

# PACS-CSにおける物性物理学研究

ExaFLOPS 201x -

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K 2011 -

T2K 2008 -

PACS-CS 2005 - 2007

CP-PACS 1996 -

QCDPAX 1989

SX9 2007

SX8 2004

SX6 2001

SX5 1998

(ES)

SX4 1994

SX3 1989

SX2 1983  
NEC

2007 -

U Tokyo

1995 - 2007 U Tsukuba

↑ CNT, Si, ...

*Days of vectorization  
and computations*

1985 - 1995 NEC

1983 - 1984 IBM  
*myself*



*Lattice QCD*

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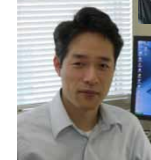
*Lattice QCD*

QCDPAX 1989

Real-Space  
Density-  
Functional-  
Theory  
(RSDFT) Code for  
Unprecedentedly  
Large-Scale  
Calculations

2007 -

U Tokyo



1995 - 2007 U Tsukuba



*CNT, Si,...*

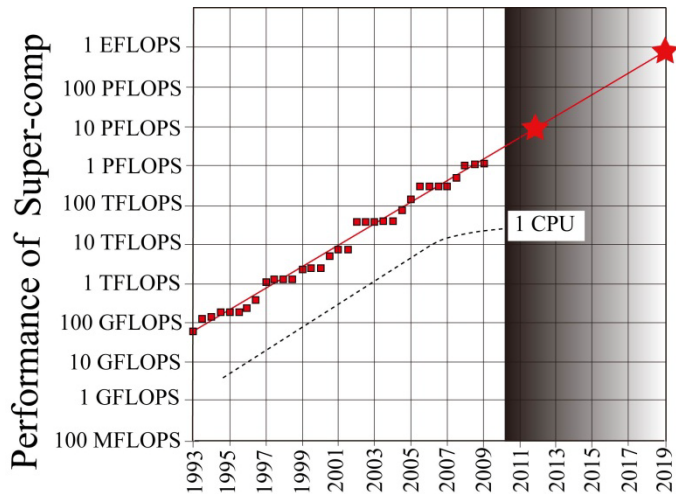
*Days of vectorization  
and computations*

1985 - 1995 NEC

1983 - 1984 IBM  
*myself*

# Computics: Fusion of Materials Science and Computer Science

## Computers that are changing rapidly



- ✓ Saturation of performance of a single computation processor  
**Breakdown of Moore's Scaling Law**
- ✓ Multi-core massively parallel architecture in the next generation  
**Kobe-K: 80,000CPU x 8 cores**
- ✓ Accelerator introduced in the next next generation  
**Supercomputer that is a monster**

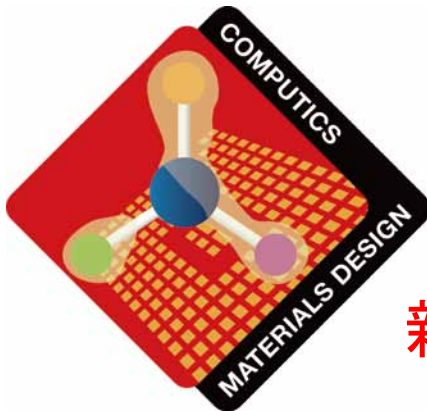
## Collaboration between Computational Materials Science and Computer Science is Imperative

- Developments of Mathematical Methodology and Algorithms Suitable to Current Hard wares
- Tuning of High Performance Computation codes based on deep knowledge of hard wares

# Computics: Fusion of Materials Science and Computer Science

Computics,

A New Approach to  
Materials Science in 21<sup>st</sup> Century,  
in addition to Mathematics  
that plays an important role from  
Newton's Principia



新学術領域研究: <http://computics-material.jp/>

# Development and Status of Computational Materials Science

## 80s ~ Success of Computational Science Approach

*Density Functional Theory (DFT)* (W. Kohn: Nobel Prize 1998)

*Car-Parrinello Molecular Dynamics (CPMD)* (1985)

*Various Developments in Quality and Quantity*

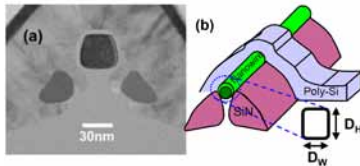
Clarification and Prediction of Material Properties and their Dynamics based on First Principles of Quantum Theory

## 90s ~ Limitation of DFT (Strongly Correlated System, Excitation Spectrum)

*From Now On!*

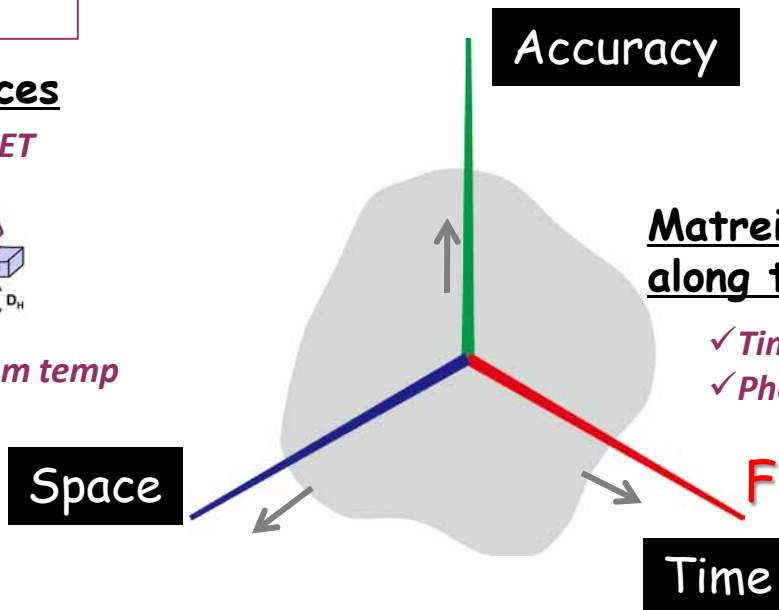
### Future in nano devices

*Next-Generation NWFET*



*Quantum effects at room temp*

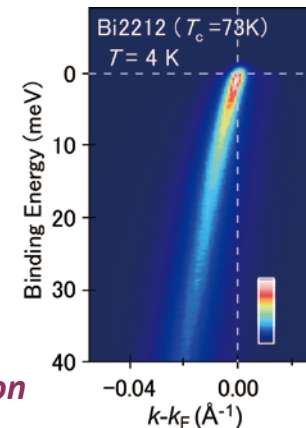
Larger scale:  
Nano world =  
100,000-atom world



