Nucleon Matrix Elements with Domain Wall Fermions

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RBC calculations of nucleon structure: form factors, moments of structure functions and nucleon decay matrix elements,

- using DWF and DBW2 actions.
- Based on the works by Yasumichi Aoki, Tom Blum, Kostas Orginos, Shoichi Sasaki, ...

Domain wall fermions (DWF) preserves almost exact chiral symmetry on the lattice:

- by introducing a fictitious fifth dimension in which the symmetry violation is exponentially suppressed.

DBW2 ("doubly blocked Wilson 2") action improves approach to the continuum:

- by adding rectangular (2×1) Wilson loops to the action.

By combining the two, the "residual mass," which controls low energy chiral behavior, is driven to

- $am_{\rm res} \sim O(10^{-4})$ or $m_{\rm res} <$ MeV.

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Numerical calculation: we like to have

- good chiral behavior, *i.e.* close enough to the continuum, and
- big enough volume.

Improved gauge actions help both. DBW2[1], in particular,

$$S_G = \beta[c_0 \sum W_{1,1} + c_1 \sum W_{1,2}],$$

with $c_0 + 8c_1 = 1$, and $c_1 = -1.4069$.

Fermion action: DWF,

Quenched calculation: about 400 lattices, complete,

- $\beta = 0.87$, at the chiral limit, $am_\rho = 0.592(9)$ (so $a^{-1} \sim 1.3\text{GeV}$),
- $L_s = 16$, $M_5 = 1.8$, $am_{\text{res}} \sim 5 \times 10^{-3}$,
- $8^3 \times 24 \times 16$ ($\sim (1.2\text{fm})^3$) and $16^3 \times 32 \times 16$ ($\sim (2.4\text{fm})^3$) volumes,
- $m_N/m_\rho \sim 1.3$.

Dynamical calculation ($N_f = 2$): about 50 lattice at each of $m_f a = 0.04$, 0.03, and 0.02, ongoing,

- $\beta = 0.8$ (m_ρ and Sommer scales agree with $a^{-1} \sim 1.7\text{GeV}$),
- $L_s = 12$, $M_5 = 1.8$, $m_{\text{res}} \sim 2.5$ MeV,
- $16^3 \times 32 \times 16$ ($\sim (2.0\text{fm})^3$) volume,
- $m_N/m_\rho \sim 1.35$.

[1] QCD-TARO collaboration, Nucl. Phys. B577, 263 (2000). See also RBC collaboration, Phys. Rev. D69, 074504 (2004); hep-lat/0211023.

Axial charge: from neutron β decay, we know $g_V = G_F \cos\theta_c$ and $g_A/g_V = 1.2670(30)$[2]:

- $g_V \propto \lim_{q^2\to 0} g_V(q^2)$ with $\langle n|V_\mu^-(x)|p\rangle = i\bar{u}_n[\gamma_\mu g_V(q^2) + q_\lambda \sigma_{\lambda\mu} g_M(q^2)]u_p e^{-iqx}$,
- $g_A \propto \lim_{q^2\to 0} g_A(q^2)$ with $\langle n|A_\mu^-(x)|p\rangle = i\bar{u}_n\gamma_5[\gamma_\mu g_A(q^2) + q_\mu g_P(q^2)]u_p e^{-iqx}$.

On the lattice, in general, we calculate the relevant matrix elements of these currents

- with a lattice cutoff, $a^{-1} \sim$ 1-2 GeV,
- and extrapolate to the continuum, $a \to 0$,

introducing lattice renormalization: $g_{V,A}^{\text{renormalized}} = Z_{V,A} g_{V,A}^{\text{lattice}}$.

Also, unwanted lattice artefact may result in unphysical mixing of chirally distinct operators.

DWF makes g_A/g_V particularly easy, because:

- the chiral symmetry is almost exact, and
- maintains $Z_A = Z_V$, so that $g_A^{\text{lattice}}/g_V^{\text{lattice}}$ directly yields the renormalized value.

[2]The Particle Data Group.

Historically

- NR quark model gives 5/3,
- MIT bag model gives 1.07,
- lattice calculations with Wilson or clover fermions typically underestimates by up to 25 %:

type	group	fermion	lattice	β	volume	configs	$m_\pi L$	g_A
quenched	KEK[a]	Wilson	$16^3 \times 20$	5.7	$(2.2\text{fm})^3$	260	≥ 5.9	0.985(25)
	Liu et al[b]	Wilson	$16^3 \times 24$	6.0	$(1.5\text{fm})^3$	24	≥ 5.8	1.20(10)
	DESY[c]	Wilson	$16^3 \times 32$	6.0	$(1.5\text{fm})^3$	1000	≥ 4.8	1.074(90)
	LHPC-SESAM[d]	Wilson	$16^3 \times 32$	6.0	$(1.5\text{fm})^3$	200	≥ 4.8	1.129(98)
	QCDSF[e]	Wilson	$24^3 \times 48$	6.2	$(1.6\text{fm})^3$	O(300)		1.14(3)
			$32^3 \times 48$	6.4	$(1.6\text{fm})^3$	O(100)		
			$16^3 \times 32$	6.0	$(1.5\text{fm})^3$	O(500)		
	QCDSF-UKQCD[f]	Clover	$24^3 \times 48$	6.2	$(1.6\text{fm})^3$	O(300)		1.135(34)
			$32^3 \times 48$	6.4	$(1.6\text{fm})^3$	O(100)		
full($N_f = 2$)	LHPC-SESAM[d]	Wilson	$16^3 \times 32$	5.5	$(1.7\text{fm})^3$	100	≥ 4.2	0.914(106)
	SESAM[g]	Wilson	$16^3 \times 32$	5.6	$(1.5\text{fm})^3$	200	≥ 4.5	0.907(20)

[a] M. Fukugita, Y. Kuramashi, M. Okawa and A. Ukawa, Phys. Rev. Lett. 75, 2092 (1995).

[b] K.F. Liu, S.J. Dong, T. Draper and J.M. Wu, Phys. Rev. D49, 4755 (1994).

[c] M. Göckeler et al, Phys. Rev. D53, 2317 (1996).

[d] D. Dolgov et al, hep-lat/0201021.

[e] S. Capitani et al, Nucl. Phys. B (Proc. Suppl.) 79, 548 (1999).

[f] R. Horsley et al, Nucl. Phys. B (Proc. Suppl.) 94, 307 (2001).

[g] S. Güsken et al, Phys. Rev. D59, 114502 (1999)

– with $Z_A \neq Z_V$ and other renormalization complications.

Our formulation follows the standard one,

- Two-point function: $G_{_N}(t) = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}}\langle TB_1(x)B_1(0)\rangle]$, using $B_1 = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$ for proton,
- Three-point functions,
  - vector: $G_{_V}^{u,d}(t,t') = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}'}\sum_{\vec{x}}\langle TB_1(x')V_t^{u,d}(x)B_1(0)\rangle]$,
  - axial: $G_{_A}^{u,d}(t,t') = \frac{1}{3}\sum_{i=x,y,z}\mathrm{Tr}[(1+\gamma_t)\gamma_i\gamma_5\sum_{\vec{x}'}\sum_{\vec{x}}\langle TB_1(x')A_i^{u,d}(x)B_1(0)\rangle]$.

  with fixed $t' = t_{\mathrm{source}} - t_{\mathrm{sink}}$ and $t < t'$.
- From the lattice estimate

$$g_{_\Gamma}^{\mathrm{lattice}} = \frac{G_{_\Gamma}^u(t,t') - G_{_\Gamma}^d(t,t')}{G_{_N}(t)},$$

  with $\Gamma = V$ or A, the renormalized value

$$g_{_\Gamma}^{\mathrm{ren}} = Z_{_\Gamma} g_{_\Gamma}^{\mathrm{lattice}},$$

  is obtained.
- Non-perturbative renormalizations, defined by

$$[\bar{u}\Gamma d]_{\mathrm{ren}} = Z_{_\Gamma}[\bar{u}\Gamma d]_0,$$

  satisfies $Z_{_A} = Z_{_V}$ well, so that

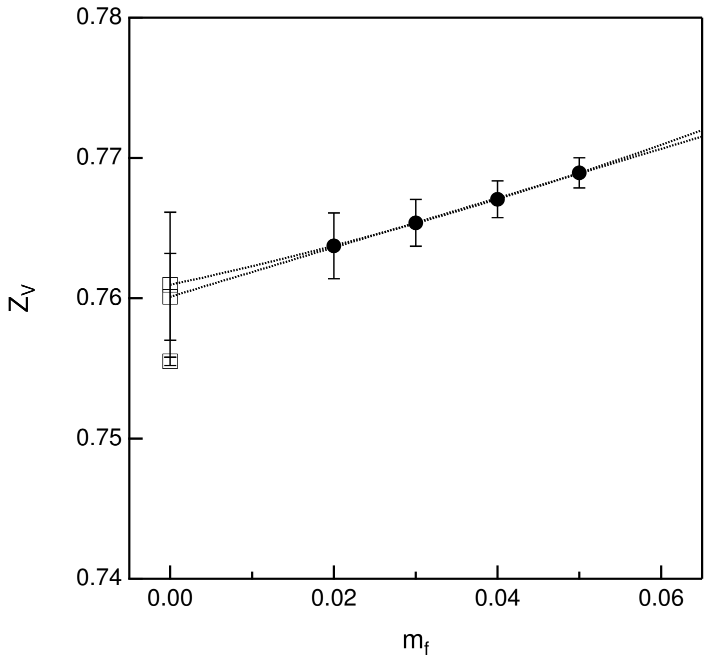
$$\left(\frac{g_{_A}}{g_{_V}}\right)^{\mathrm{ren}} = \left(\frac{G_{_A}^u(t,t') - G_{_A}^d(t,t')}{G_{_V}^u(t,t') - G_{_V}^d(t,t')}\right)^{\mathrm{lattice}}.$$

