

# Calculating $\epsilon'/\epsilon$ using staggered fermions

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# Collaboration

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- W. Lee (Seoul National University)
- G. Fleming (Jefferson Lab)
- T. Bhattacharya (Los Alamos National Lab)  
R. Gupta (Los Alamos National Lab)  
G. Kilcup (Ohio State University)
- QCDSP support: N. Christ, G. Fleming, G. Liu,  
R. Mawhinney, L. Wu.



## $Re(A_2)$ , $Re(A_0)$ , and $\epsilon'/\epsilon$ (Exp)

- $\langle (\pi\pi)_I | H_W | K^0 \rangle = A_I e^{i\delta_I}$
- $\omega = \frac{ReA_2}{ReA_0} = 0.045$
- $ReA_2 = 1.50 \times 10^{-8} GeV$
- $ReA_0 = 33.3 \times 10^{-8} GeV$
- $\epsilon'/\epsilon = -\frac{\omega}{\sqrt{2}|\epsilon|} \left[ \frac{ImA_0}{ReA_0} - \frac{ImA_2}{ReA_2} \right]$   
KTeV  $\rightarrow Re(\epsilon'/\epsilon) = (20.7 \pm 2.8) \times 10^{-4}$   
NA48  $\rightarrow Re(\epsilon'/\epsilon) = (15.3 \pm 2.6) \times 10^{-4}$



# Main goals of the staggered $\epsilon'/\epsilon$ project.

- Check the results obtained using the **quenched** Domain-Wall fermions:  $\rightarrow$  **negative**  $\epsilon'/\epsilon$  (CP-PACS, RBC).
- Calculate  $\epsilon'/\epsilon$  in the **full** QCD.
  - **Quenching**  $\rightarrow$  main systematic errors.
  - Using **the improved actions**  $\rightarrow$  **HYP** or  $\overline{\text{Fat7}}$ .
- Comparison with experiment  $\rightarrow$  new physics (?)



# Why staggered fermions?

- Advantage:
  1. Conserved  $U(1)_V \otimes U(1)_A$ .
  2. Quark mass is protected (No Residual Mass).
  3. Computationally cheap ( $< 50 \times 4$ ).
  4. Discretization error =  $\mathcal{O}(a^2)$ .
  5. Easy to improve (no extra cost).
- Disadvantage
  1. Broken  $SU(4)$  Flavor (Taste) Symmetry.
  2. Operator mixing matrix of  $65536 \times 65536$ .
  3. NPR is impractical.



# The First Stage

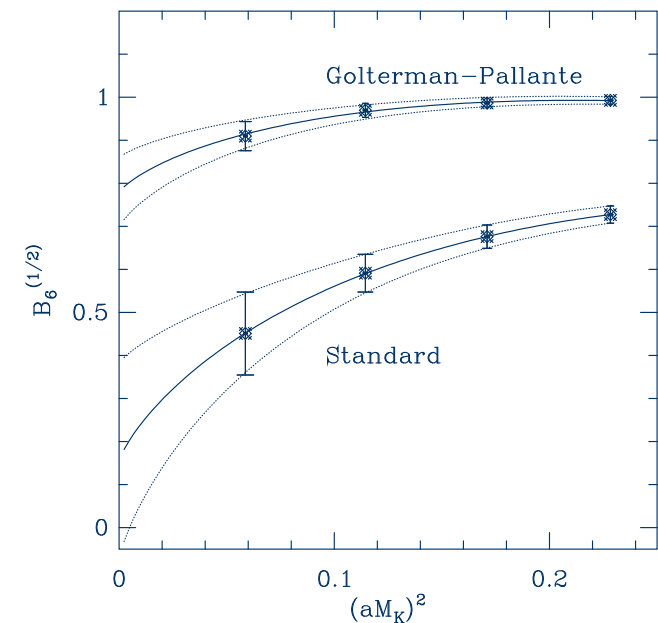
## Perturbative matching for unimproved staggered operators

- Penguin diagrams (Sharpe, Patel, 1993)  
→ small ( $\approx 2\%$ ).
- Current-current diagrams (W. Lee, 2001)  
→ large ( $\approx 100\%$  for  $[P \times P][P \times P]$ ).
- Hence, we must improve the staggered fermion action and operators to reduce the perturbative correction.



## The Second Stage:

- Numerical study with unimproved staggered
- Check the previous work (Kilcup, Pekurovsky).
- Use fully one-loop matched operators.
- Nucl. Phys. B (Proc. Suppl.) 106, 311; hep-lat/0111004.
- Large quenching uncertainty.





# The Third Stage:

## Improving staggered fermions

- AsqTad: Fat7 + Lepage + Naik
- Fat7
- Fat7 + Lepage
- Hypercubic (HYP)
- Other possibilities:  $\overline{\text{Fat7}}$ , APE



# Main goal of improvement:

Reduce perturbative corrections

- **Key idea:**

Suppress the taste changing interaction of high momentum gluons with quarks, using the Symanzik improvement program and smearing.

- **Numerical Studies find:**

- Taste symmetry breaking substantially reduced.  
(Hasenfratz, Knechtli)
- Largest reduction in HYP and APE.



# Bilinear Operator Renormalization

- W. Lee and S. Sharpe, PRD 66, (2002), 114501.
- Fat7 and HYP are the best in reducing the size of one-loop corrections. ( $\leq 10\%$  level)
- Comparable to those for Wilson and Domain Wall Fermions.
- This leads us to choosing HYP for our numerical study.



# The Fourth Stage: Study on HYP

- W. Lee, PRD 66 (2002) 114504.
- Properties:
  - Theorem 1  $SU(3)$  Projections.
  - Theorem 2 Triviality of renormalization.
  - Theorem 3 Multiple  $SU(3)$  Projection.
  - Theorem 4 Uniqueness
  - Theorem 5 Equivalence ( $HYP \leftrightarrow \overline{Fat7}$ )
- $|C_{HYP}| \ll |C_{thin}|$ ,  $SU(3)$  Projection  $\leftrightarrow$  T.I. for doublers.
- Renormalization simplified by  $\langle A_\mu A_\nu \rangle \rightarrow \langle B_\mu B_\nu \rangle$



# Current-current Diagrams for $\overline{\text{HYP/Fat7}}$

- W. Lee and S. Sharpe, PRD68, (2003) 054510; hep-lat/0306016.
- $\overline{\text{HYP/Fat7}} \approx 10\%$ .
- Tadpole improvement:  
 $C_N = 13.159$   
 $C_H = 1.4051$

Finite correction to  $(\mathcal{O}_3)_{II}$

Operators	$[S \times P][S \times P]_{II}$
<b>NAIVE</b>	$2 \times (95.6 - 6C_N)$
$\overline{\text{HYP/Fat7}}$	$2 \times (19.1 - 6C_H)$

Operators	$[P \times P][P \times P]_{II}$
<b>NAIVE</b>	$2 \times (111.3 - 2C_N)$
$\overline{\text{HYP/Fat7}}$	$2 \times (6.9 - 2C_H)$



# Penguin Diagrams for HYP

- K. Choi and W. Lee: hep-lat/0309070.
- Theorem 1 (Equivalence):

At the one loop level, diagonal mixing coefficients are identical between unimproved and improved staggered operators of HYP (I), HYP (II), Fat7, Fat7+Lepage, and  $\overline{\text{Fat7}}$ .

- Note that **AsqTad** is **NOT** included in the list.
- Off-diagonal mixing vanishes for Fat7, HYP (II), and  $\overline{\text{Fat7}}$ .



# The Fifth Stage:

## Numerical Study with HYP staggered

- W. Lee NPB (P.S.) 128 (2004) 125, hep-lat/0310047;  
T. Bhattacharya, *et al.*, hep-lat/0309105.
- $\beta = 6.0$ ,  $16^3 \times 64$  lattice, 218 confs.
- Bare quark mass shifted by  $\approx 2.5$ .  
 $Z_m \cong 2.5 \rightarrow Z_m \cong 1$ .
- Small perturbative correction ( $\approx 10\%$ ).
- Reduced taste symmetry breaking.