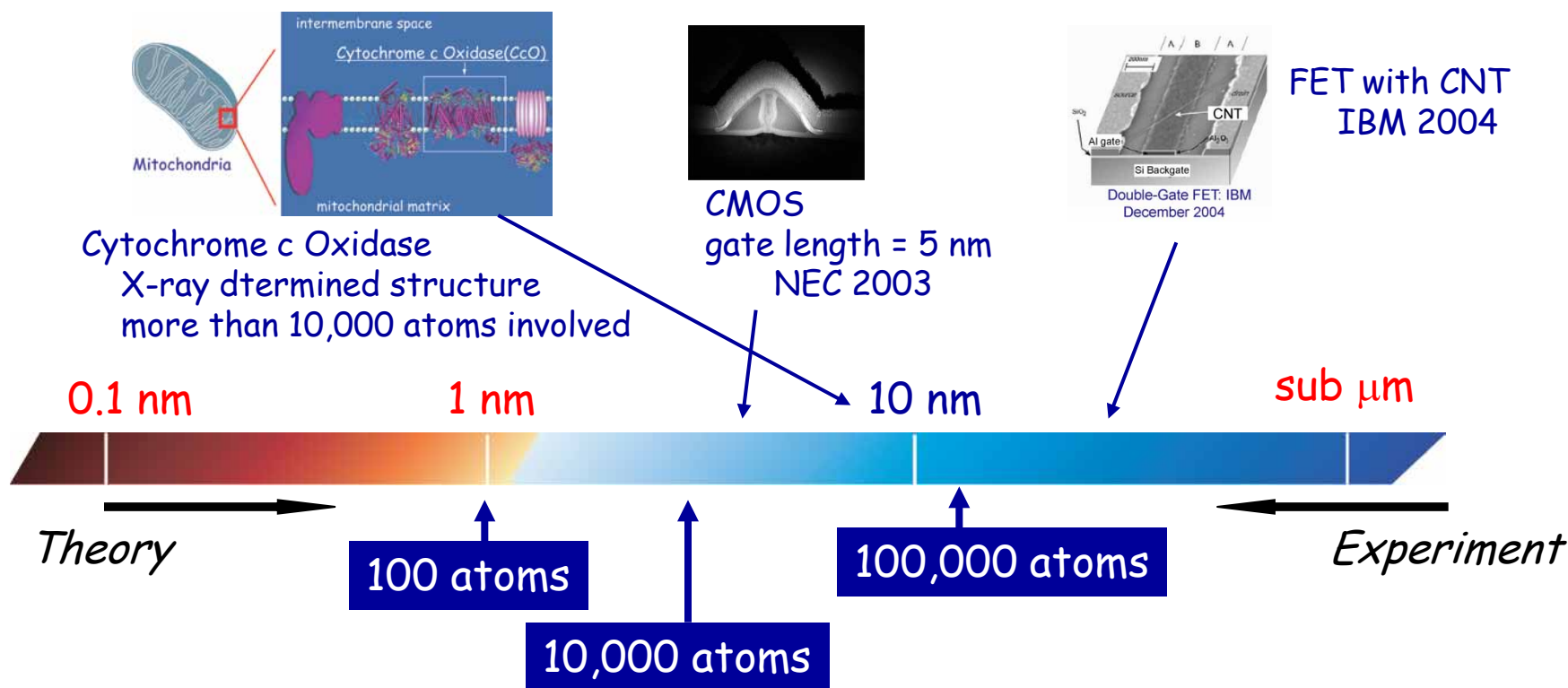
Materials design  
along time axes

- ✓ Time-resolved ARPES
- ✓ Photo-induced function

From sub-Femto sec with electrons to Nano sec with ions



# Why Large-Scale Density Functional (DFT) Calculations?



Quantum-theory-based large-scale DFT calculations and experiments meet together in Nano World !

# Density Functional Theory (DFT)

Total energy of a material is a functional of electron density:

$$\begin{aligned} E[n(\mathbf{r})] &= \langle \Psi | \mathbf{H} | \Psi \rangle = \langle \Psi | \mathbf{T} + V_{\text{nucl}} + V_{\text{ee}} | \Psi \rangle \\ &= T_{\text{S}}[n(\mathbf{r})] + \int v_{\text{nucl}}(\mathbf{r})n(\mathbf{r})d\mathbf{r} + \frac{1}{2} \iint \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}d\mathbf{r}' + E_{\text{XC}}[n(\mathbf{r})] \end{aligned}$$

When the electron density is written as  $n(\mathbf{r}) = \sum_{i: \text{occupied}} |\varphi_i(\mathbf{r})|^2$

we obtain Euler equation (Kohn-Sham) equation,

$$\left[ -\frac{1}{2}\nabla^2 + v_{\text{eff}}(\mathbf{r}; n(\mathbf{r})) \right] \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r}) \quad v_{\text{eff}}(\mathbf{r}) = v_{\text{nucl}}(\mathbf{r}) + \int \frac{n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}' + \frac{\delta E_{\text{XC}}[n]}{\delta n(\mathbf{r})}$$

Forces acting on nuclei  $\mathbf{R}$  are calculated as

$$F_u = -\frac{d}{dR_u} (E[n] + E_{\text{nucl-nucl}}) = \underbrace{\left[ \frac{\partial E}{\partial R_u} \right]_{\varphi}}_{\text{Hellmann-Feynman Force}} + \sum_i \int \underbrace{\left( \frac{\delta E}{\delta \varphi_i} \frac{d\varphi_i}{dR_u} + \frac{\delta E}{\delta \varphi_i^*} \frac{d\varphi_i^*}{dR_u} \right)}_{\text{Pulay Correction}} d\mathbf{r}$$

# Solving Kohn-Sham Equation

Introducing complete basis set  $\chi_n(\mathbf{r})$  to expand Kohn-Sham wave function:

$$\varphi_j(\vec{r}) = \sum_n c_{jn} \chi_n(\vec{r})$$

to convert the differential equation to a matrix equation, and then solve it by iterative techniques. Most convenient basis set has been plane wave basis set:

$$\chi_n(\vec{r}) = e^{i(\vec{k} + \vec{G}_n) \cdot \vec{r}}$$

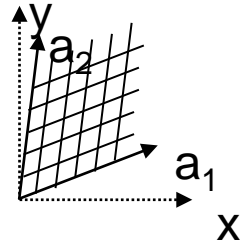
In the plane-wave scheme, several quantities such as  $n(\mathbf{r})$ ,  $v(\mathbf{r})$  etc is Fourier-transformed and Inverse-Fourier-transformed frequently.

**FFT: a heavy communication burden on parallel-architecture computers**



# Solving Kohn-Sham Equation Using Real-Space Finite-Difference Scheme

Introducing mesh points with spacing  $H$  in real space,

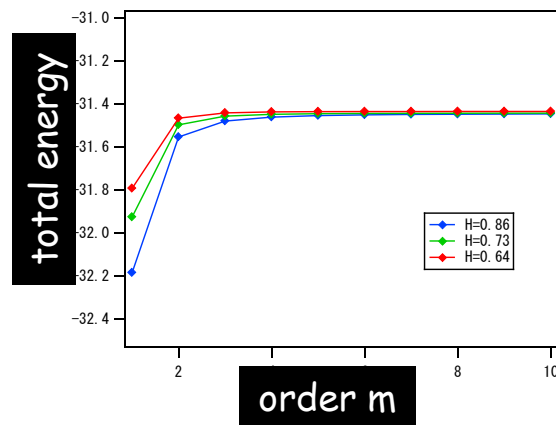


Kohn-Sham differential equation is converted to a **m-th** order finite-difference equation:

$$\frac{\partial^2}{\partial x^2} \varphi_j(x, y, z) = \sum_{m=-M}^M c_m \varphi_j(x + mH, y, z)$$

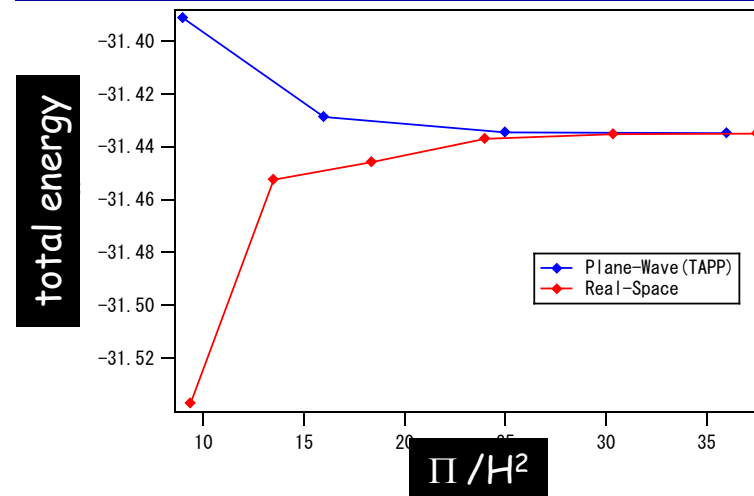
All the quantities computed at each mesh point, KS Hamiltonian expressed as a matrix.