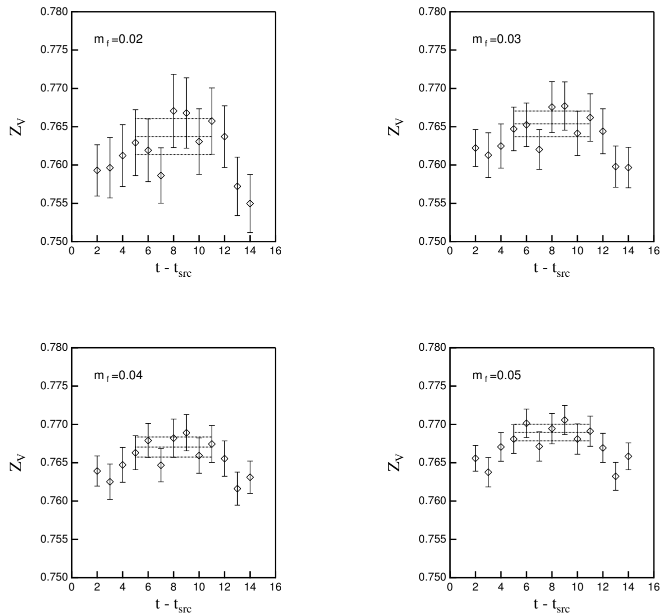
$g_{_A}$ is also described as $\Delta u - \Delta d$.

Numerical calculations with Wilson (single plaquette) gauge action:

- RIKEN-BNL-Columbia QCDSP,
- 400 gauge configurations, using a heat-bath algorithm,
- $\beta = 6.0$, $16^3 \times 32 \times 16$, $M_5 = 1.8$,
- source at $t = 5$, sink at 21, current insertions in between.

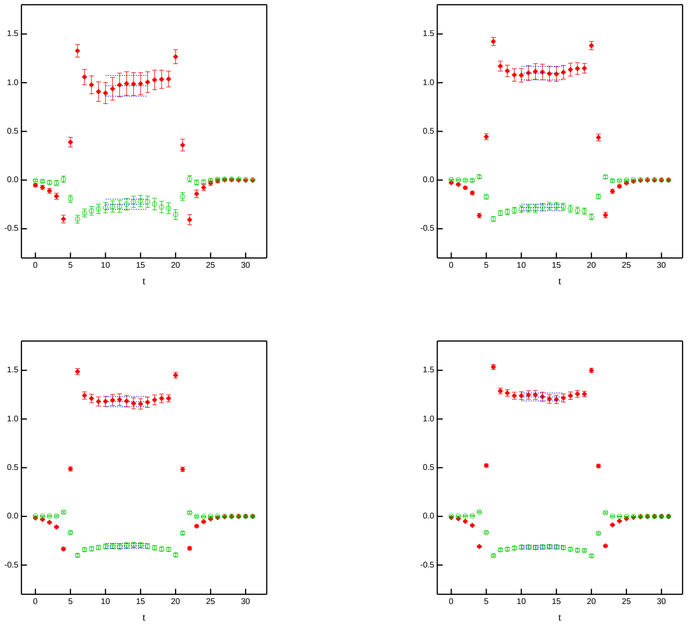
$Z_{_V} = 1/g_{_V}^{\rm lattice}$ is well-behaved,

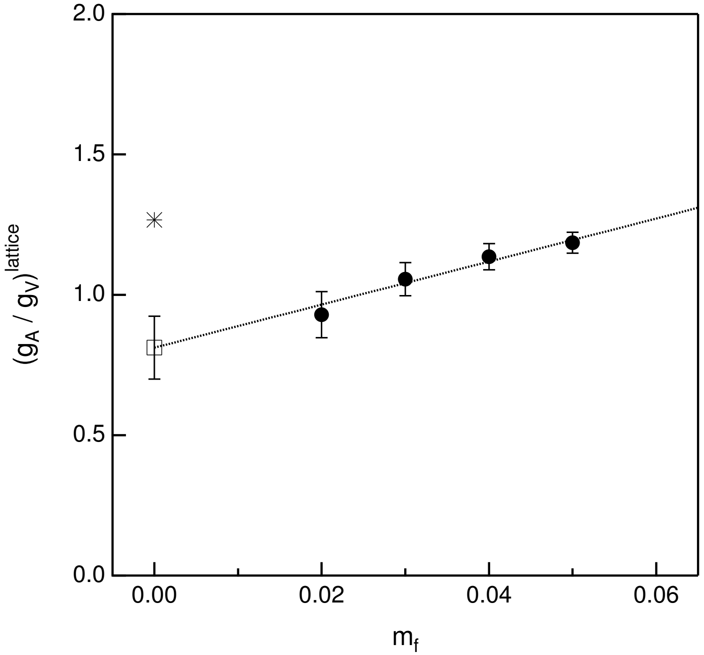




- the value 0.764(2) at $m_f = 0.02$ agrees well with $Z_{_A} = 0.7555(3)$ from
  - $\langle A_\mu^{\rm conserved}(t)\bar{q}\gamma_5 q(0)\rangle = Z_{_A}\langle A_\mu^{\rm local}(t)\bar{q}\gamma_5 q(0)\rangle$ (RBC hep-lat/0007038, to appear in Phys. Rev. D),
- linear fit gives $Z_{_V} = 0.760(7)$ at $m_f = 0$, and quadratic fit, 0.761(5).

Δu, Δd, and g_A/g_V (averaged in $10 \le t \le 16$):





- linear extrapolation yields 0.81(11) at $m_f = 0$, and simlarly small values for
  - $\Delta q/g_V = 0.49(12)$ and
  - $(\delta q/g_V)^{\rm lattice} = 0.47(10)$ (with a preliminary $Z_T \sim 1.1$).
- While relevant three-point functions are well behaved in DWF, and $Z_V = Z_A$ is well satisfied, 0.760(7) and 0.7555(3).

Why so small?

- finite lattice volume [3],
- excited states (small separation between $t_{\rm source}$ and $t_{\rm sink}$),
- quenching (zero modes, absent pion cloud, ...).

To investigate size-dependence, we simultaneously need

- good chiral behavior, *i.e.* close enough to the continuum, and
- big enough volume.

Improved gauge actions help both. DBW2[4], in particular,

$$S_G = \beta[c_0 \sum W_{1,1} + c_1 \sum W_{1,2}],$$

with $c_0 + 8c_1 = 1$ and $c_1 = -1.4069$:

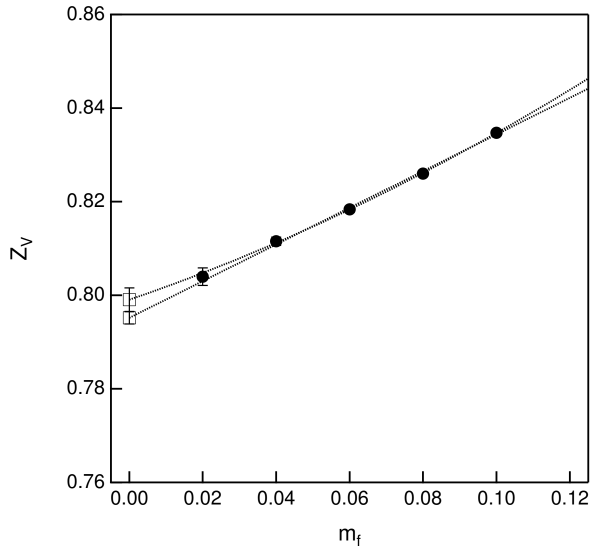
- very small residual chiral symmetry breaking, $am_{\rm res} < 10^{-3}$,
- at the chiral limit, $am_\rho = 0.592(9)$ (so $a^{-1} \sim 1.3$GeV), $m_\rho/m_N \sim 0.8$,
- $m_\pi(m_f = 0.02) \sim 0.3a^{-1}$.

[3] R.L. Jaffe, Phys .Lett. B529:105, 2002; hep-ph/0108015. See also T.D. Cohen, Phys. Lett. B529:50, 2002; hep-lat/0112014.
[4] QCD-TARO collaboration, Nucl. Phys. B577, 263 (2000); RBC collaboration, in preparation.

