキテクチャ上でのHigh Performance
- ◆ 10,000 - 100,000原子系の電子状態、構造決定が可能に



新学術領域研究：<http://computics-material.jp/>