Check of finite difference



Check of mesh spacing

⇒ systematic accuracy improvement

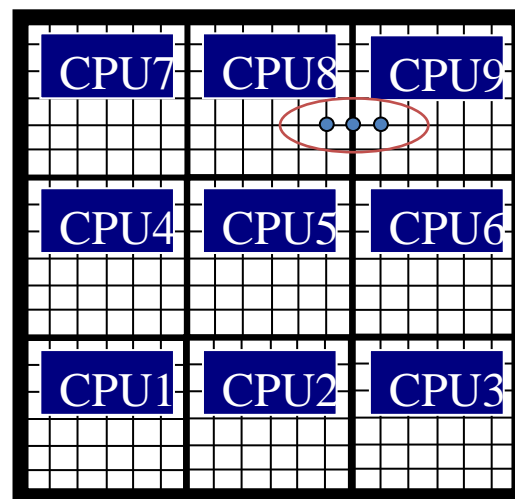


# Advantages of Real-Space DFT (RSDFT) in Parallel Computing

- Almost free from FFT, reducing communication burden
  - ⇒ high efficiency
- Flexible boundary condition to wave-functions
  - ⇒ targets expanded including charged objects
- Utilize inherent locality of the system
  - ⇒ prospect to efficient  $O(N)$  scheme

On multi-core parallel machines:

- Huge number of 3D mesh points divided into moderate-size cells
- Each cell treated by a single CPU
- Also, electron states are grouped and each group is treated by a single CPU or a core : **Hybrid parallelization**
- **MPI for CPU and OpenMP for core**



# Some Details and Their Optimization

- Residual minimization as well as conjugate-gradient minimization to solve KS equation
- Change of algorithm for Gram-Schmidt to utilize BLAS\*-level3 matrix-matrix operations
- Divide-and-Conquer algorithm to solve secular equation in subspace

## SCF(get-selfconsistent-field) loop

Get-KS wavefunction-for-a-fixed-potential loop  $O(N^2)$

conj-grad or residual-min

**gram-schmidt orthonormalization**  $O(N^3)$

**diagonalization of a partial matrix**  $O(N^3)$

(occupied states)

potential (charge) updated

## Get-wavefunction loop

conj-grad

or

residual-min

minimize  $\langle \phi | H | \phi \rangle$

or

minimize  $\| H | \phi \rangle - e | \phi \rangle \|^2$

Hamiltonian

Hamiltonian

operation

operation

**gram-schmidt**  $O(N^3)$

required

residual vector

next guess

& next guess

Order  $N^3$  Calculations, but the  $N^3$  part is highly optimized!

\* BLAS = Basic Linear Algebra Subprograms

# Gram-Schmidt Orthonormalization: Active use of Level 3 BLAS in $O(N^3)$ Computation

$$\psi'_1 = \psi_1$$

$$\psi'_2 = \psi_2 - \psi'_1 \langle \psi'_1 | \psi_2 \rangle$$

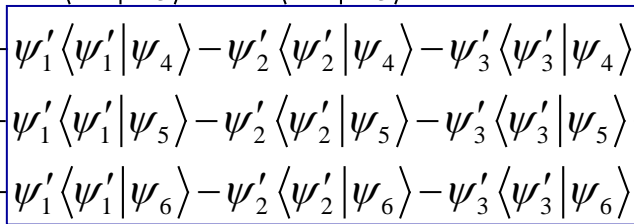
$$\psi'_3 = \psi_3 - \psi'_1 \langle \psi'_1 | \psi_3 \rangle - \psi'_2 \langle \psi'_2 | \psi_3 \rangle$$

$$\psi'_4 = \psi_4 - \psi'_1 \langle \psi'_1 | \psi_4 \rangle - \psi'_2 \langle \psi'_2 | \psi_4 \rangle - \psi'_3 \langle \psi'_3 | \psi_4 \rangle$$

$$\psi'_5 = \psi_5 - \psi'_1 \langle \psi'_1 | \psi_5 \rangle - \psi'_2 \langle \psi'_2 | \psi_5 \rangle - \psi'_3 \langle \psi'_3 | \psi_5 \rangle - \psi'_4 \langle \psi'_4 | \psi_5 \rangle$$

$$\psi'_6 = \psi_6 - \psi'_1 \langle \psi'_1 | \psi_6 \rangle - \psi'_2 \langle \psi'_2 | \psi_6 \rangle - \psi'_3 \langle \psi'_3 | \psi_6 \rangle - \psi'_4 \langle \psi'_4 | \psi_6 \rangle - \psi'_5 \langle \psi'_5 | \psi_6 \rangle$$

Part of the calculations can be performed as Matrix x Matrix operations!



Performance of Gram-Schmidt Routine  
(on 1000 CPUs of Tsukuba PACS-CS)

Theoretical Peak Performance	Our RSDFT (Operation)	Our RSDFT (Operation & Communication)
5.6 GFLOPS/cpu	4.3 GFLOPS/cpu	3.5 GFLOPS/cpu

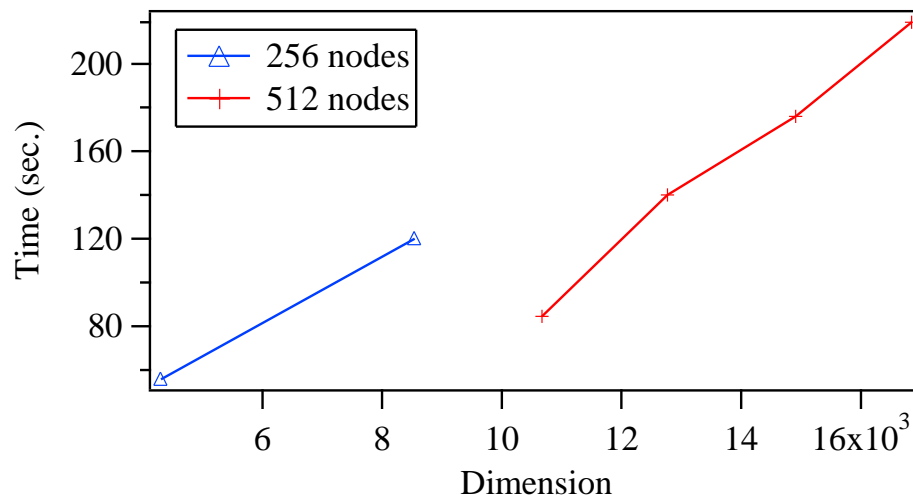
$O(N^3)$  part computed at 80% of the theoretical peak performance!