DBW2 calculations are performed at $a \sim 0.15$ fm ($\beta = 0.87$) with both wall and sequential sources on

- $8^3 \times 24 \times 16$ ($\sim (1.2\text{fm})^3$), 400 configurations (wall) and 160 (sequential),
- $16^3 \times 32 \times 16$ ($\sim (2.4\text{fm})^3$), 100 configurations (wall and sequential),
- source-sink separation of about 1.5 fm,
- $m_f = 0.02, 0.04, \ldots$: $m_\pi \geq 390\text{MeV}$, $m_\pi L \geq 4.8$ and 2.4.

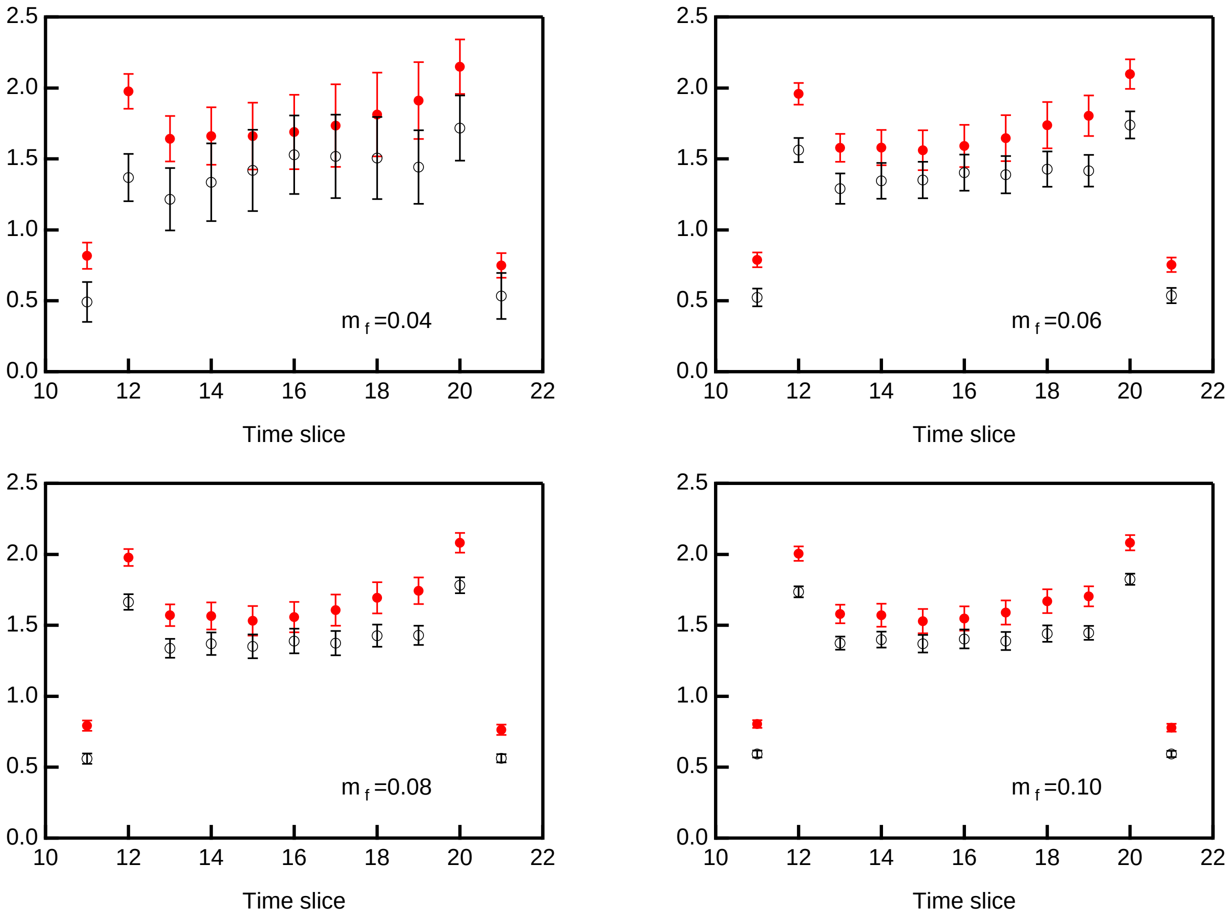
Renormalization factors: $\mathcal{O}^{\text{ren}}(\mu) = Z_{\mathcal{O}}(a\mu)\mathcal{O}^{\text{lattice}}(a)$.



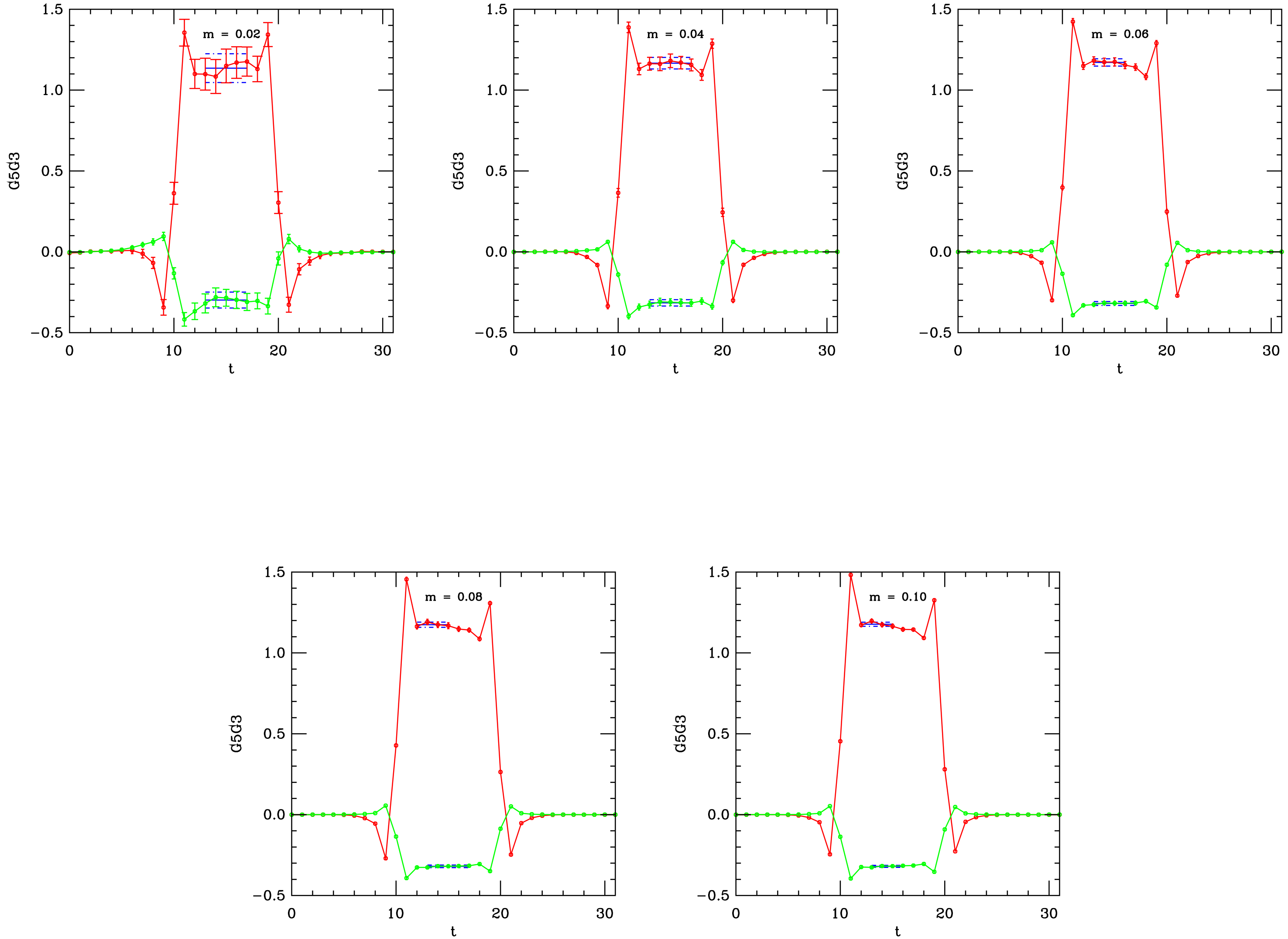
- Z_V shows slight quadratic dependence on m_f as expected: $V_\mu^{\text{conserved}} = Z_V V_\mu^{\text{local}} + \mathcal{O}(m_f^2 a^2)$,
  - yielding a value $Z_V = 0.784(15)$,
  - agrees well with $Z_A = 0.77759(45)$ [5].

[5]RBC Collaboration, in preparation: this value is obtained from a relation $\langle A_\mu^{\text{conserved}}(t)[\bar{q}\gamma_5 q](0)\rangle = Z_A \langle A_\mu^{\text{local}}(t)[\bar{q}\gamma_5 q](0)\rangle$.

