

On-shell improvement of the massive Wilson quark action

Y.Kuramashi (U.Tsukuba)

in collaboration with

S.Aoki (U.Tsukuba)

Y.Kayaba (U.Tsukuba)

N.Yamada (RIKEN-BNL)

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§1. Introduction

$m_b a \sim 1 - 2$ in quenched QCD

2 - 3 in full QCD

- static approximation ($m_Q \rightarrow 0$)
- NRQCD ($1/m_Q$ expansion)
- nonrelativistic interpretation of Wilosn/SW results
(Fermilab interpretation)

relativistic approach

cutoff effects

$$(m_Q a)^n$$

$$(a\Lambda_{\text{QCD}})^k$$

$$(m_Q a)^n \cdot (a\Lambda_{\text{QCD}})^k$$

relativistic on-shell improvement in the massive case

$$S_{\text{eff}} = S_{\text{cont}} + \sum_{k \geq 1} a^k \int d^4x C_{4+k,i}(g, m_Q a) \mathcal{O}_{4+k,i}(x)$$

→ remove $(m_Q a)^n a\Lambda_{\text{QCD}}$ errors

remaining cutoff effects are $O((a\Lambda_{\text{QCD}})^2)$

$$a^{-1} \sim 3\text{Gev}, \Lambda_{\text{QCD}} \sim 300\text{MeV} \longrightarrow (a\Lambda_{\text{QCD}})^2 \sim 1\%$$

extend Symanzik's improvement program to the massive case

§2. Relativistic on-shell improvement

§3. Determination of improvement coefficients in the action

§4. Improvement of the axial current

§5. Numerical simulations

§6. Summary

§2. Relativistic on-shell improvement

cutoff effects

$$(m_Q a)^n, (a\Lambda_{\text{QCD}})^k, (m_Q a)^n \cdot (a\Lambda_{\text{QCD}})^k$$

we assume

$$f_0(m_Q a) > f_1(m_Q a) a\Lambda_{\text{QCD}} > f_2(m_Q a) (a\Lambda_{\text{QCD}})^2 > \dots$$

$f_i(m_Q a)$ have Taylor expansions around $m_Q a = 0$

Symanzik's improvement program

$$S_{\text{eff}} = S_{\text{cont}} + \sum_{k,i \geq 1} a^k \int d^4x C_{4+k,i}(g) \mathcal{O}_{4+k,i}(x)$$

allowed operators with axis interchange symmetry

$$\text{dim.3 : } \mathcal{O}_3(x) = \bar{q}(x)q(x)$$

$$\text{dim.4 : } \mathcal{O}_4(x) = \bar{q}(x)\not{D}q(x)$$

$$\text{dim.5 : } \mathcal{O}_{5a}(x) = \bar{q}(x)D_\mu^2 q(x)$$

$$\mathcal{O}_{5b}(x) = i\bar{q}(x)\sigma_{\mu\nu}F_{\mu\nu}q(x)$$

$$\text{dim.6 : } \mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x)$$

$$\mathcal{O}_{6b}(x) = \bar{q}(x)D_\mu^2\not{D}q(x)$$

⋮

generic form of the improved quark action

$$S_q^{\text{imp}} = \sum_x [c_3\mathcal{O}_3(x) + c_4\mathcal{O}_4(x) + c_{5a}\mathcal{O}_{5a}(x) + c_{5b}\mathcal{O}_{5b}(x) \\ + c_{6a}\mathcal{O}_{6a}(x) + c_{6b}\mathcal{O}_{6b}(x) + \dots],$$

allowed operators with axis interchange symmetry

$$\text{dim.3 : } \mathcal{O}_3(x) = \bar{q}(x)q(x)$$

$$\text{dim.4 : } \mathcal{O}_4(x) = \bar{q}(x)\not{D}q(x)$$

$$\text{dim.5 : } \mathcal{O}_{5a}(x) = \bar{q}(x)D_\mu^2 q(x) \rightarrow \text{redundant}$$

$$\mathcal{O}_{5b}(x) = i\bar{q}(x)\sigma_{\mu\nu}F_{\mu\nu}q(x) \rightarrow O(a\Lambda_{\text{QCD}})$$

$$\text{dim.6 : } \mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x) \text{ still } O((a\Lambda_{\text{QCD}})^2) \text{ for } m_Q a \sim O(1)?$$

$$\mathcal{O}_{6b}(x) = \bar{q}(x)D_\mu^2 \not{D}q(x) \quad \partial_0 \sim O(m_Q), \quad \partial_i \sim O(\Lambda_{\text{QCD}})$$