## $Re(A_2)$ , $Re(A_0)$ , and $\epsilon'/\epsilon$ (Exp)

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# Standard model $H_W$ (Theory)

- $H_W = \frac{G_F}{\sqrt{2}} V_{us} V_{ud} \sum_i \left( z_i(\mu) + \tau y_i(\mu) \right) Q_i(\mu)$
- Current-current:  $Q_1, Q_2 = (\bar{s}u)_L(\bar{u}d)_L$
- QCD Penguin:  $Q_3, Q_4, Q_5, Q_6 = (\bar{s}d) \sum_q (\bar{q}q)$
- EW Penguin:  $Q_7, Q_8, Q_9, Q_{10} = (\bar{s}d) \sum_q e_q (\bar{q}q)$
- Group representation:
  - $Q_1, Q_2, Q_9, Q_{10} \in (27_L, 1_R) + (8_L, 1_R)$
  - $Q_3, Q_4, Q_5, Q_6 \in (8_L, 1_R)$
  - $Q_7, Q_8 \in (8_L, 8_R)$



## $Re(A_0)$ and $Re(A_2)$ (Theory)

- Standard model:

$$ReA_I = \frac{G_F}{\sqrt{2}} |V_{ud}V_{us}| \left[ \sum_{i=1,2} z_i \langle Q_i \rangle_I + Re(\tau) \sum_{i=3}^{10} y_i \langle Q_i \rangle_I \right]$$

- $\langle Q_i \rangle_I \equiv \langle (\pi\pi)_I | Q_i | K^0 \rangle$
- $z_i, y_i \approx 1$  or  $\alpha = 1/129$
- $Re(\tau) = -Re(\lambda_t/\lambda_u) = 0.002$
- Hence, only  $\langle Q_1 \rangle_I$  and  $\langle Q_2 \rangle_I$  are important in  $ReA_I$ .

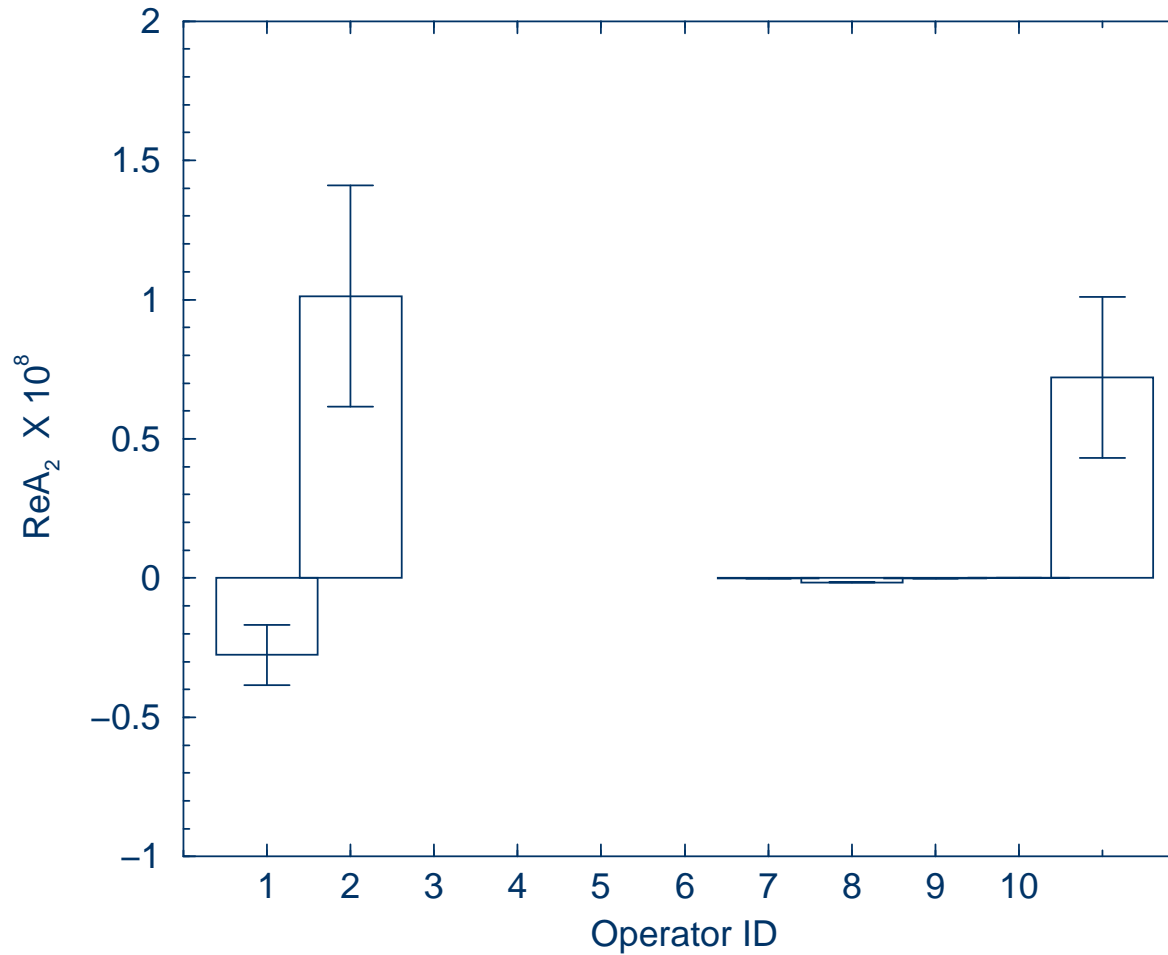


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$ReA_2 \times 10^8$

Linear fit in the chiral limit



$$ReA_2 = 1.50 \times 10^{-8} GeV \text{ (Experiment)}, \mu = 1/a$$

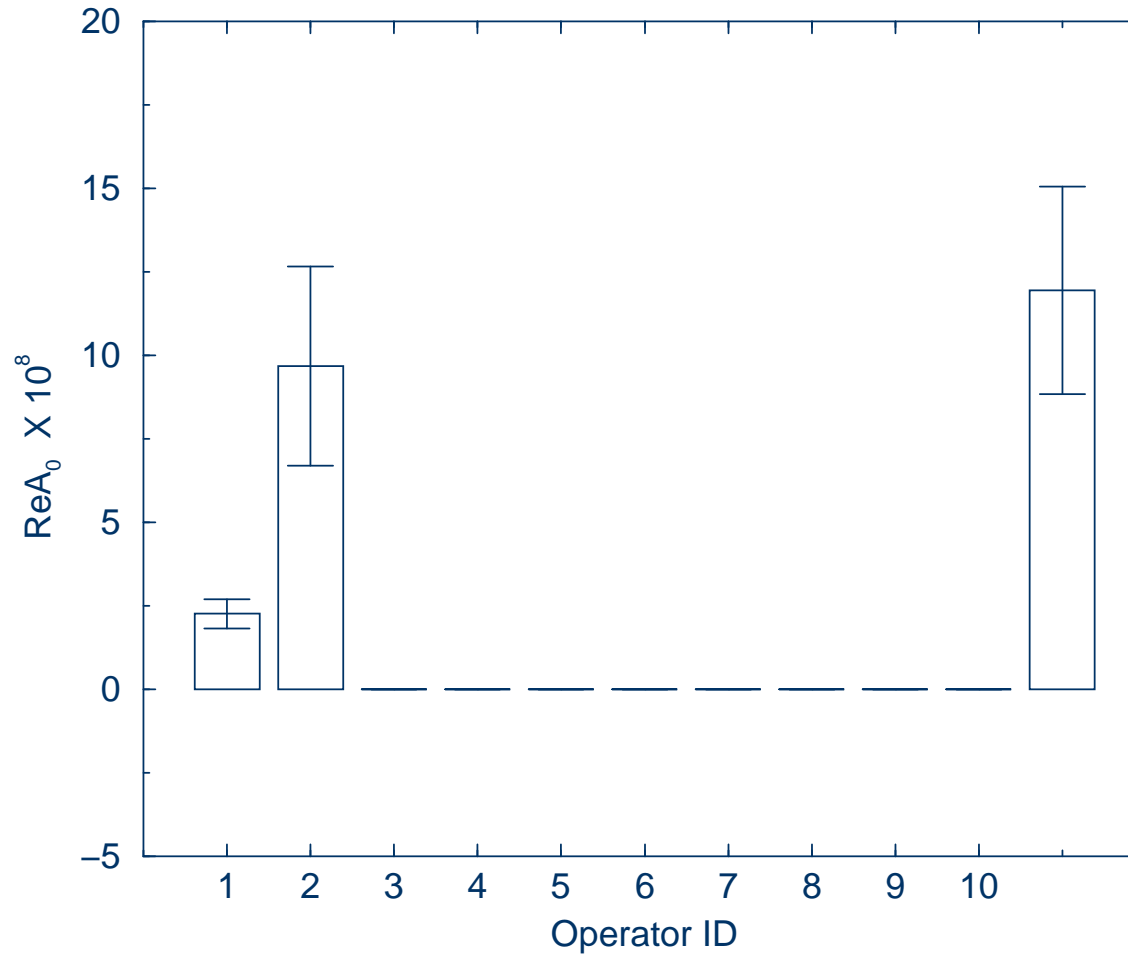


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$\text{Re}A_0 \times 10^8$

Linear fit in the chiral limit (std)