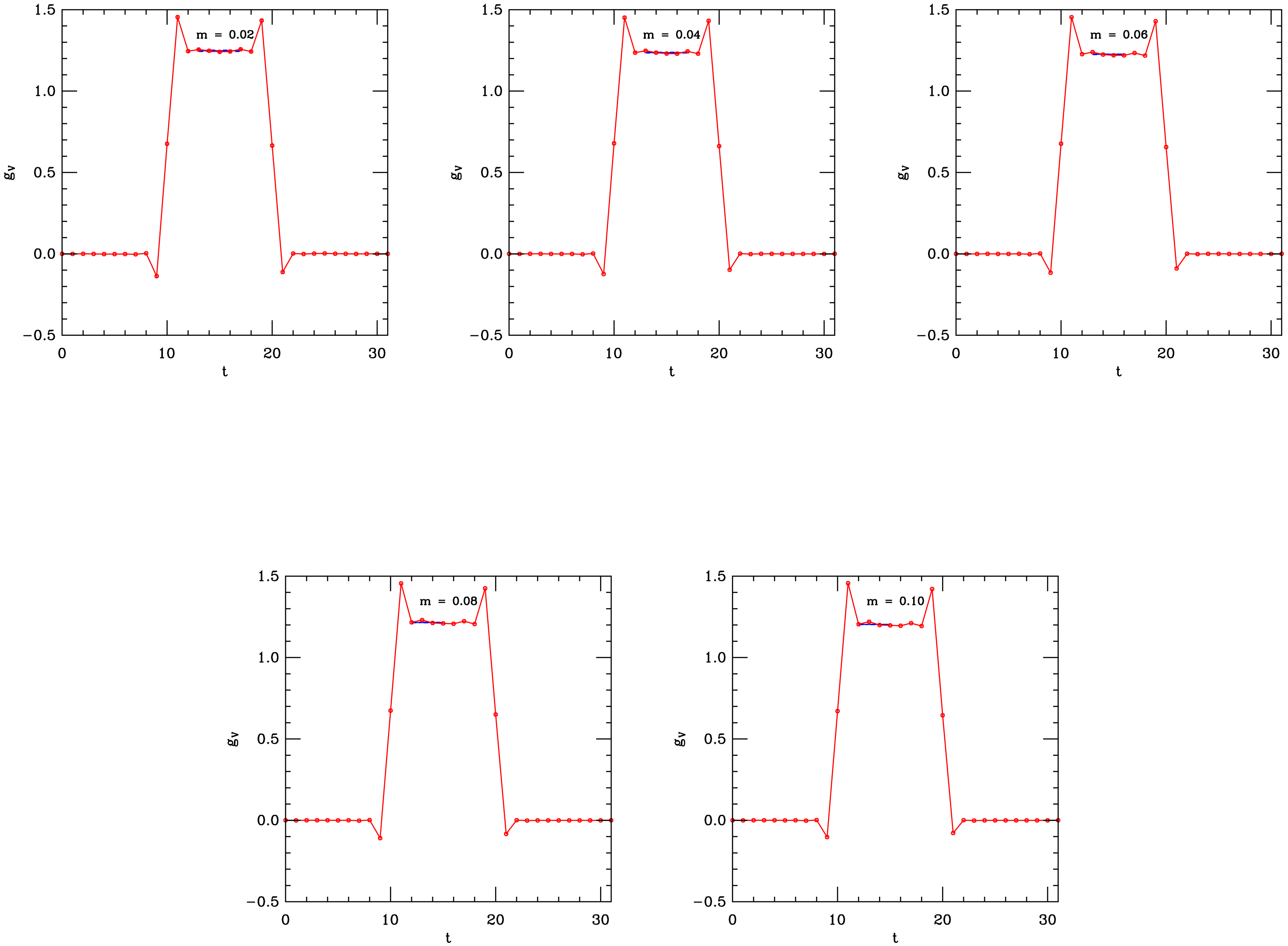
Bare g_A^{lattice} from wall source show volume dependence at medium m_f ($(2.4\text{fm})^3$ (filled) and $(1.2\text{fm})^3$ (open) volumes):



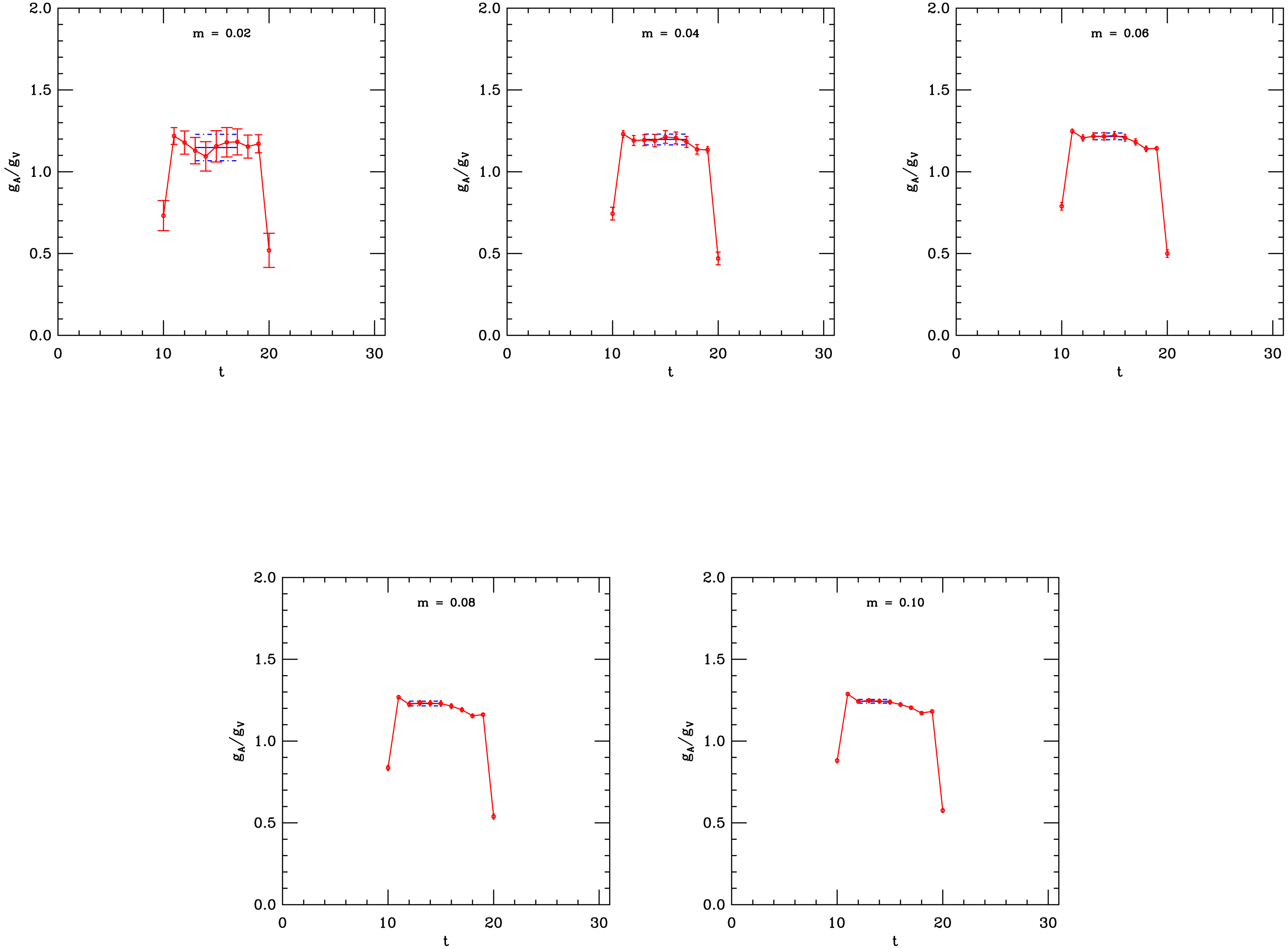
Bare $\Delta u^{\mathrm{lattice}}$ and $\Delta d^{\mathrm{lattice}}$ from sequential source ($(2.4\mathrm{fm})^3$):



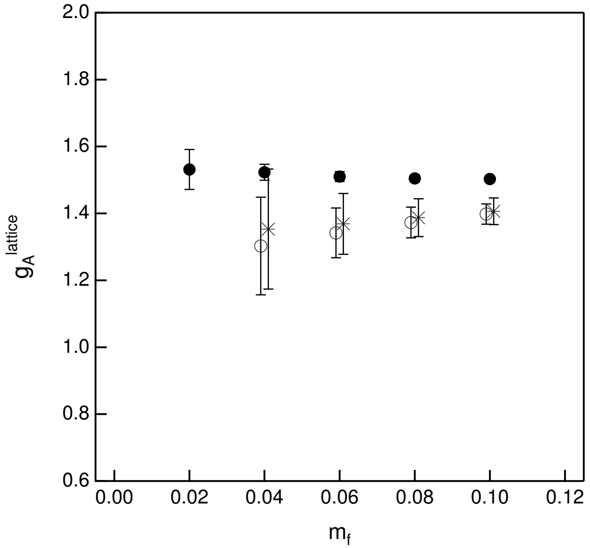
Bare $g_V^{\rm lattice}$ from sequential source $((2.4{\rm fm})^3)$:

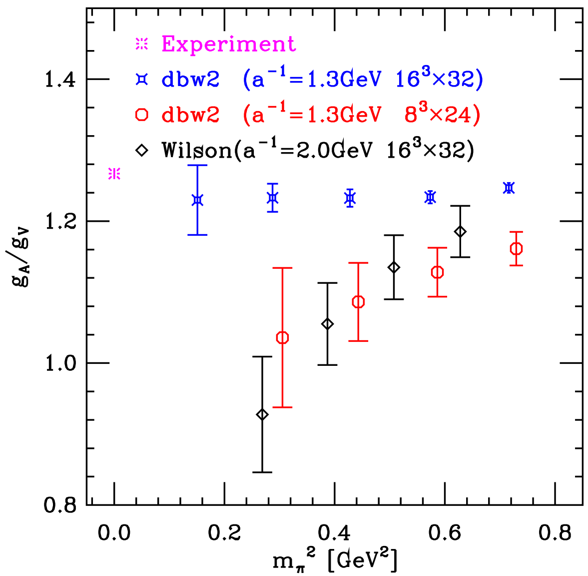


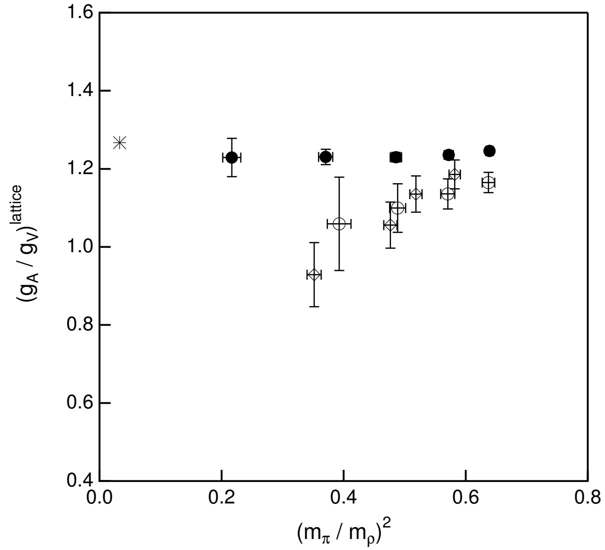
$(g_A/g_V)^{\rm lattice}$ from sequential source $((2.4{\rm fm})^3)$:



$(g_A/g_V)^{\text{lattice}} = (g_A/g_V)^{\text{ren}}$: m_f and volume dependence in bare and physical scales (m_ρ and Sommer):

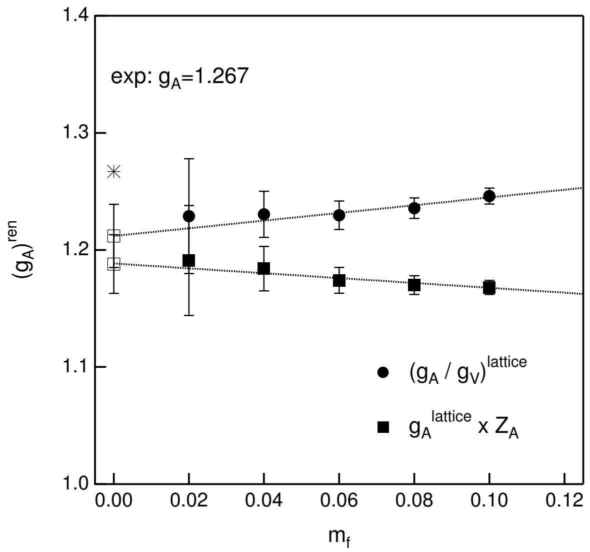






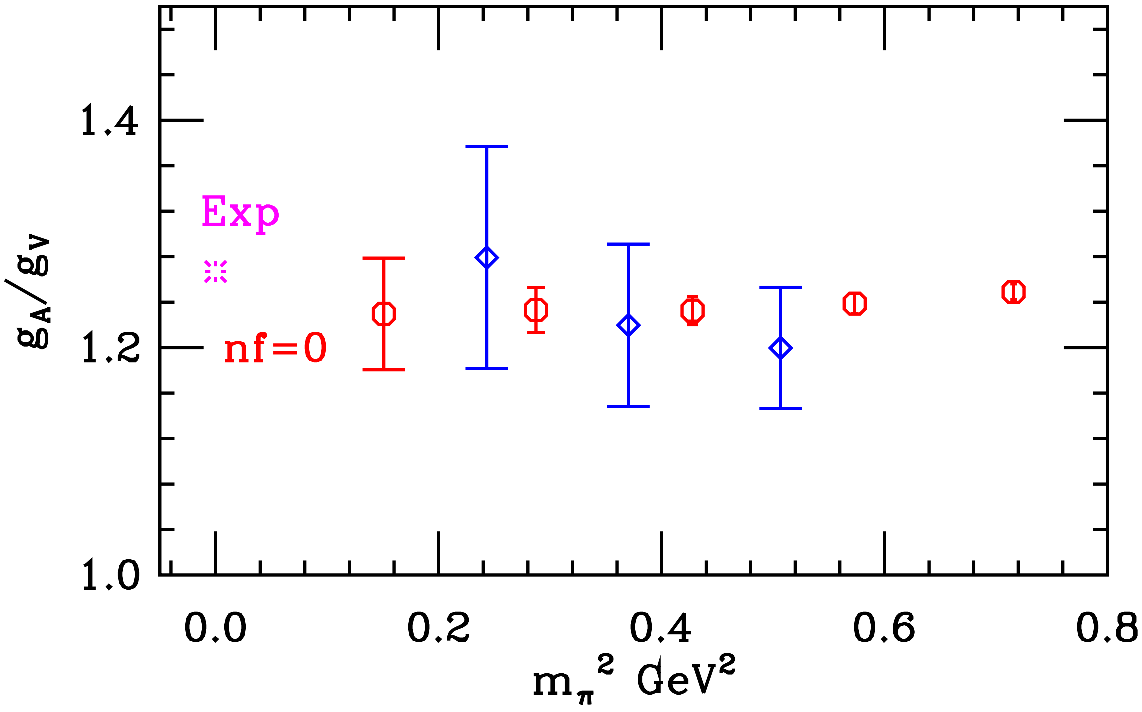
- Clear volume dependence is seen between $(2.4\text{fm})^3$ and $(1.2\text{fm})^3$ volumes.
- The large volume results (sequential)
  - show a very mild m_f dependence,
  - extrapolate to about 8 % under estimation, $g_A = 1.15(11)$.

Alternatively we can use $g_A^{\text{lattice}} \times Z_A$:



agree well with $(g_A/g_V)^{\text{lattice}}$ in the chiral limit, and and an expected difference seen away from there.

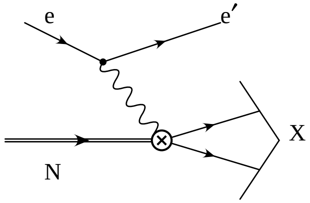
New, this year, of axial charge: dynamical result seems to follow the quenched [6].



[6] Note the lattice scales obtained from m_ρ and Sommer scale agree, with $a^{-1} \sim 1.7$ GeV.

Structure functions: measured in deep inelastic scatterings (and RHIC/Spin):

- DIS



$$\left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}$$
$$W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$$

$$W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu\right)\left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x,Q^2)}{\nu}$$
$$W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q \cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$$

with $\nu = q \cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

- The same structure funtions appear in RHIC/Spin (which also provides $h_1(x,Q^2)$).

Moments of the structure functions are accessible on the lattice:

$$2\int_0^1 dx x^{n-1} F_1(x,Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2),$$
$$\int_0^1 dx x^{n-2} F_2(x,Q^2) = \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2),$$
$$2\int_0^1 dx x^{n} g_1(x,Q^2) = \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),$$
$$2\int_0^1 dx x^{n} g_2(x,Q^2) = \frac{1}{2}\frac{n}{n+1} \sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n \rangle_q(\mu)$, $\langle x^n \rangle_{\Delta q}(\mu)$ and d_n are forward nucleon matrix elements of certain local operators.

Lattice operators:

- Unpolarized (F_1/F_2):

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} -(\text{trace})\right]q$$

On the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

- Polarized (g_1/g_2) and transversity (h_1):

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} -(\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1}-S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} -(\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu-S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} -(\text{traces})]q$$

On the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

- Higher moment operators mix with lower dimensional ones.
- Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

Renormalization: $\mathcal{O}^{\rm ren} = Z_{\mathcal{O}}(a\mu)\mathcal{O}^{\rm lat}(a)$,

- lattice complications: operator mixing from broken Lorentz or chiral symmetry,
- NPR is required when mixing with lower dimensional operator occurs.

We calculate $Z_{\mathcal{O}}(a\mu)$ non-perturbatively in RI/MOM scheme[7] with perturbative matching to $\overline{\rm MS}$.