# Divide and Conquer for Subspace Diagonalization

## 1. Calculation of Matrix Elements

**BLAS-3**

## 2. Diagonalization



PDSYEVD  
(Solver in SCALAPACK)  
using Divide & Conquer

## 3. Vector (Wavefunction) Rotation

**BLAS-3**

*Now, Imamura Algorithm*  
*In future,*  
*Sakurai-Sugiura Algorithm*

# RSDFT on K(京) at Kobe

- ✓ 100,000-atom DFT calculations using hundreds of thousands cores with unprecedentedly efficient performance have been done
- ✓ 10,000-atom DFT calculations have become just an overnight task

*In collaboration with,*

*Jun-ichi Iwata (Univ Tokyo)*

*Yukihiro Hasegawa (Riken)*

*Miwako Tsushi (Univ Tsukuba)*

*Daisuke Takahashi (Univ Tsukuba)*

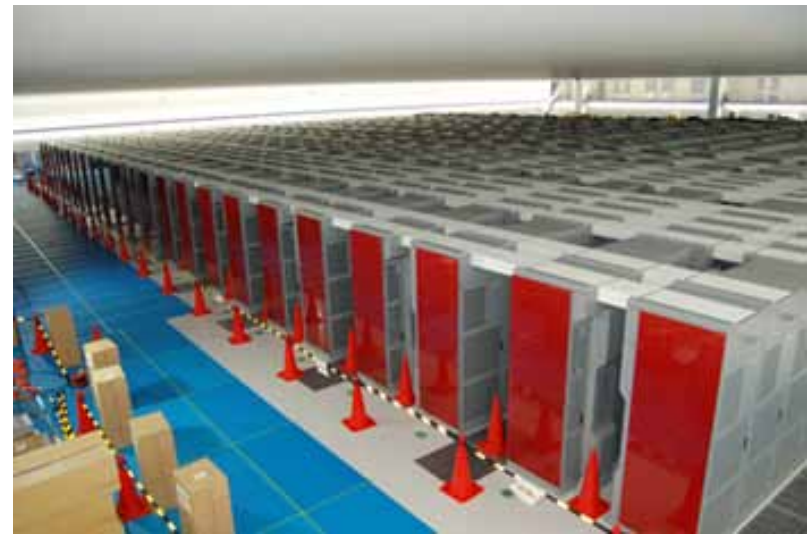
*Kazuo Minami (Riken)*

*Taisuke Boku (Univ Tsukuba)*

*Shinnosuke Furuya (Univ Tokyo)*

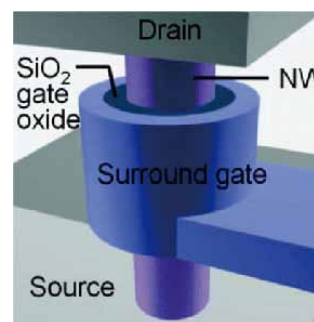
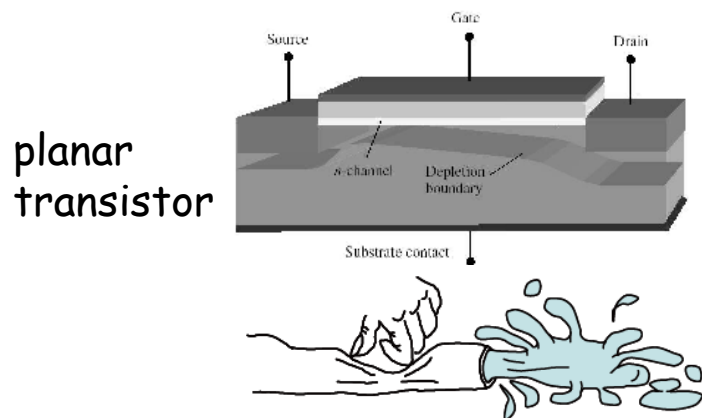
*Mitsuo Yokokawa (Riken)*

*'''  
who are in the fields of  
materials science & computer science.*



# Prediction of Electron States of Si Nanowires

*Si Nanowire, a Booster  
in the next-generation semiconductor technology*  
*More Moore → More than Moore*



Surrounding gate transistor

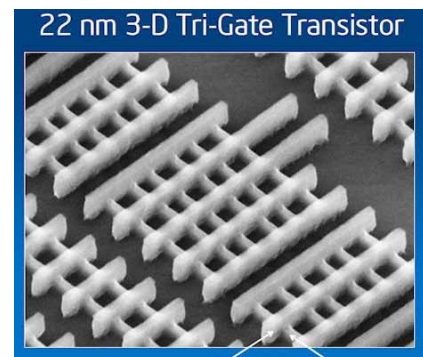


Gate Controllability

→ Suppress short-channel effects

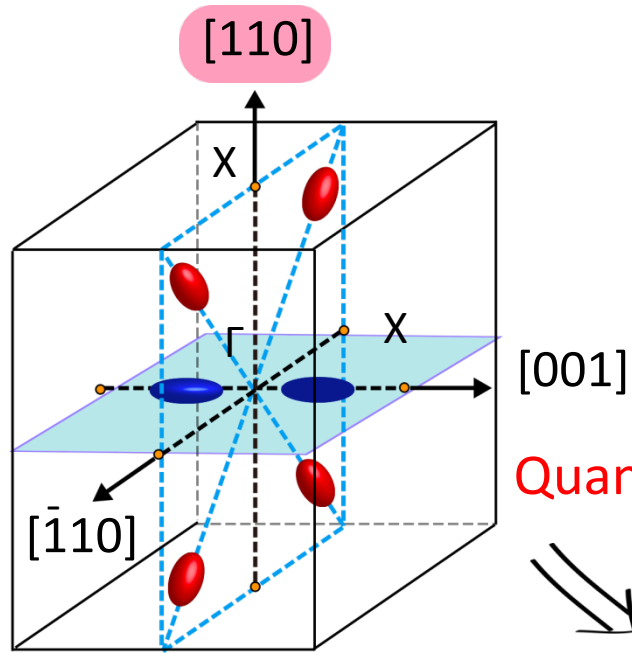
Suppress leaks at off state

→ save energy



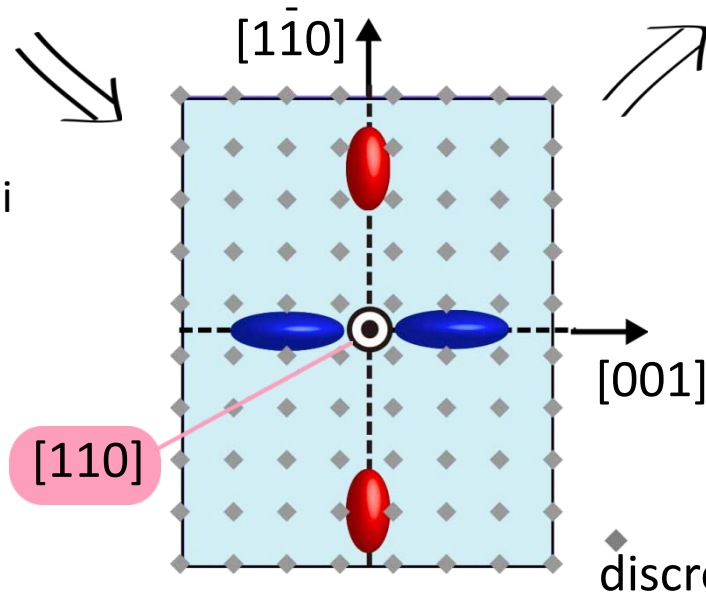
*Actually tri-gate by  
Intel in 2011*

# Effects of quantum confinement of [110]-SiNWs



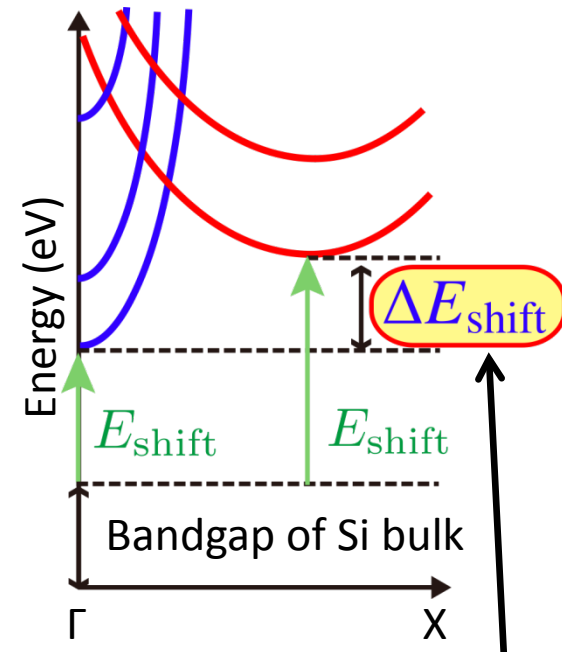
Brillouin Zone (BZ) of bulk Si and six conduction band valleys