$$\text{Re}A_0 = 33.3 \times 10^{-8} \text{GeV (Experiment)}, \mu = m_c$$

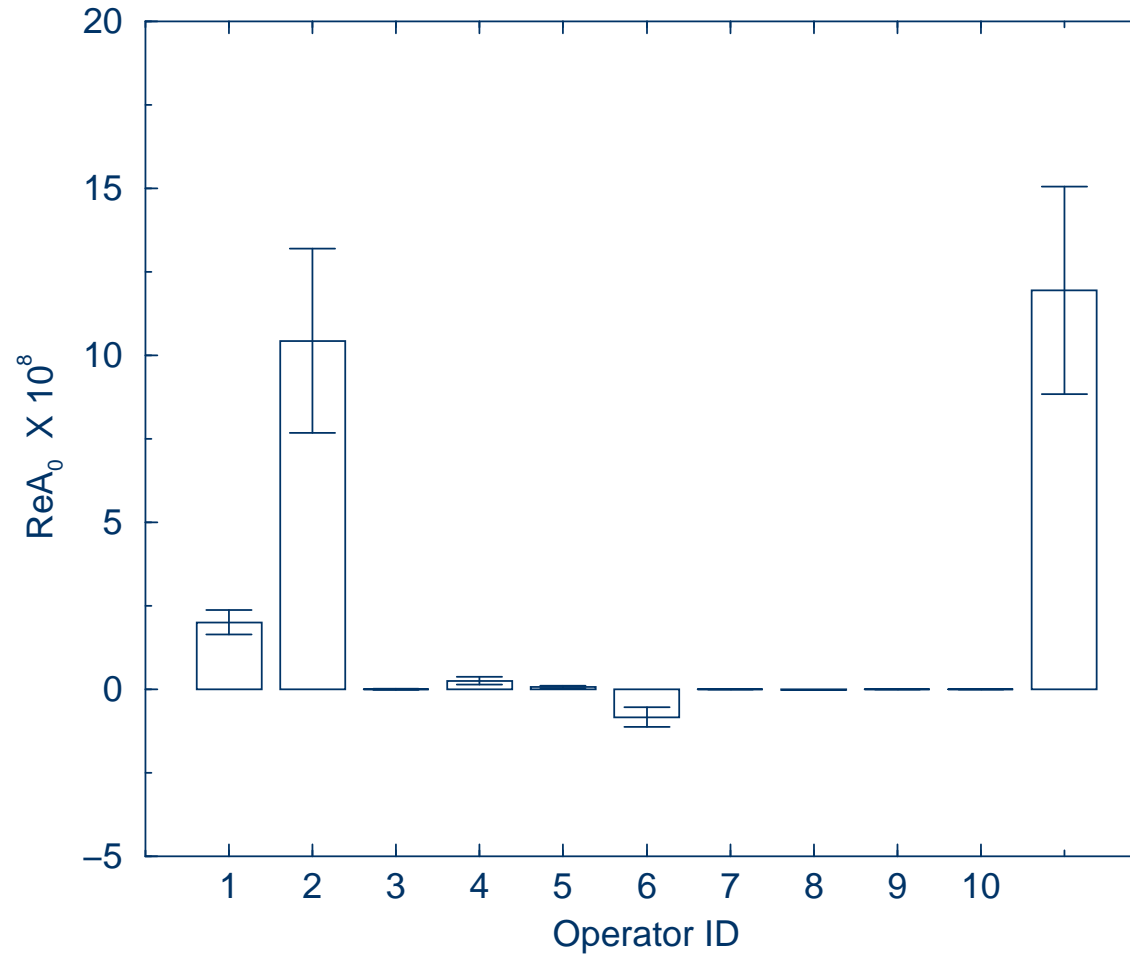


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$ReA_0 \times 10^8$

Linear fit in the chiral limit (std)



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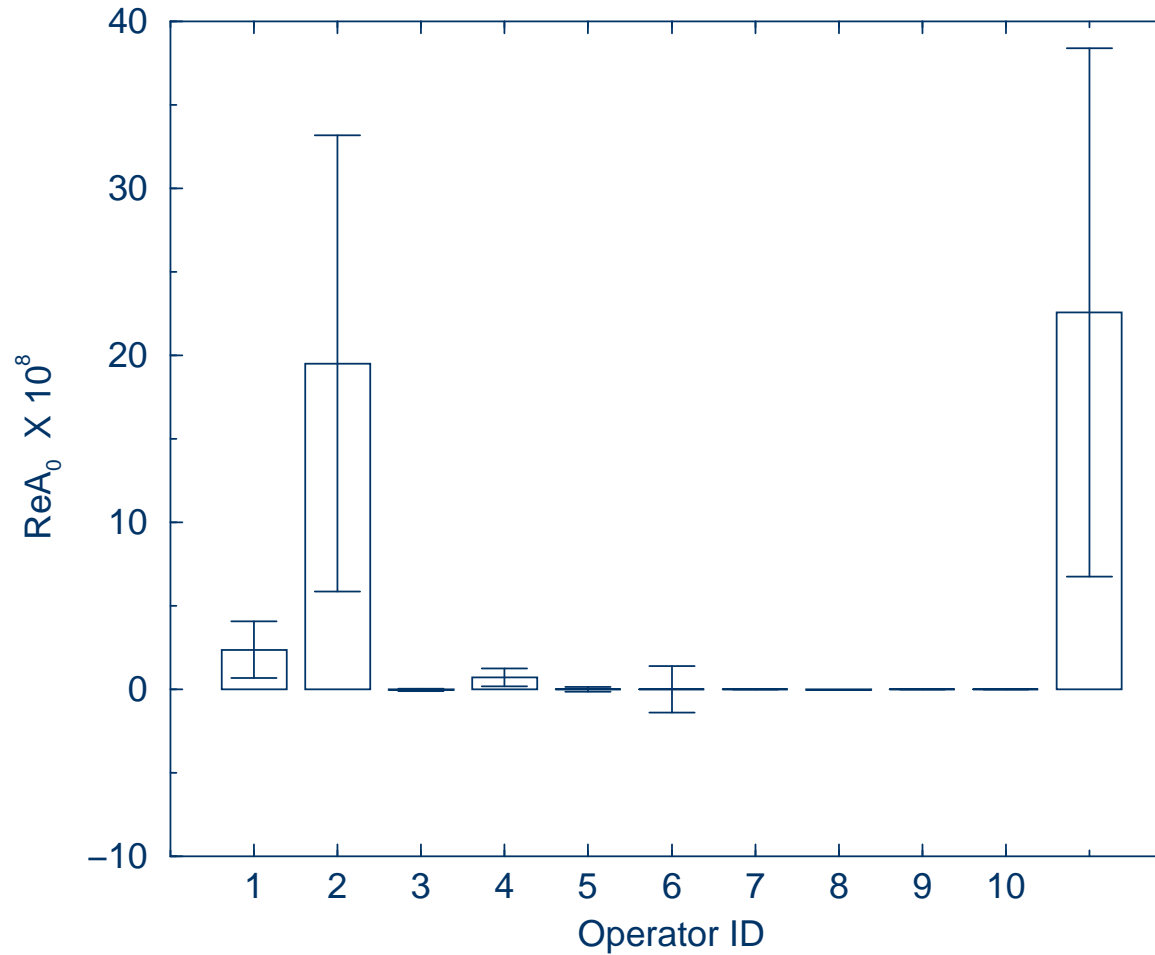


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$ReA_0 \times 10^8$

Lin-log fit in the chiral limit (std)