⋮

generic form of the improved quark action

$$S_q^{\text{imp}} = \sum_x [c_3\mathcal{O}_3(x) + c_4\mathcal{O}_4(x) + c_{5a}\mathcal{O}_{5a}(x) + c_{5b}\mathcal{O}_{5b}(x) \\ + c_{6a}\mathcal{O}_{6a}(x) + c_{6b}\mathcal{O}_{6b}(x) + \dots],$$

contribution of $\mathcal{O}_{6a}(x) = \bar{q}(x)\gamma_\mu D_\mu^3 q(x)$

$$\begin{aligned}
 a^2 \bar{q}(x)\gamma_0 D_0^3 q(x) &= -\frac{1}{a}(m_Q a)^3 \bar{q}(x)q(x) \\
 &\quad - (m_Q a)^2 \bar{q}(x)\gamma_i D_i q(x) \\
 &\quad + a(m_Q a) \bar{q}(x) D_i^2 q(x) + O((\Lambda_{\text{QCD}} a)^2). \\
 a^2 \bar{q}(x)\gamma_0 D_0^3 q(x) &\sim O((a\Lambda_{\text{QCD}})^2)
 \end{aligned}$$

with eq. of motion: $\gamma_0 D_0 + \gamma_i D_i + m_Q = 0$

expressed in terms of lower dimensional operators

→ asymmetries with $(m_Q a)^n$ in coefficients

$$\begin{aligned}
 \bar{q}(x)\gamma_0 D_0 q(x) &\leftrightarrow \bar{q}(x)\gamma_i D_i q(x) \\
 \bar{q}(x)\gamma_0 D_0^2 q(x) &\leftrightarrow \bar{q}(x)\gamma_i D_i^2 q(x)
 \end{aligned}$$

axis interchange symmetry is not retained any more

general form of quark action for $O(a\Lambda_{\text{QCD}})$ improvement

$$S_q^{\text{RHQ}} = \sum_x \left[m_0 \bar{q}(x)q(x) + \bar{q}(x)\gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x)\gamma_i D_i q(x) \right. \\ \left. - \frac{r_t a}{2} \bar{q}(x) D_0^2 q(x) - \frac{r_s a}{2} \sum_i \bar{q}(x) D_i^2 q(x) \right. \\ \left. - \frac{iga}{2} c_E \sum_i \bar{q}(x) \sigma_{0i} F_{0i} q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{q}(x) \sigma_{ij} F_{ij} q(x) \right],$$

r_t is redundant

ν, r_s, c_E, c_B should be adjusted in a $m_Q a$ dependent way

$\nu - 1, r_s - r_t, c_B - c_E$ represent contributions from higher dimensional ops. without space-time rotational symmetry

cf. r_t and r_s are redundant in Fermilab approach

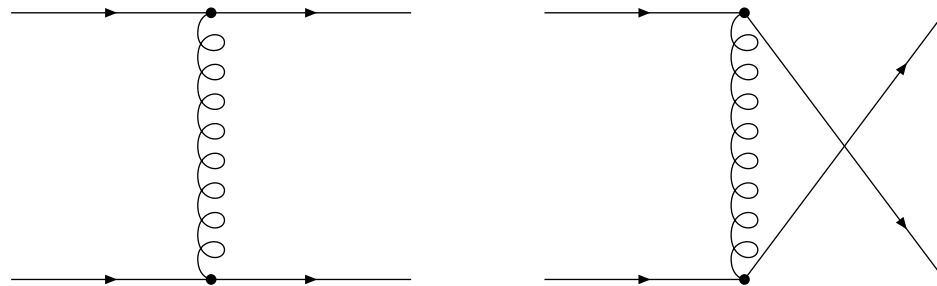
§3. Determination of improvement coefficients

§3-1. $m_Q a$ corrections at tree level

"... on-shell quantities (particle energies, scattering amplitudes, normalized matrix elements of local composite fields between particle states etc.) ..."