- compute off-shell matrix element of the operator, $\mathcal{O}$, in Landau gauge,
- impose a MOM scheme condition $\mathrm{Tr}\, V_{\mathcal{O}}(p^2)\Gamma\big|_{p^2=\mu^2} \dfrac{Z_{\mathcal{O}}}{Z_q} = 1$,
  - $V_{\mathcal{O}}(p^2)$ is the relevant amputated vertex,
  - Γ is an appropriate projector,
- extrapolate to the chiral limit, defining the RI scheme,
- in an appropriate window, $\Lambda_{\rm QCD} \ll \mu^2 \ll a^{-1}$, a scale invariant

$$Z_{\rm rgi} = \frac{Z(\mu^2)}{C(\mu^2)}$$

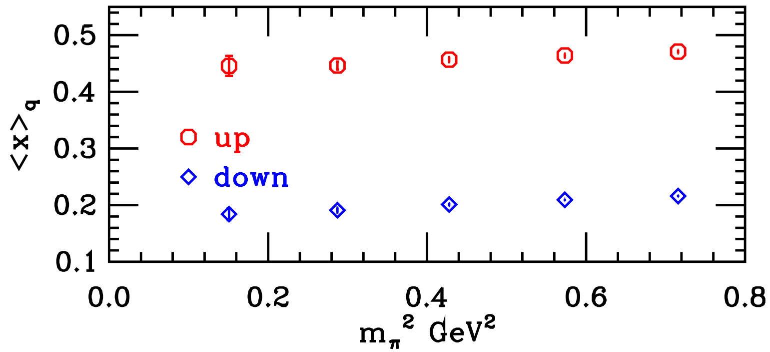
  is obtained, with the operator running $C(\mu^2)$ in the continuum perturbation theory.
- Now we can perturbatively match to e.g. $\overline{\rm MS}$.

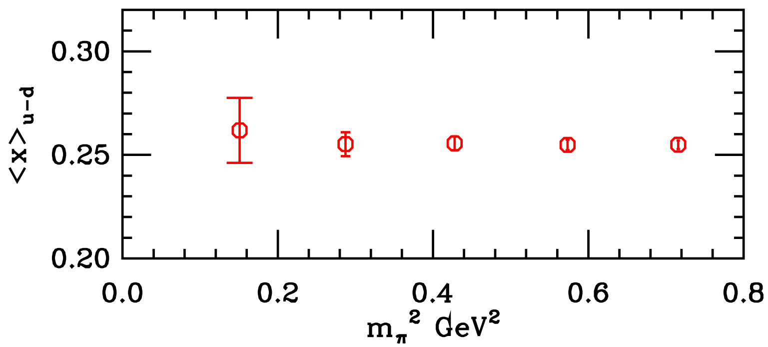
Works nicely with DWF.

[7] Martinelli et. al, Nucl. Phys. B455, 81 (1995).

Quark density $\langle x\rangle_{u-d}$, calculated with $\mathcal{O}^q_{44} = \bar{q}\left[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overleftrightarrow{D}_k\right] q$.

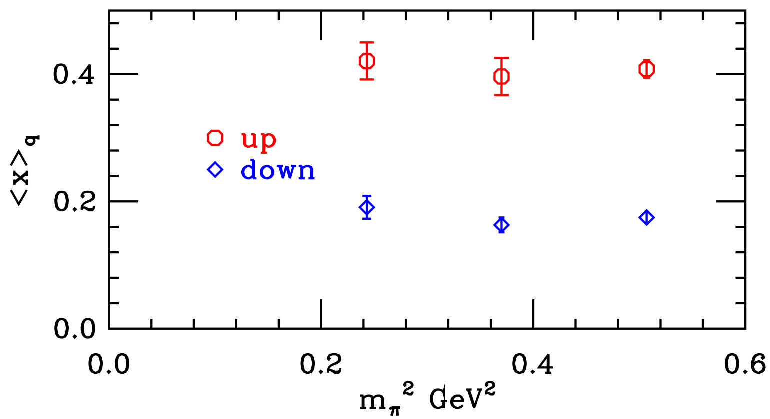
- Quenched calculation complete with NPR,

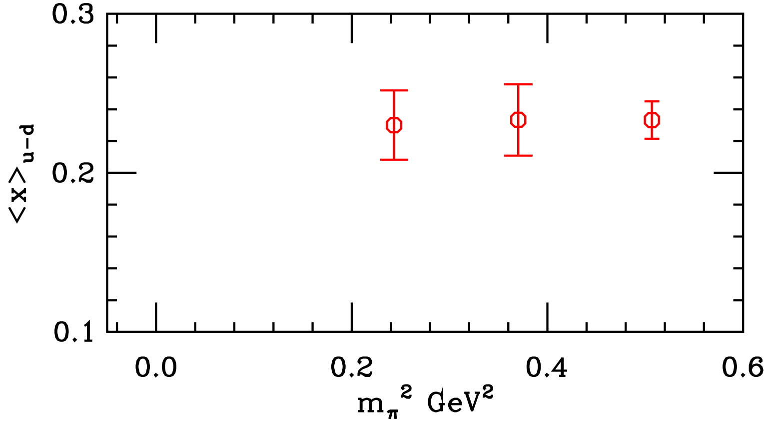




  - $Z = 1.02(10)$, with $\overline{\text{MS}}$ 2 GeV, 2-loop running,
  - no curvature seen in the chiral limit,
  - $\langle x\rangle_u / \langle x\rangle_d = 2.41(4)$ at the chiral limit.

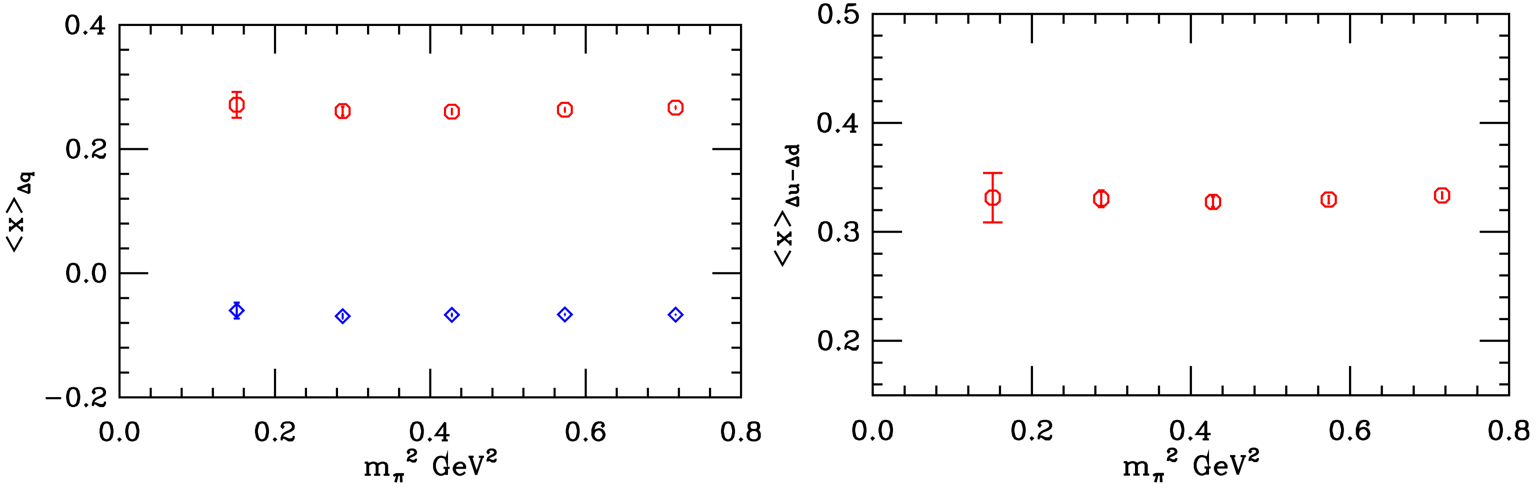
- Dynamical calculation ongoing, lacks NPR,





Polarization, $\langle x \rangle_{\Delta u - \Delta d}$, calculated with $\mathcal{O}^{5q}_{34} = \frac{1}{4}\bar{q}\gamma_5 \left[\gamma_3 \overleftrightarrow{D}_4 + \gamma_4 \overleftrightarrow{D}_3\right] q$.

- Quenched calculation complete with NPR,

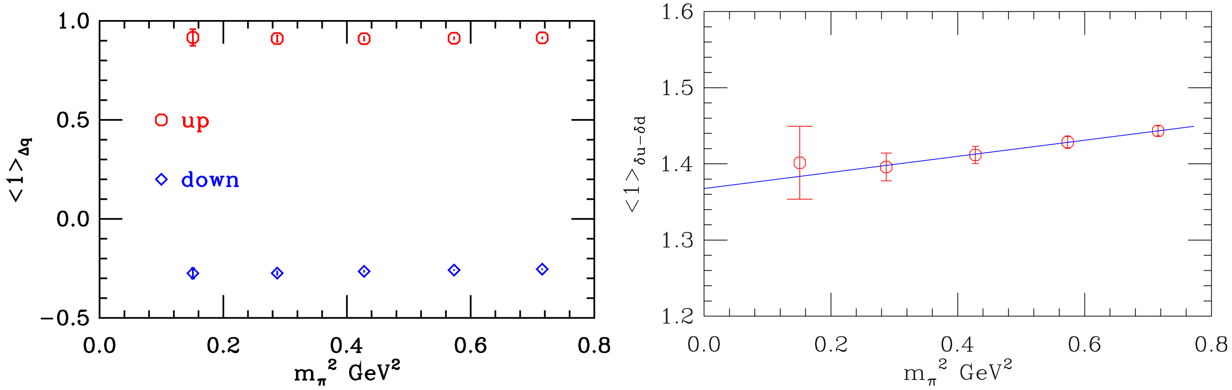


  - $Z = 1.02(9)$, with $\overline{\text{MS}}$ 2 GeV, 2-loop running,
  - no curvature seen in the chiral limit.