$ReA_0 = 33.3 \times 10^{-8} GeV$  (Experiment),  $\mu = 1/a$

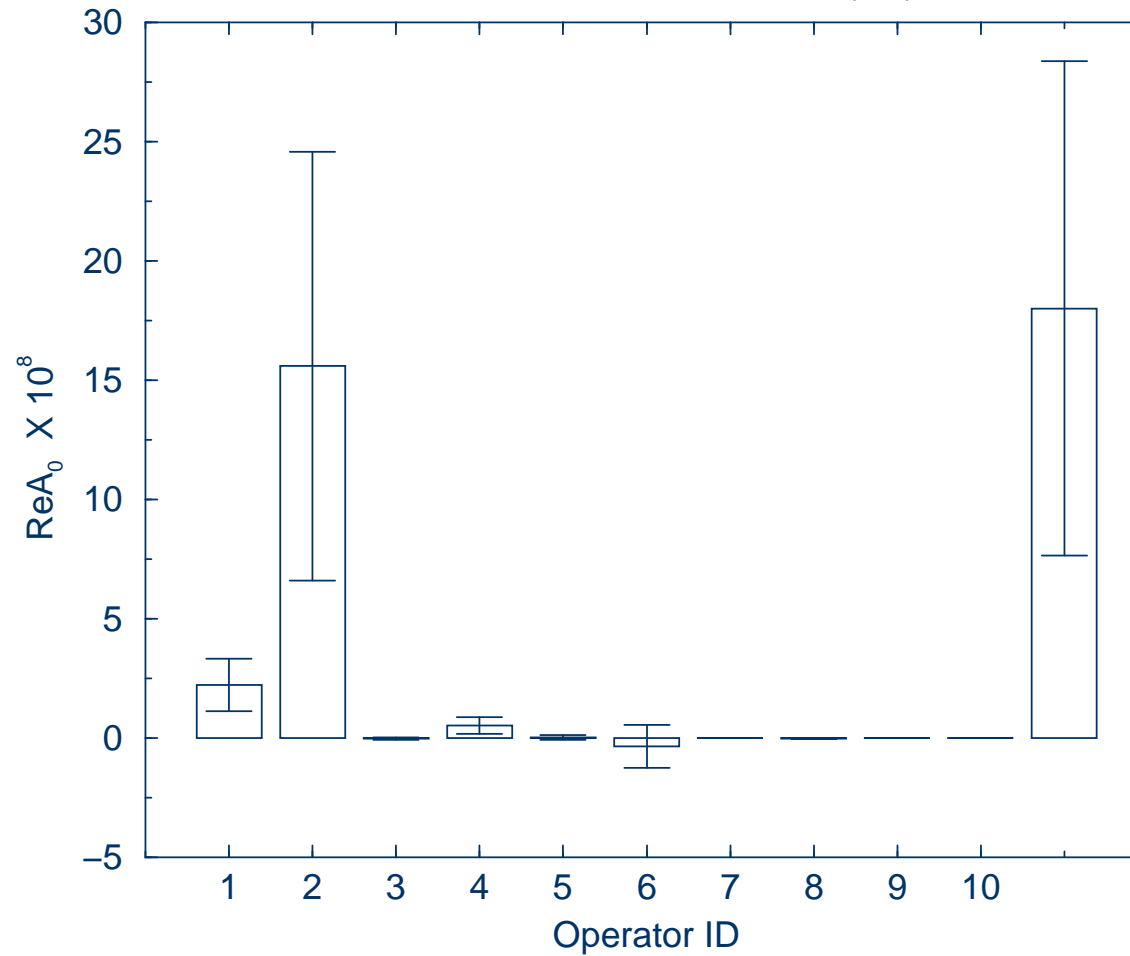


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$ReA_0 \times 10^8$

Quadratic fit in the chiral limit (std)



$$ReA_0 = 33.3 \times 10^{-8} GeV \text{ (Experiment)}, \mu = 1/a$$



## $\epsilon'/\epsilon$ (Theory)

- Standard model:

$$\epsilon'/\epsilon = \text{Im}(V_{ts}^* V_{td}) \left[ P^{(1/2)} - P^{(3/2)} \right]$$

$$P^{(1/2)} = r \sum_{i=3}^{10} y_i(\mu) \langle Q_i \rangle_0(\mu) (1 - \Omega_{\eta+\eta'})$$

$$P^{(3/2)} = \frac{r}{\omega} \sum_{i=3}^{10} y_i(\mu) \langle Q_i \rangle_2(\mu)$$

$$r = \frac{G_F \omega}{2|\epsilon| \text{Re} A_0}$$



## $\epsilon'/\epsilon$ (Theory)

- NO contribution from  $\langle Q_1 \rangle_I$  and  $\langle Q_2 \rangle_I$ .
- $P^{(1/2)}$  is dominated by  $\langle Q_6 \rangle$ .
- $P^{(3/2)}$  is dominated by  $\langle Q_8 \rangle$ .



## Lattice version of $Q_6$ and $Q_5$

- Flavor symmetry of quenched QCD =  $SU(3|3)$
- Flavor symmetry of full QCD =  $SU(3)$
- Hence, one may choose a singlet in  $SU(3)_R$  or a singlet in  $SU(3|3)_R$  in quenched QCD.
- The standard operator  $\in SU(3)_R$ .
- Golterman-Pallante operator  $\in SU(3|3)_R$ .

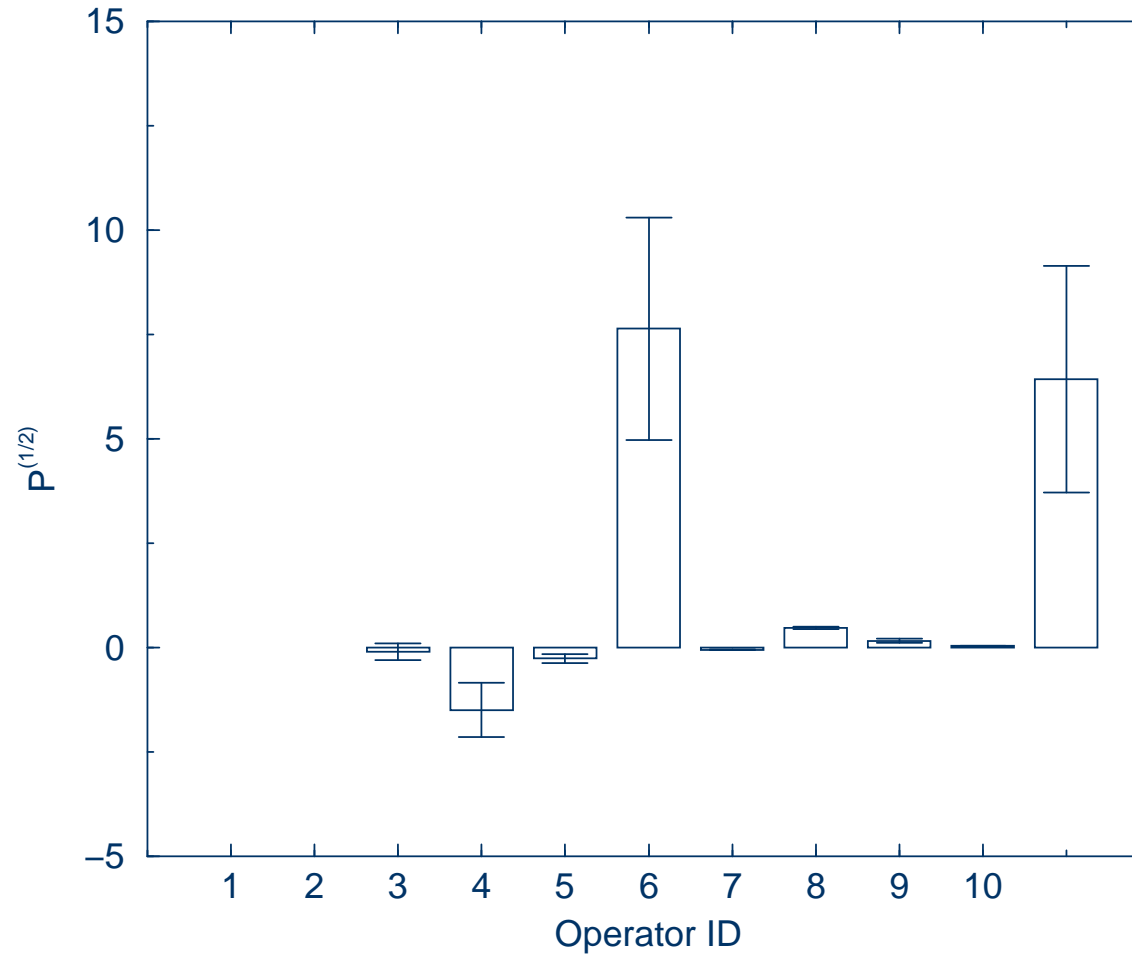


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# $P^{(1/2)}$ (Standard op)