Quantum confinement effects



Positions of allowed wave numbers  
depend on cross-sectional morphology

Conduction band structure of [110]-SiNW



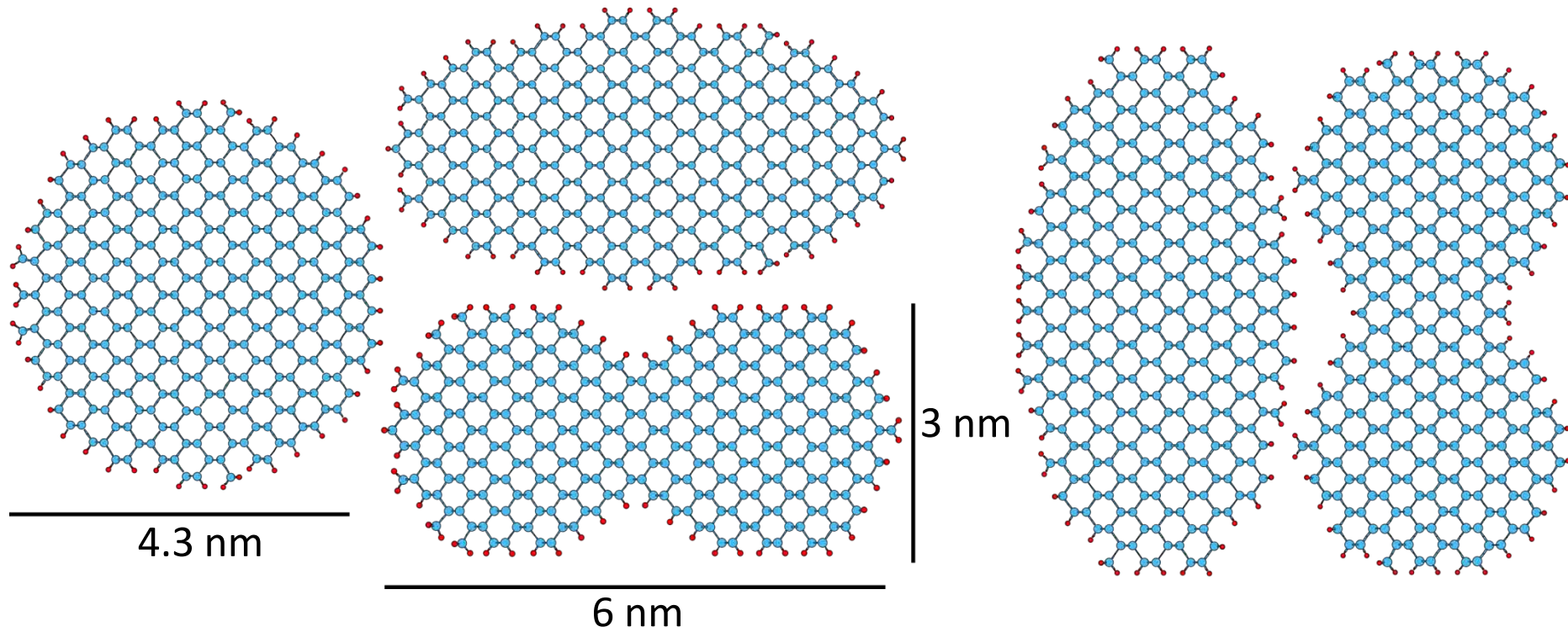
This quantity is decisive in the number of the conduction channels.



# Cross Sectional Shapes, Diameters, Directions,, What is the best Choice?

$[110]$   
 $[110]$   $[001]$

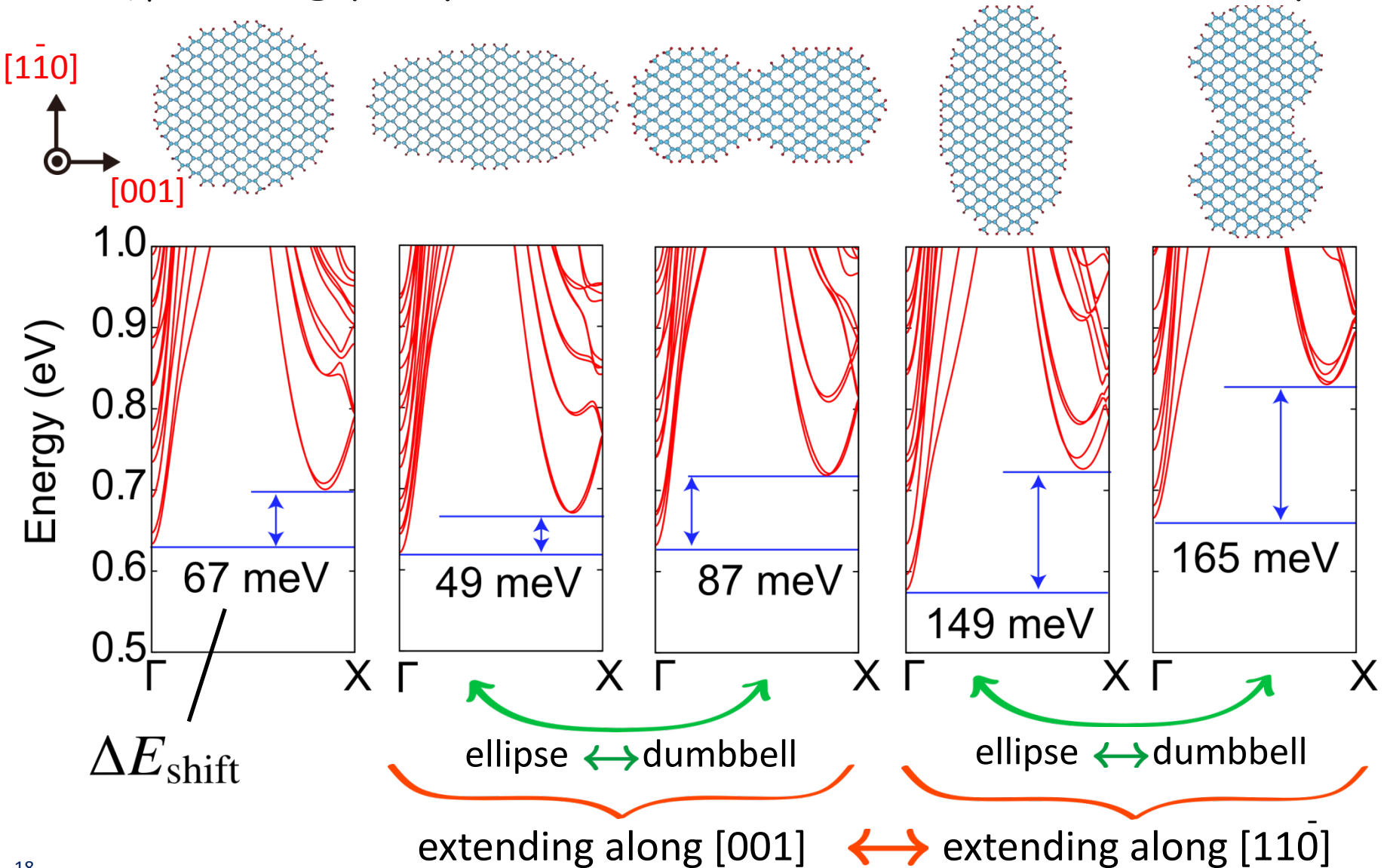
Considering Si (110) Nano Wire,  
*Directions inequivalent to each other*