- Dynamical calculation ongoing.

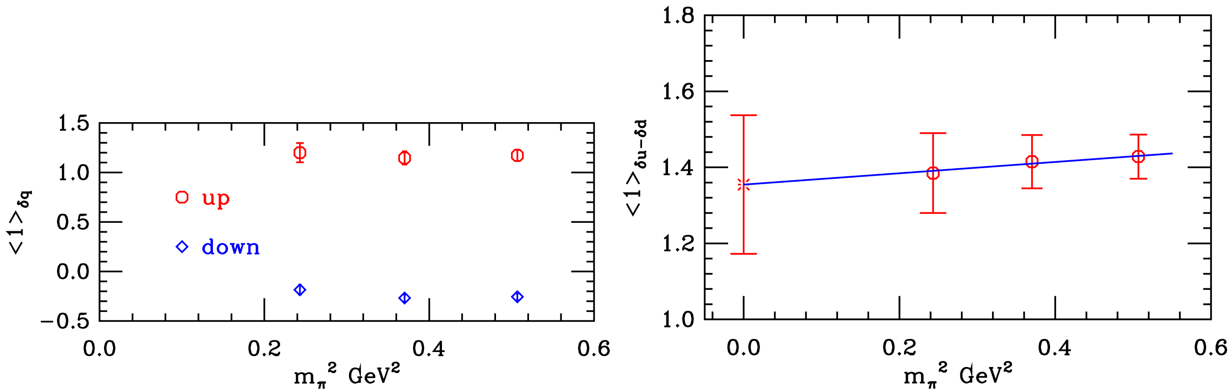
Transversity, $\langle 1 \rangle_{\delta u - \delta d}$, calculated with $\mathcal{O}_{34}^{\sigma q} = \bar{q}\gamma_5\sigma_{34}q$.

- Quenched calculation complete with NPR,



  - $\langle 1 \rangle_{\delta u - \delta d} = 1.193(30)$, $\overline{\text{MS}}$ (2 GeV) 2-loop running,
  - QCDSF (quenched continuum): $\langle 1 \rangle_{\delta u - \delta d} = 1.214(40)$, $\overline{\text{MS}}$ (1 GeV) 1-loop perturbative.

- Dynamical calculation ongoing, lacks NPR,



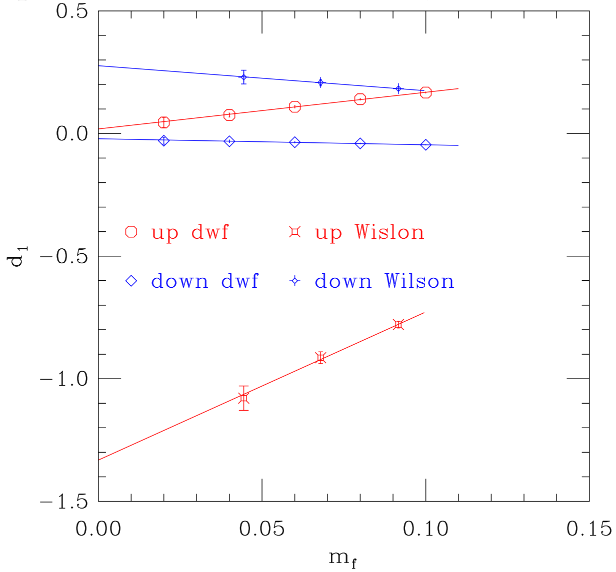
d_1: twist-3 part of g_2 ($\langle x \rangle_{\Delta q}$ is twist-2),

$$2\int_0^1 dx x^n g_2(x,Q^2) = \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d}[e^q_{2,n}(\mu^2/Q^2, g(\mu))\, d^q_n(\mu) - 2e^q_{1,n}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_{\Delta q}(\mu)],$$

calculated with $\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_{[\sigma}\overset{\leftrightarrow}{D}_{\mu_1]}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - \text{traces}\right]q.$

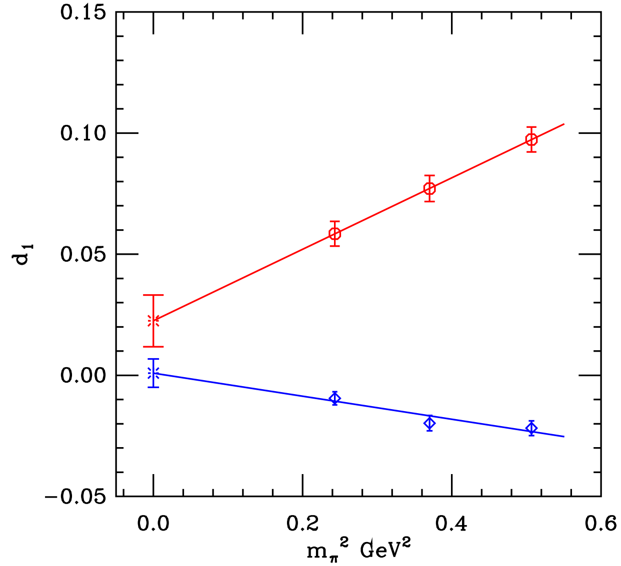
- negligible in Wandzura-Wilczek relation, $g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$,
- but need not be small in a confining theory (Jaffe and Ji, Phys. Rev. D43, 91),

quenched unrenormalized,



- small in the chiral limit (no power divergent mixing),
- disagree with Wilson fermion results (which suffer from power divergent mixing)?

dynamical unrenormalized,



- small in the chiral limit.

Nucleon decay (Yasumichi Aoki): proton decay with dimension 6 operators such as

$$\langle \pi^0 e^+ | qqql | p \rangle$$

or more precisely the hadronic matrix elements in general take the form of

$$\langle \pi^0 | i\epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L u^k | p \rangle$$

(SUSY) GUT processes: classification by $SU(3) \times SU(2) \times U(1)$ leads to a complete set of operators. Relevant for p/n decay are,

$$\begin{array}{ll} \langle \pi^0 | i\epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L u^k | p \rangle, & \langle \pi^+ | i\epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L d^k | p \rangle, \\ \langle K^0 | i\epsilon_{ijk} (u^{iT} C P_{L/R} s^j) P_L u^k | p \rangle, & \langle K^+ | i\epsilon_{ijk} (u^{iT} C P_{L/R} s^j) P_L d^k | p \rangle, \\ \langle K^+ | i\epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L s^k | p \rangle, & \langle K^0 | i\epsilon_{ijk} (u^{iT} C P_{L/R} s^j) P_L d^k | n \rangle, \\ \langle \eta | i\epsilon_{ijk} (u^{iT} C P_{L/R} d^j) P_L u^k | p \rangle, & \end{array}$$

and those obtained through the exchange of u and d.

Lattice methods:

- indirect: chiral perturbation (tree level) + low-energy constant (lattice), *ie*

$$\mathcal{L}_\chi(\text{mesons and baryons: } D, F, f_{\text{meson}}, m_{\text{baryon}}) + (\text{baryon decay interaction: } \alpha, \beta),$$

- direct: calculate all the relevant 2- and 4-point functions on the lattice.

Issues:

- direct method is about 10 times more expensive,
- indirect and direct results disagree (Gavela et al (1989)),
- |indirect| = |direct|+ ∼ 50 % (JLQCD (2000)).

Direct method:

$$\langle\pi^0|i\epsilon_{ijk}(u^{iT}CP_{L/R}d^j)P_L u^k|p\rangle = P_L[W_0(q^2) - W_q(q^2)i(\gamma q)]u_p,$$

where q is the momentum transfer of $p \to \pi^0$.

- as $i(\gamma q)v_e \sim m_e v_e$ is negligible, we need to extract W_0,
- yet the mixing of W_q is inevitable because we also need to project to positive parity proton,

$$\mathrm{tr}\left(P_L[W_0 - W_q i(\gamma q)]\frac{1+\gamma_4}{2}\right) = W_0 - iq_4 W_q,$$

- we go around this by injecting finite momentum (JLQCD, PRD 62, 014506 (2000)),

$$\mathrm{tr}\left(P_L[W_0 - W_q i(\gamma q)]\frac{1+\gamma_4}{2}i\gamma_j\right) = q_j W_q.$$

Slightly different sequential propagators are used.