Linear fit in the chiral limit



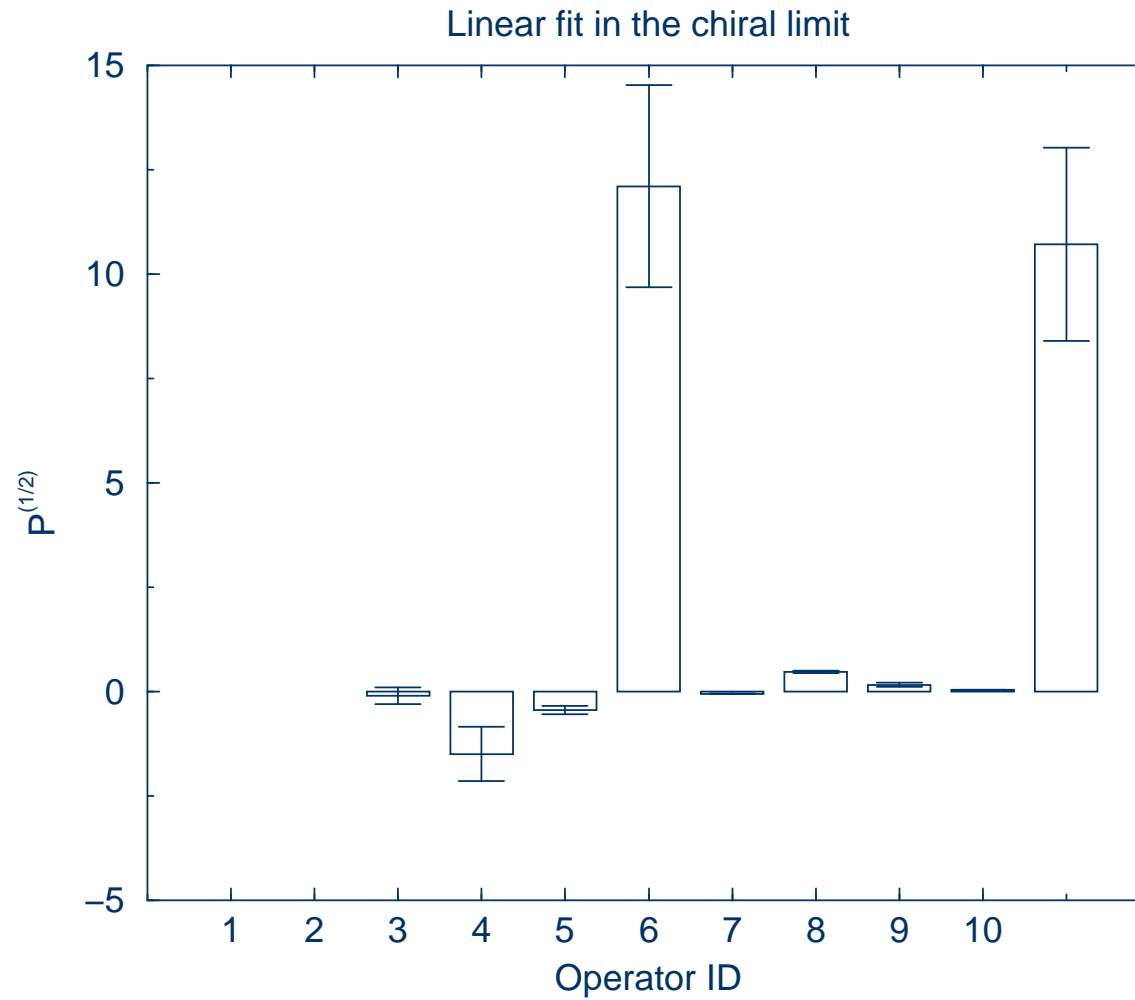
$$\mu = 1/a$$



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# $P^{(1/2)}$ (Golterman–Pallante op)



$$\mu = 1/a$$

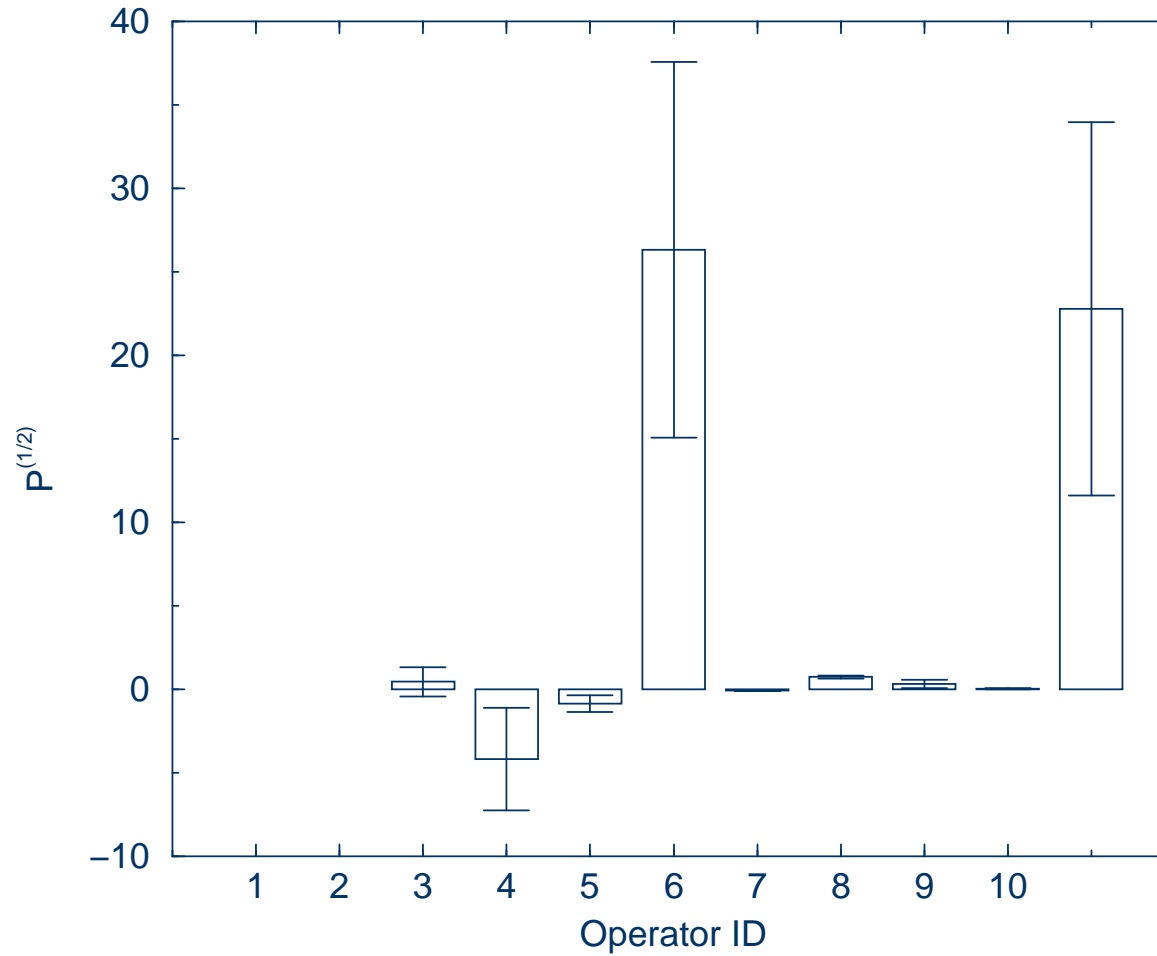


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# $P^{(1/2)}$ (Golterman–Pallante op)

Lin-log fit in the chiral limit



$$\mu = 1/a$$

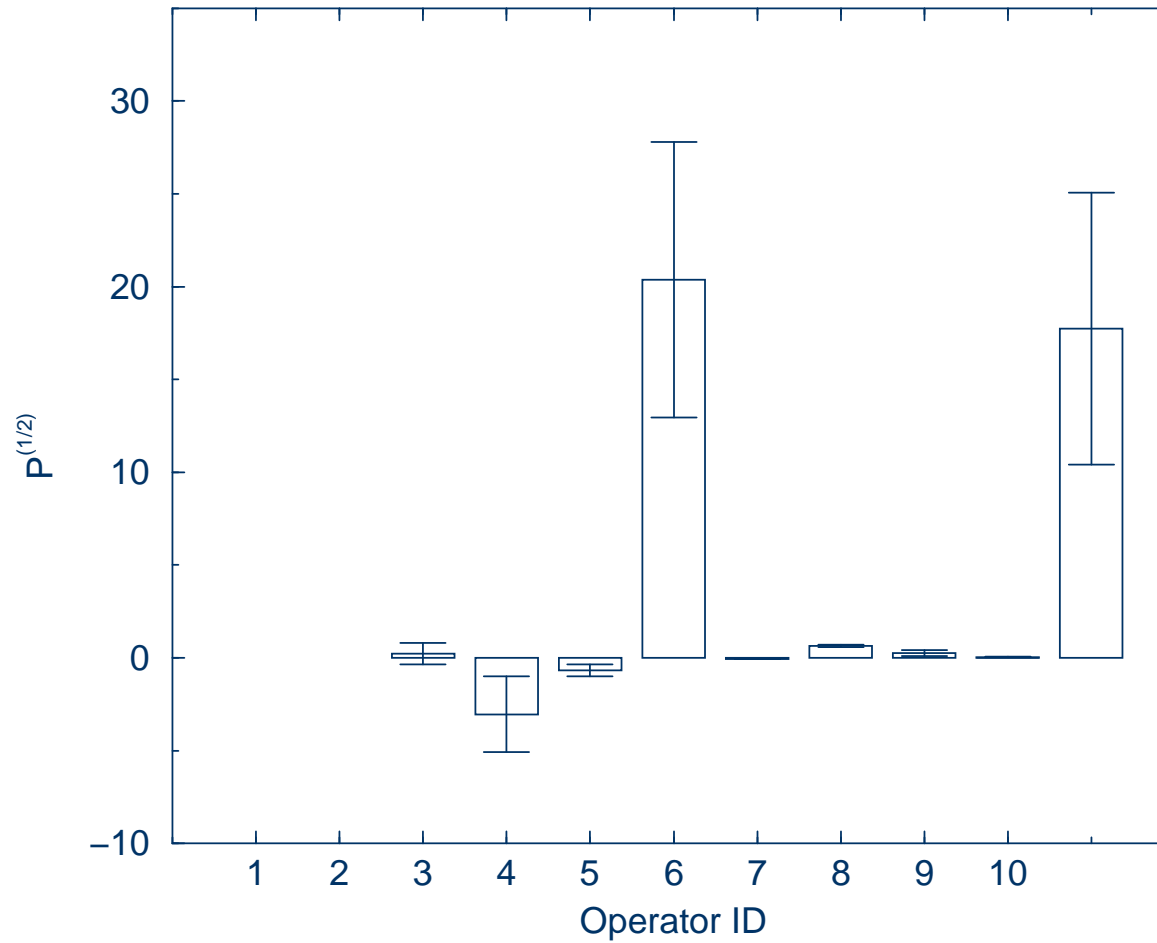


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# $P^{(1/2)}$ (Golterman–Pallante op)