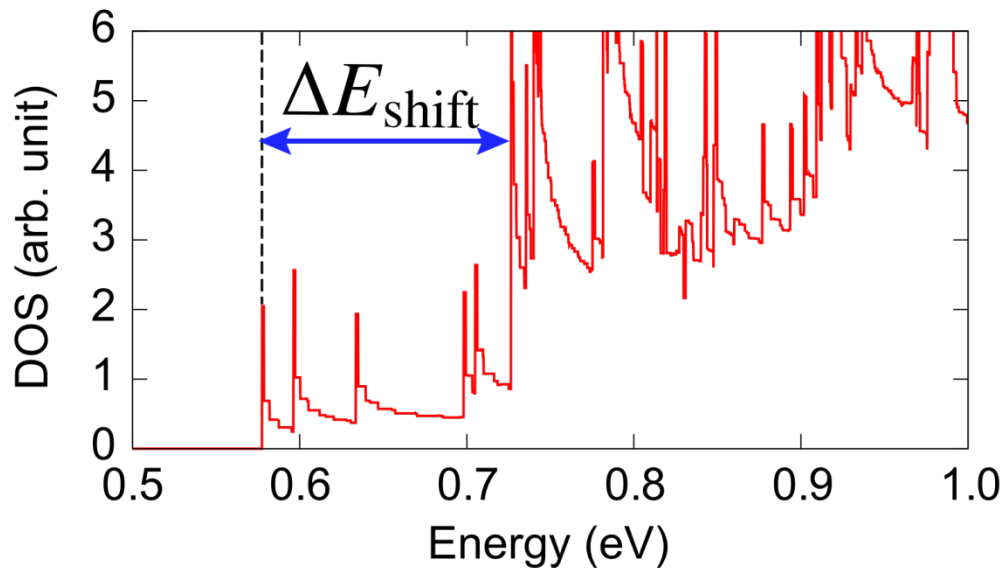
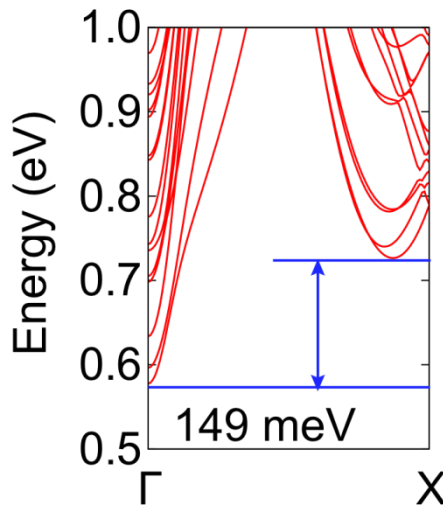
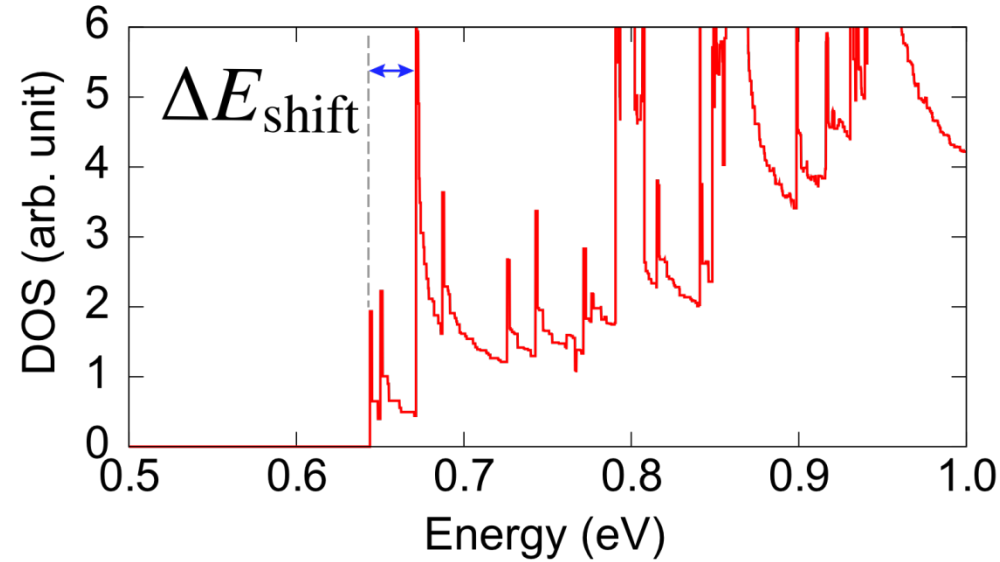
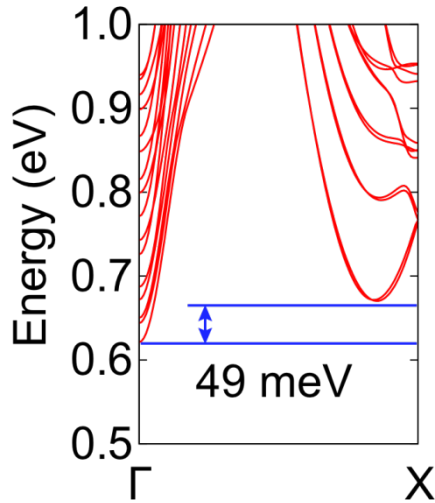
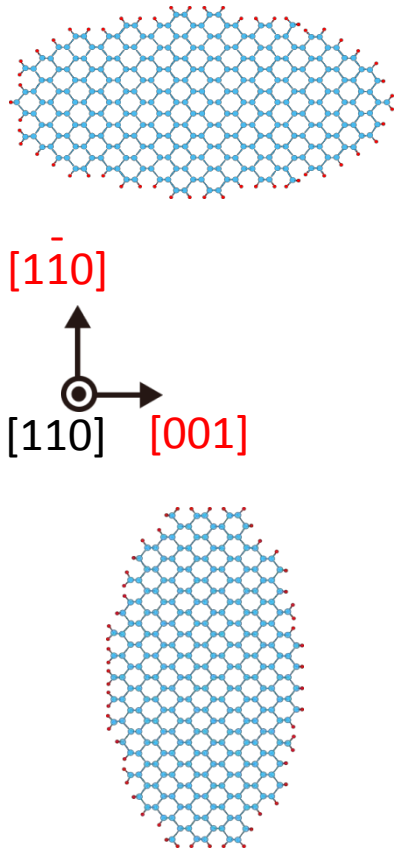
DFT could Give the Answer