Lüscher-Sint-Sommer-Weisz, NPB478(1996)365

ν , r_s , c_E , c_B can be determined by on-shell quark-quark scattering



continuum scattering amplitude should be reproduced
massless case done by Wohlert to determine c_{SW}

the on-shell improvement condition yields

$$\nu^{(0)} = \frac{\sinh(m_p^{(0)})}{m_p^{(0)}},$$

$$r_s^{(0)} = \frac{\cosh(m_p^{(0)}) + r_t \sinh(m_p^{(0)})}{m_p^{(0)}} - \frac{\sinh(m_p^{(0)})}{m_p^{(0)2}},$$

$$c_E^{(0)} = r_t \nu^{(0)},$$

$$c_B^{(0)} = r_s^{(0)},$$

points

- 4 parameters are uniquely determined
- r_t is not fixed \longrightarrow consistent with redundancy
- $\nu \rightarrow 1$, $r_s \rightarrow r_t$, $c_B \rightarrow r_t$, $c_E \rightarrow r_t$ for $m_p^{(0)} \rightarrow 0$

inverting the Wilson-Dirac operator

$$S_q^{-1}(p) = i\gamma_0 \sin(p_0) + \nu i \sum_i \gamma_i \sin(p_i) + m_0 \\ + r_t(1 - \cos(p_0)) + r_s \sum_i (1 - \cos(p_i))$$

ν , r_s are determined by demanding

$$S_q(p) = \frac{1}{Z_q^{(0)}} \frac{-i\gamma_0 p_0 - i \sum_i \gamma_i p_i + m_p^{(0)}}{p_0^2 + \sum_i p_i^2 + m_p^{(0)2}} + (\text{no pole terms}) + O((p_i a)^2)$$

around the pole

speed of light (coeffs. of $\gamma_0 p_0$ & $\gamma_i p_i$) $\rightarrow \nu^{(0)}$

dispersion relation ($E = m_p^{(0)2} + \sum_i p_i^2$) $\rightarrow r_s^{(0)}$

same results as the quark-quark scattering

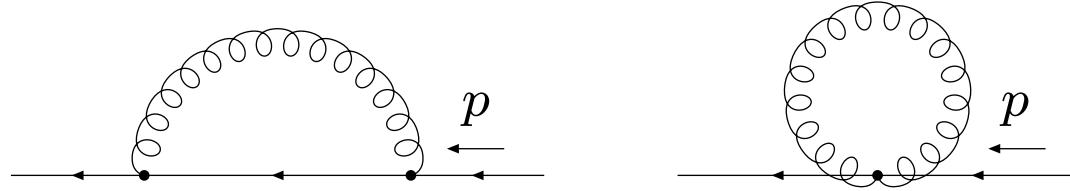
$$\nu^{(0)} = \frac{\sinh(m_p^{(0)})}{m_p^{(0)}}$$
$$r_s^{(0)} = \frac{\cosh(m_p^{(0)}) + r_t \sinh(m_p^{(0)})}{m_p^{(0)}} - \frac{\sinh(m_p^{(0)})}{m_p^{(0)2}}$$

Dirac spinor has a correct continuum form up to $O((p_i a)^2)$
necessary for correct on-shell matrix elements

next step is to determine ν , r_s , c_E , c_B at one-loop level

§3-2. $\nu(m_Q a, g)$, $r_s(m_Q a, g)$ at one-loop level

determined from quark propagator



up to one-loop level

$$S_q^{-1}(p, m) = i\gamma_0 \sin p_0 [1 - g^2 B_0(p, m)] + \nu i \sum_i \gamma_i \sin p_i [1 - g^2 B_i(p, m)]$$

$$+ m + 2r_t \sin^2 \left(\frac{p_0}{2} \right) + 2r_s \sum_i \sin^2 \left(\frac{p_i}{2} \right) - g^2 \hat{C}(p, m)$$