Quadratic fit in the chiral limit



$$\mu = 1/a$$

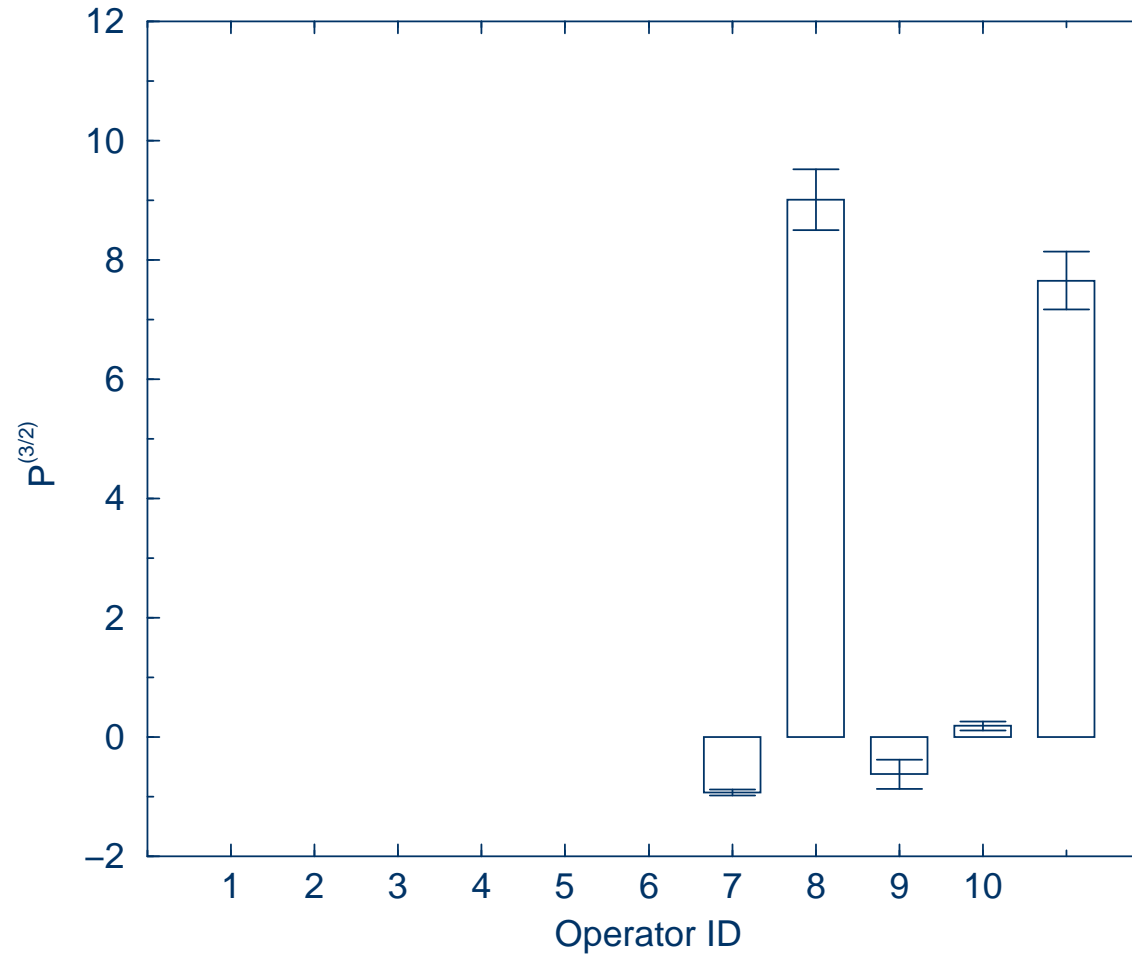


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$P^{(3/2)}$

Linear fit in the chiral limit



$$\mu = 1/a$$



## $\epsilon'/\epsilon$ ( $\mu = 1/a$ , **preliminary results**)

- Standard Operator:

$$\epsilon'/\epsilon = \begin{cases} -2.4(34) \times 10^{-4} & \text{(linear fit)} \\ -12.8(114) \times 10^{-4} & \text{(quadratic fit)} \\ -20.1(175) \times 10^{-4} & \text{(\chi-log fit)} \end{cases}$$

- Golterman-Pallante Operator:

$$\epsilon'/\epsilon = \begin{cases} +3.2(29) \times 10^{-4} & \text{(linear fit)} \\ +8.8(93) \times 10^{-4} & \text{(quadratic fit)} \\ +13.1(142) \times 10^{-4} & \text{(\chi-log fit)} \end{cases}$$



# Calculating $K \rightarrow \pi\pi$ on the lattice

- Calculate  $\langle \pi^+ | Q_i | K^+ \rangle$  and  $\langle 0 | Q_i | K^0 \rangle$  on the lattice
- Use  $\chi$ PT at leading order to obtain  $\langle \pi^+ \pi^- | Q_i | K^0 \rangle$

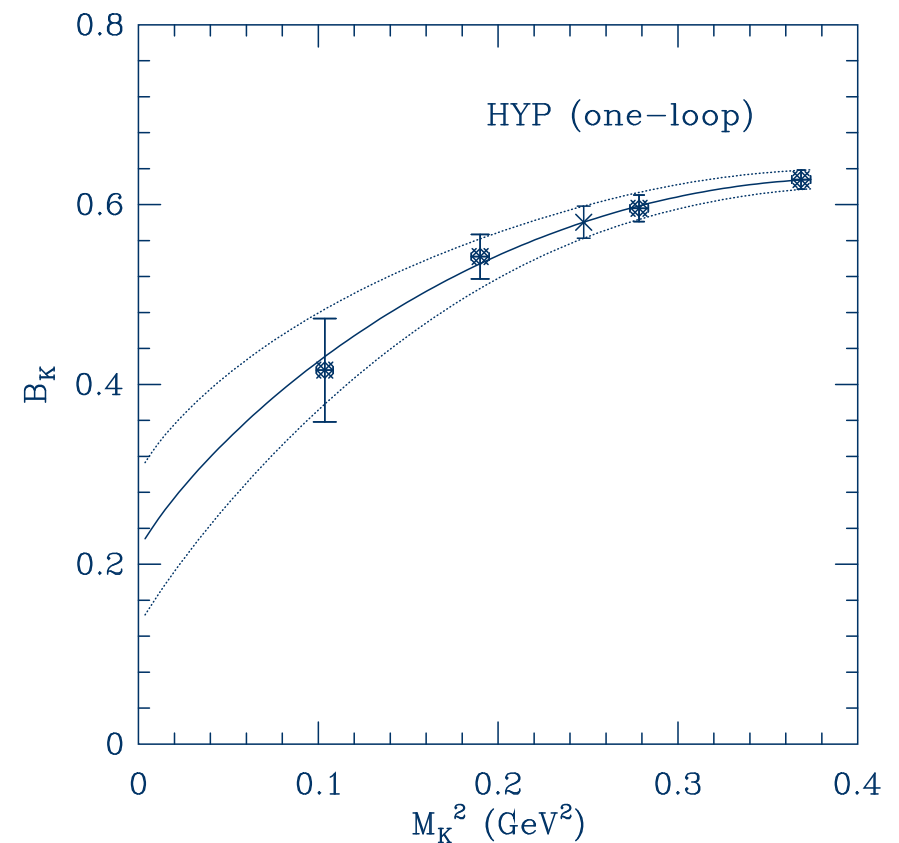
$$\left\{ \begin{array}{l} K^+ \rightarrow \pi^+ \\ K^0 \rightarrow 0 \end{array} \right\} \Longrightarrow \chi PT \Longrightarrow (K^+ \rightarrow \pi^+ \pi^-)$$

- Numerical study in quenched QCD.