# Conduction Band Structures of Various Si(110) NWs

$\Delta E_{\text{shift}}$  strongly depends on the cross-sectional nano-shapes



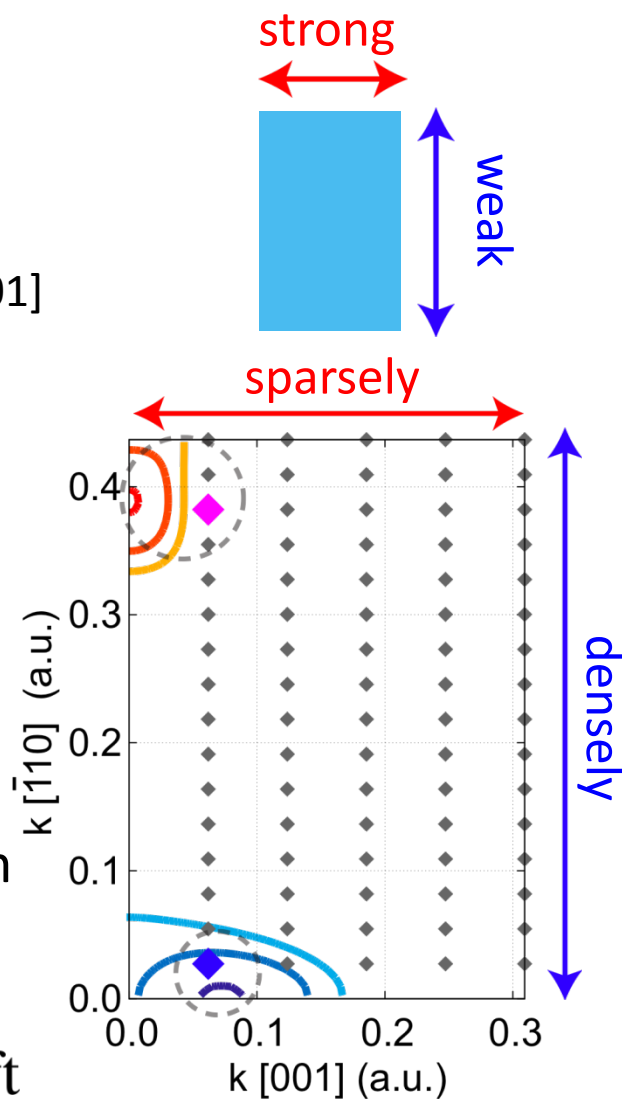
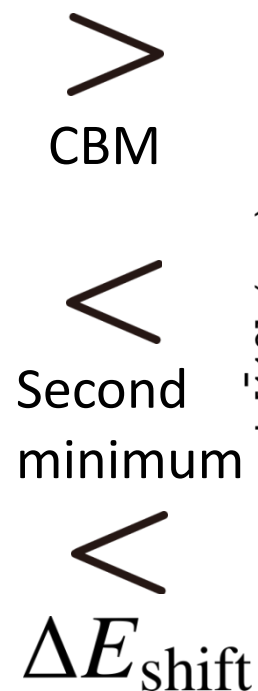
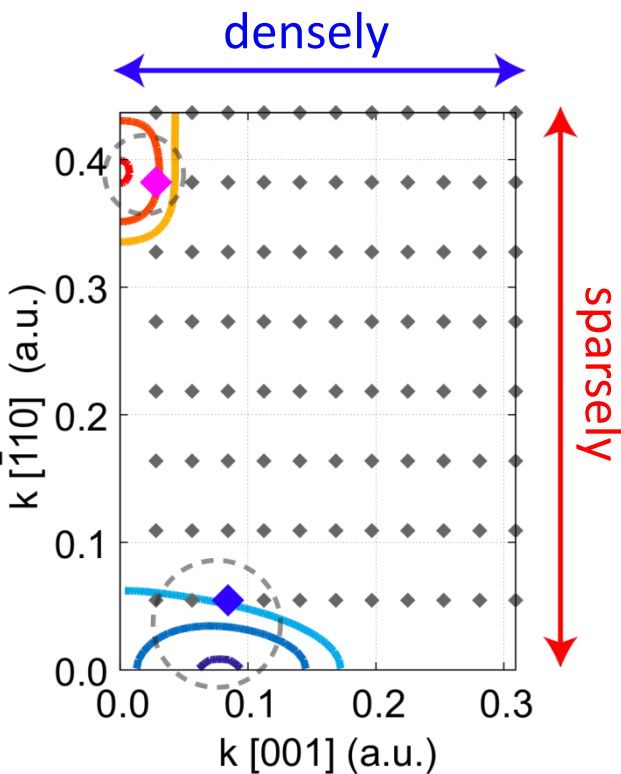
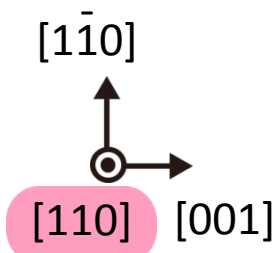
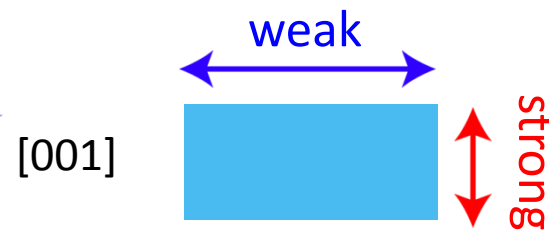
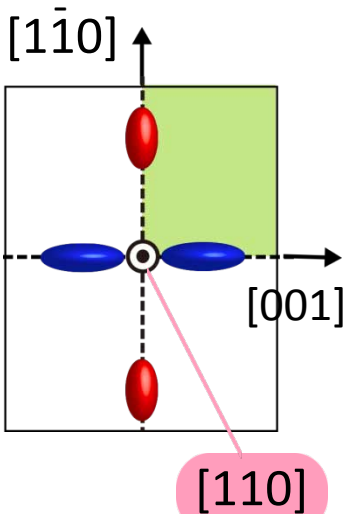
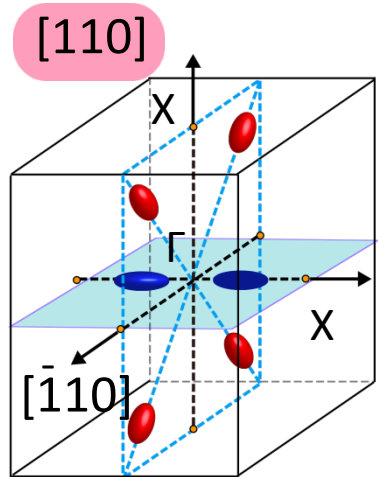
# Density of states of SiNWs



The number of channels near the CBM is strongly related to  $\Delta E_{\text{shift}}$

# Interpretation of calculated results

- ◆ discrete k
- ◆ CBM
- ◆ Second minimum



# SiNWs with sidewall roughness

Roughness makes the radius of NWs vary along circumference and wire axis.