Remaining problems:

- chiral symmetry,
  - previous studies used Wilson fermions which explicitly break chiral symmetry,

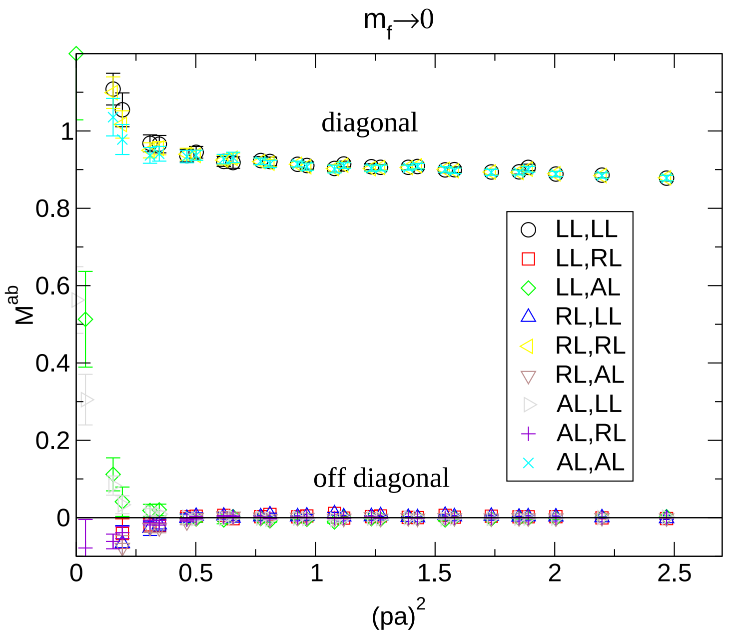
$$O_{RL}^{\rm cont} = Z O_{RL}^{\rm latt} + Z_{\rm mix} O_{LL}^{\rm latt} + Z'_{\rm mix} O_{\gamma_\mu L}^{\rm latt}$$

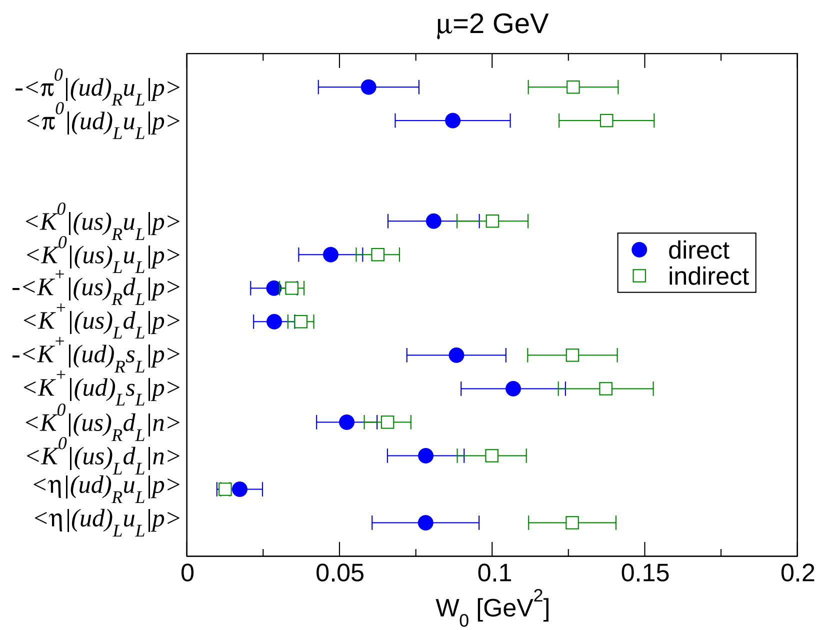
  - so the results need not match the chiral perturbation,
  - with DWF better chiral symmetry, the indirect method may work.
- $\mathcal{O}(a)$ scaling violation,
- quenched approximation.

DWF:

- good chiral symmetry, $O_{RL}^{\rm cont} = Z O_{RL}^{\rm latt}$,
  - should match the chiral perturbation at finite a,
  - if the low-energy coefficients are calculated on the lattice,
  - note f_π and g_A $(=D+F)$ are consistent with experiment within a few % even at finite a,
- scaling violation starts at $\mathcal{O}(a^2)$,

Renormalization: NPR works well, one-loop matching from MOM to $\overline{\mathrm{MS}}$(NDR), two-loop running to 2 GeV.

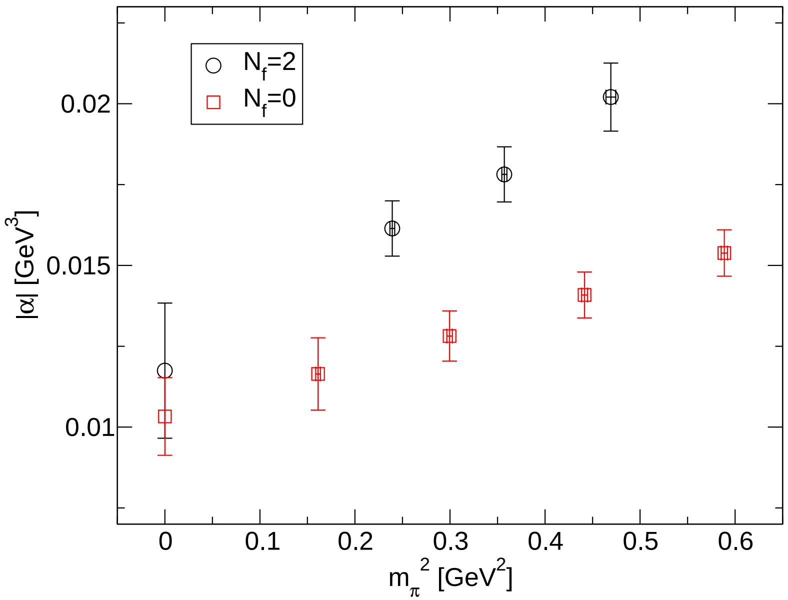




Quenched results: the direct and indirect methods disagree with each other. We have to

- follow through the direct method, or
- work out higher order chiral pertubation.

Quenching error: estimated in the indirect method, appear small from $\frac{1}{2}m_s \leq m_{\rm sea} \leq m_s$.



The dynamical result shows stronger dependence on m_π, but the extrapolation to the chiral limit is consistent with that of the quenched within ~ 20 % error.

Summary of the low energy parameter of nucleon decay at the renormalization scale $\mu = 2$ GeV. Quoted errors for DWF are statistical only. $\alpha + \beta = 0$ within the error.

Fermion	Wilson[a]	DWF	
N_f	0	0	2
a [fm]	0	0.15	0.12
$\lvert\alpha\rvert$ [GeV^3]	$0.0090(09)(^{+5}_{-19})$	0.010(1)	0.012(2)
$\lvert\beta\rvert$ [GeV^3]	$0.0096(09)(^{+6}_{-20})$	0.011(1)	0.012(2)

[a] Tsutsui *et al.*, [CP-PACS Collaboration], arXiv:hep-lat/0402026.

Need to explore much lighter quark mass with dynamical flavors. The direct method is favored.

New, this year, are

- axial charge
  - dynamical result seems to follow the quenched,
- quark density $\langle x \rangle_{u-d}$,
  - quenched calculation complete with NPR (no curvature seen in the chiral limit),
  - dynamical calculation ongoing, lacks NPR,
- polarization $\langle x \rangle_{\Delta u - \Delta d}$,
  - quenched calculation complete with NPR (no curvature seen in the chiral limit),
  - dynamical calculation ongoing, lacks NPR,
- transversity, $\langle x \rangle_{\delta u - \delta d}$,
  - quenched calculation complete with NPR,
  - dynamical calculation ongoing, lacks NPR,
- d_1: twist-3 part of g_2 ($\langle x \rangle_{\Delta q}$ is twist-2),
  - negligible in Wandzura-Wilczek relation of g_1 and g_2,
  - but need not be small in a confining theory (Jaffe and Ji, Phys. Rev. D43, 91),
  - small in the chiral limit in both quenched and dynamical (unrenormalized),
  - disagree with quenched Wilson fermion results (which suffer from power divergent mixing)?
- Nucleon decay:
  - quenched calculation complete with NPR, in favor of the direct method,
  - dynamical calculation well under way.

Conclusions

- Quenched calculations are almost complete with NPR.
- $N_f = 2$ dynamical calculations are well under way.
- Axial charge: dynamical result seems to follow the quenched,
  - seem to agree well with the experiment,
  - no curvature seen down to 390 MeV pion mass.
- Moments of structure functions: quenched results almost complete with NPR,
  - no curvature seen in $\langle x\rangle_{u-d}$, $\langle x\rangle_{\Delta u-\Delta d}$ and $\langle 1\rangle_{\delta u-\delta d}$ down to 390 MeV pion mass,

  dynamical calculations are ongoing,
  - d_1 in the chiral limit seems small in both quenched and dynamical.
- Nucleon decay: quenched calculation almost complete with NPR,
  - favors the direct method,

  dynamical calculation well under way.

Immediate futre

- Publish quenched results for structure functions and nucleon decay.
- Finish ongoing dynamical calculations (QCDSP/QCDOC).
- Explore lighter quark mass and (2+1)-flavor dynamical (QCDOC).
- Turn on observables with finite momentum: some form factors, e.g. F_1, F_2, g_P and electric dipole and higher moments of the structure functions.