# $B_K$

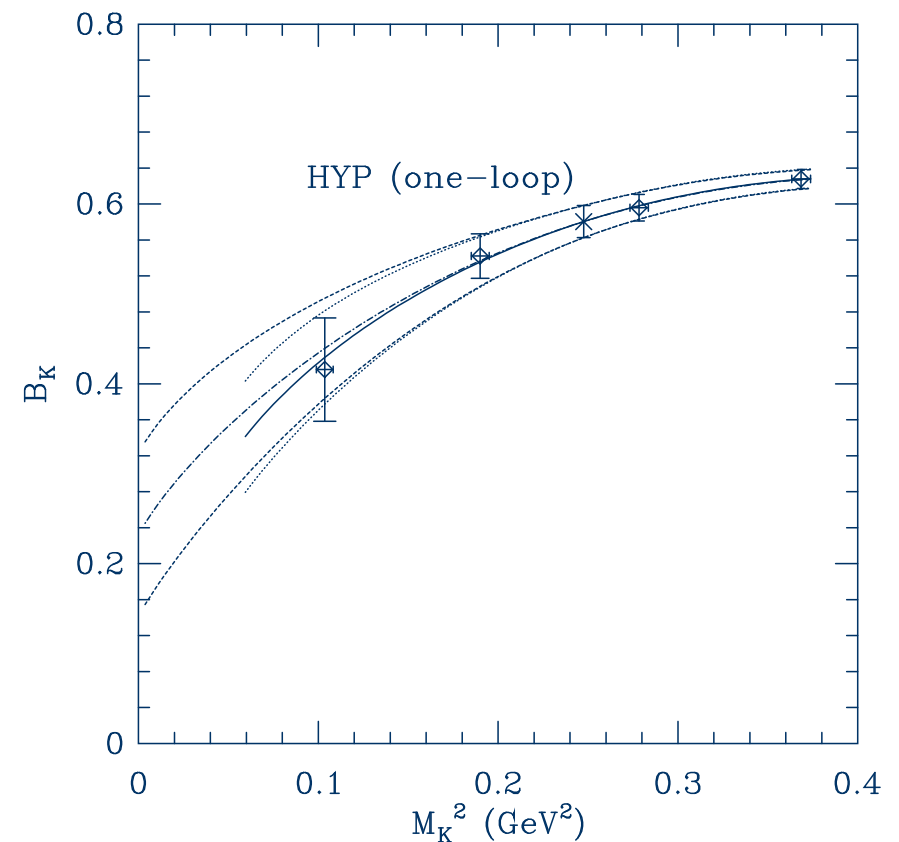
- Fit to quenched  $\chi$ PT.
- $B_K(m_K, \mu = 2\text{GeV}) = 0.580(18)$
- Consistent with JLQCD results at  $a = 0$ .
- $B_K(0, RGI) = 0.298(117)$
- Consistent with  $1/N_c$  results.





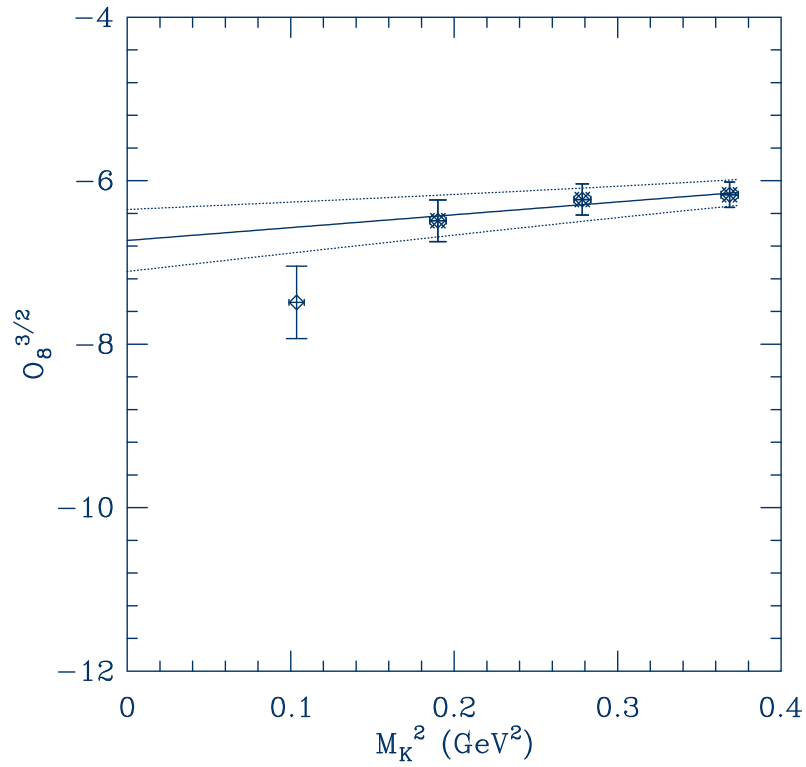
# Finite volume effect on $B_K$

- Fit to quenched  $\chi$ PT with finite volume corrections.
- $B_K(m_K, 2\text{GeV}) = 0.580(18)(41)$
- $B_K(0, RGI) = 0.321(125)(175)$
- Consistent with  $1/N_c$  results.