⇒ Lorentzian autocorrelation function

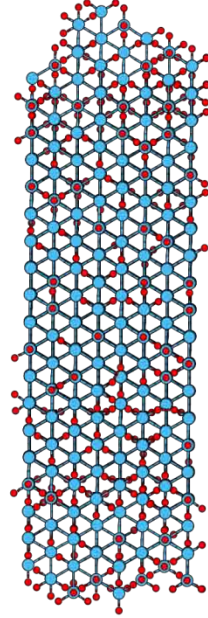
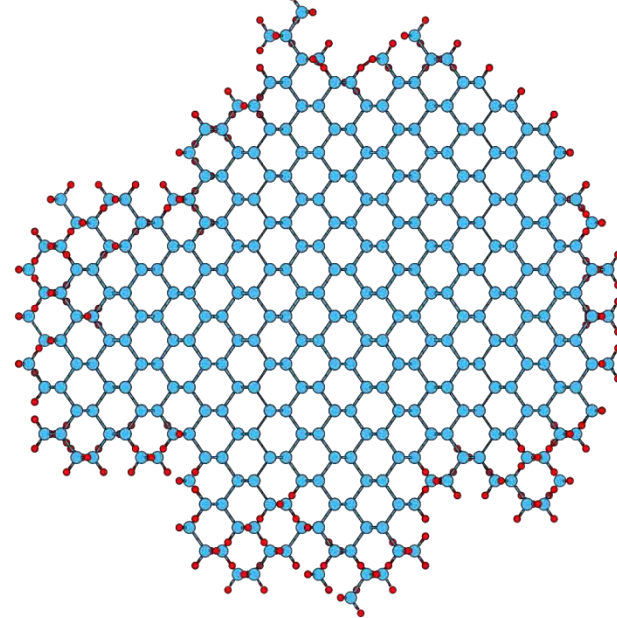
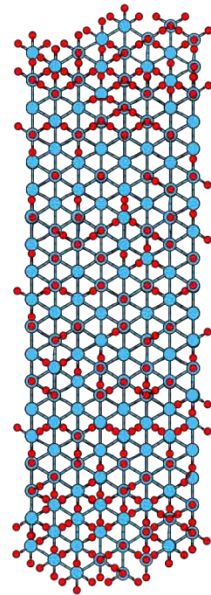
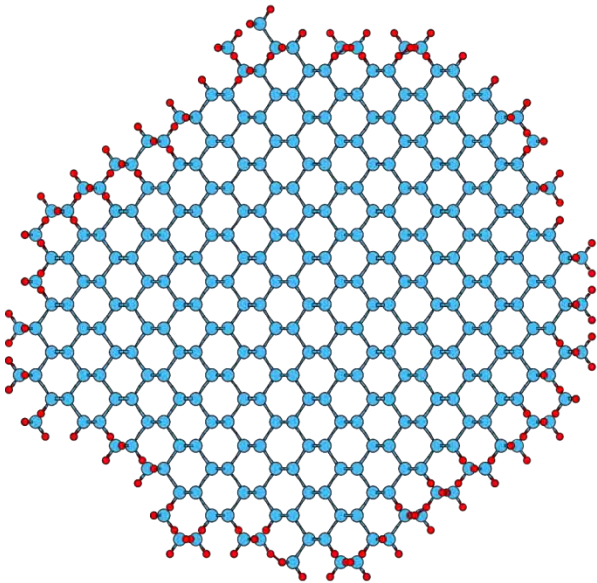
$$\langle \delta R(\mathbf{r}') \delta R(\mathbf{r}' - \mathbf{r}) \rangle = \Delta^2 e^{-\frac{r}{L_r}} \leftarrow \text{correlation length : } L_r = 0.54 \text{ nm}$$

$$(\delta R(\mathbf{r}) = R(\mathbf{r}) - R_0, R_0 = 2.2 \text{ nm})$$

This model is obtained by experimental data of Si(100)-SiO<sub>2</sub> interface.

$$\Delta = 0.2 \text{ nm}$$

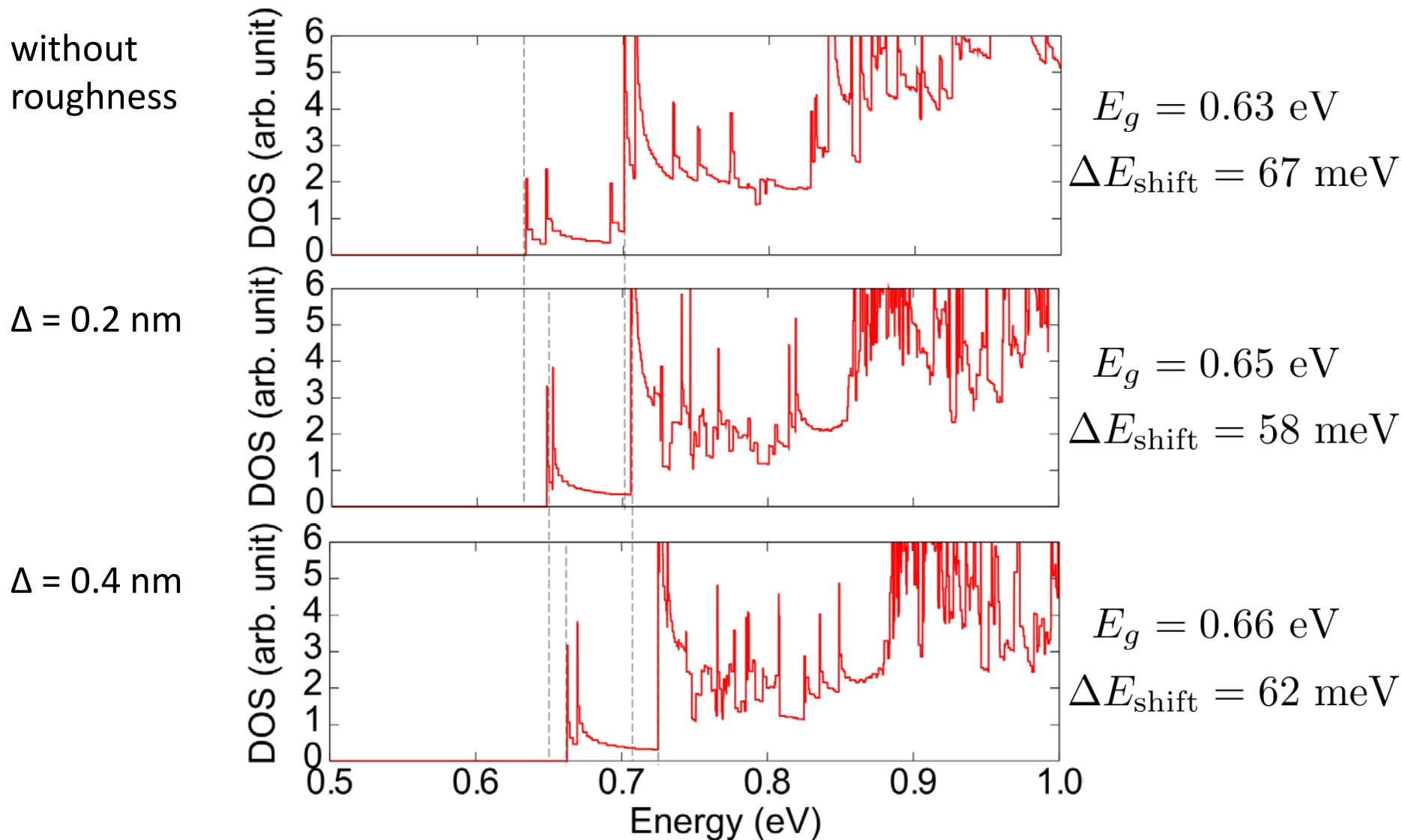
$$\Delta = 0.4 \text{ nm}$$



cross-sectional view

side view

# Effects of DOS on surface roughness

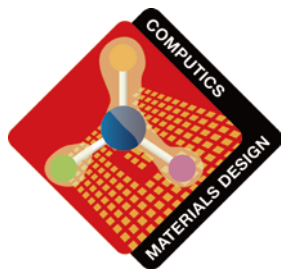
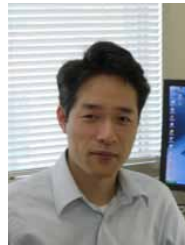


Effects of roughness are **minor** when  $\Delta$  is smaller than 0.4 nm

# PACS-CSにおける物性物理学研究

—RSDFTを中心に—

- ◆ 物質科学と計算機科学の共同での、RSDFTコードの開発：超並列マルチコア・アーキテクチャ上でのHigh Performance
- ◆ 10,000 - 100,000原子系の電子状態、構造決定が可能に



新学術領域研究：<http://computics-material.jp/>