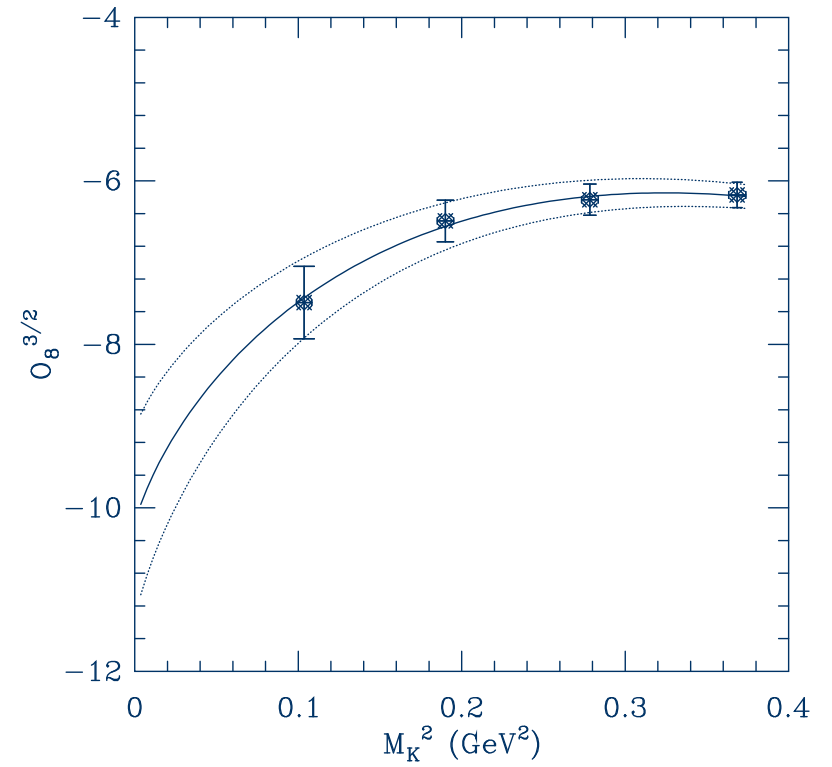




$$\langle Q_8^{\Delta I=3/2} \rangle$$



Linear fit



Chiral Log fit

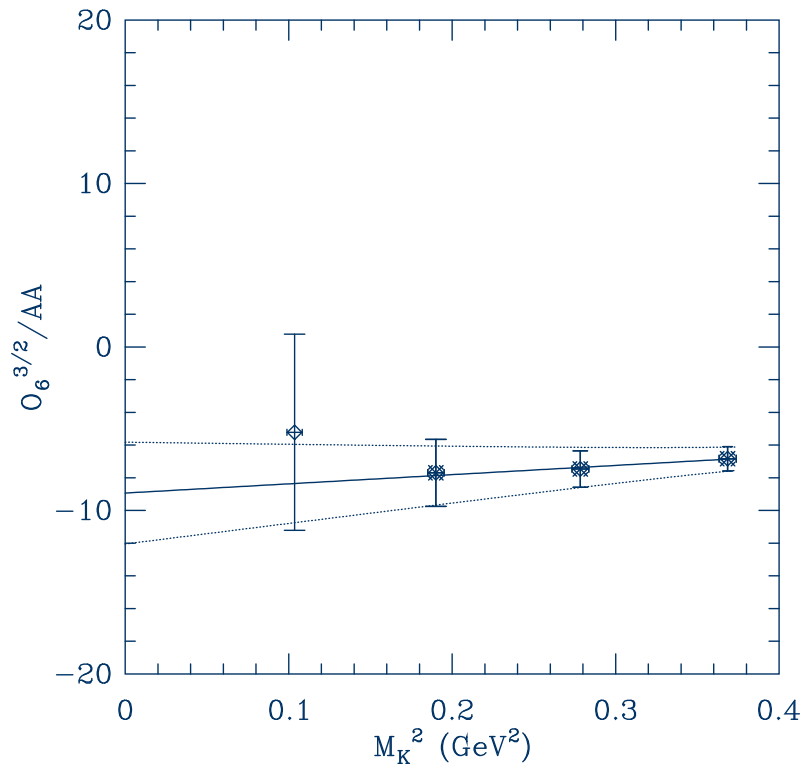


## Lattice version of $Q_6$

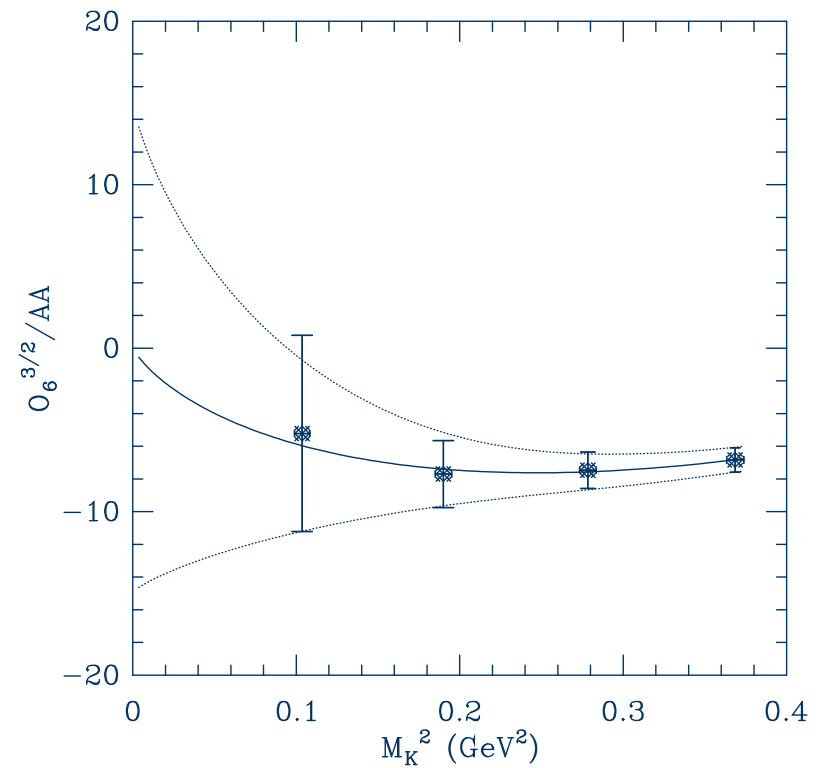
- Flavor symmetry of quenched QCD =  $SU(3|3)$
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- Hence, one may choose a singlet in  $SU(3)_R$  or a singlet in  $SU(3|3)_R$  in quenched QCD.
- The standard operator  $\in SU(3)_R$ .
- Golterman-Pallante operator  $\in SU(3|3)_R$ .



$$\frac{\langle Q_6^{\Delta I=1/2} \rangle}{m_K^2 f^2} \text{ (standard op)}$$



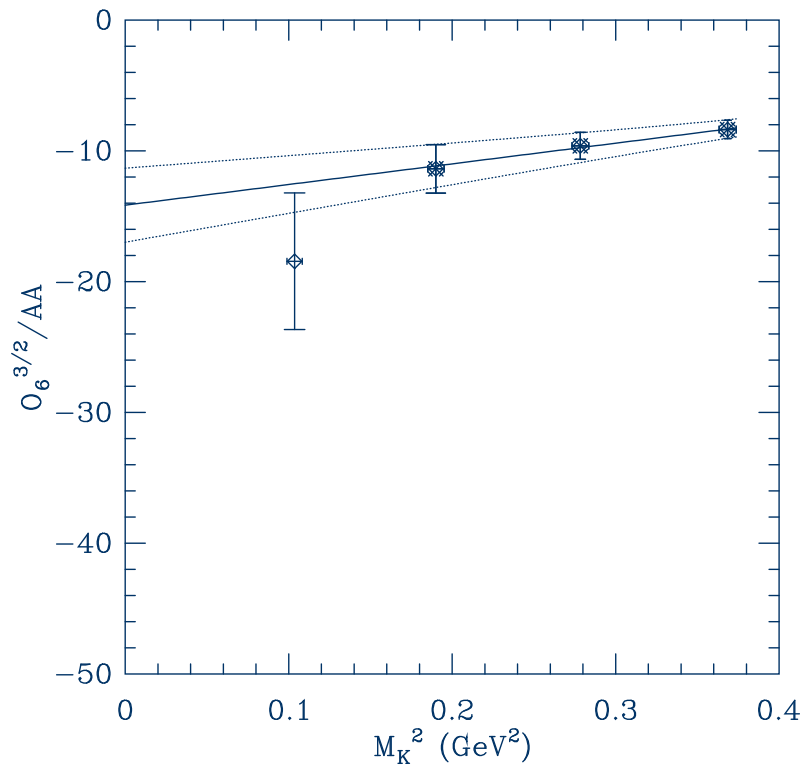
Linear fit



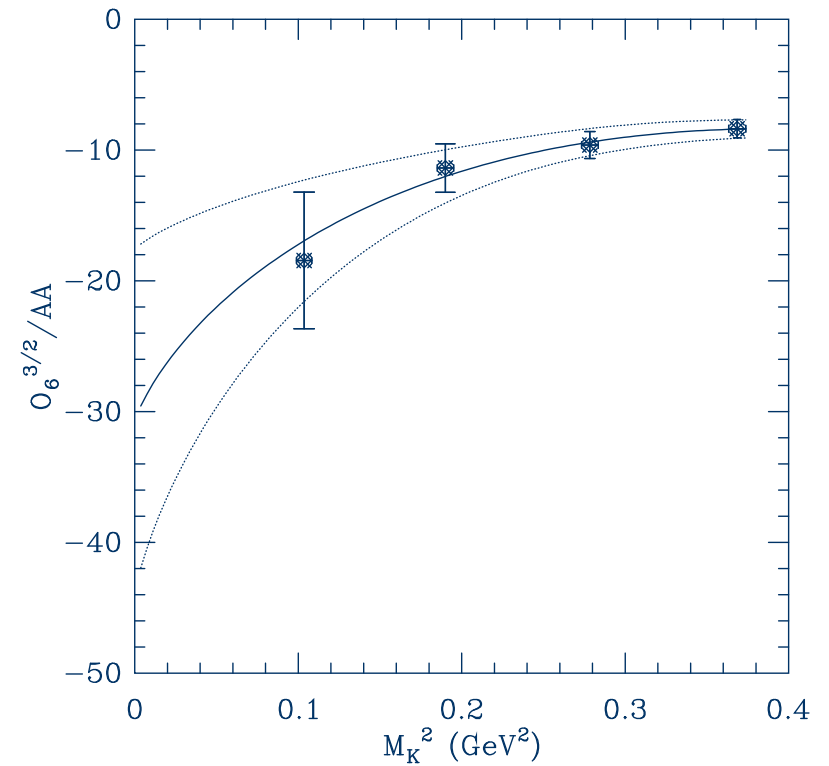
Chiral Log fit



# $\frac{\langle Q_6^{\Delta I=1/2} \rangle}{m_K^2 f^2}$ (Golterman-Pallante op)



Linear fit



Chiral Log fit



# Summary and Conclusion

- HYP/ $\overline{\text{Fat7}}$  resolves the long-standing problem of large perturbative corrections.
- The perturbative calculation for HYP/ $\overline{\text{Fat7}}$  is extended to current-current diagrams and penguin diagrams.
- This allows full matching between lattice and continuum matrix elements.
- The Golterman-Pallante method prefers positive  $\epsilon'/\epsilon$ .
- The standard method prefer negative  $\epsilon'/\epsilon$ .
- This tells us that uncertainty from the quenched approximation is large for  $\epsilon'/\epsilon$ .



- We are extending the current calculation to include more quark masses and to accumulate higher statistics.
- In order to constrain the form of the fitting functions, we need results of the staggered chiral perturbation.
- We need to check the finite volume effect (1.6 fm  $\rightarrow$  2.4 fm).
- We need to extend it to weaker gauge couplings to check the scaling violation, which is expected to be quite small.
- We need to calculate  $\epsilon'/\epsilon$  in partially quenched QCD and full QCD.
- We need to include the next leading order in  $\chi$ PT to



check the size of NLO correction.



## RG evolution for $N_f = 3$

- For  $N_f = 3$ , there is a removable singularity.
- $2\beta_0 + \gamma_8^{(0)} - \gamma_7^{(0)} = 0 \rightarrow$  singularity.
- Both the denominator and numerator vanish.
- Final analytical form contains

$$\ln \left( \frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right), \quad \left[ \ln \left( \frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right) \right]^2$$