with

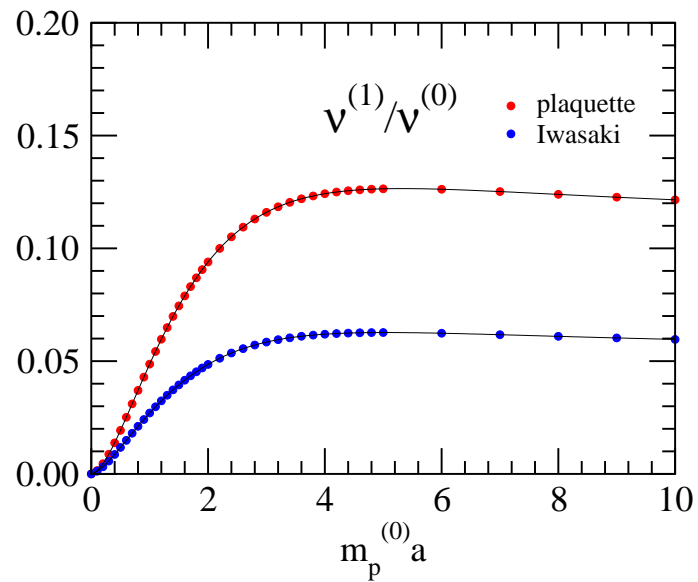
$$m = m_0 - g^2 C(p=0, m=0)$$

$$\hat{C}(p, m) = C(p, m) - C(p=0, m=0)$$

parameter $\nu = \nu^{(0)} + g^2\nu^{(1)}$
 adjust speed of light

$$\nu[1 - g^2 B_i(p^*, m)] = \frac{\sinh(m_p)}{m_p} [1 - g^2 B_0(p^*, m)]$$

$$p^* \equiv (p_0 = im_p, p_i = 0)$$

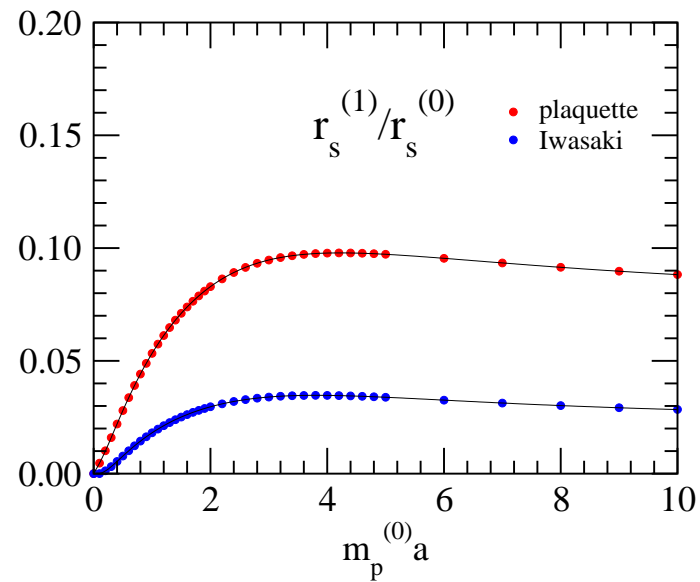


parameter $r_s = r_s^{(0)} + g^2 r_s^{(1)}$

dispersion relation

$$E = m_p^2 + F(\nu, r_s) \sum_i p_i^2 + O(p_i^4)$$

$$\rightarrow F(\nu, r_s) = 1$$



§3-3. $c_B(m_Q a, g)$, $c_E(m_Q a, g)$ at one-loop level

c_B , c_E from on-shell quark-quark scattering at tree level

→ extend to one-loop level

one possible strategy

Schrödinger functional with large quark mass?

unwilling to try

§3-3. $c_B(m_Q a, g)$, $c_E(m_Q a, g)$ at one-loop level

c_B , c_E from on-shell quark-quark scattering at tree level

→ extend to one-loop level

one possible strategy

Schrödinger functional with large quark mass?

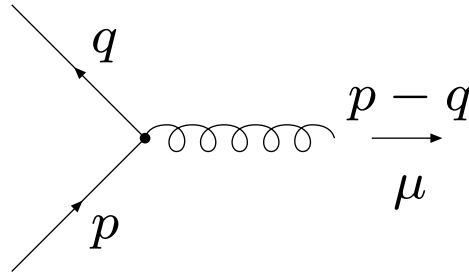
unwilling to try

another strategy

consider ordinary PT with fictitious gluon mass as infrared regulator

we first test this method in massless case

determination of c_{SW} at tree level

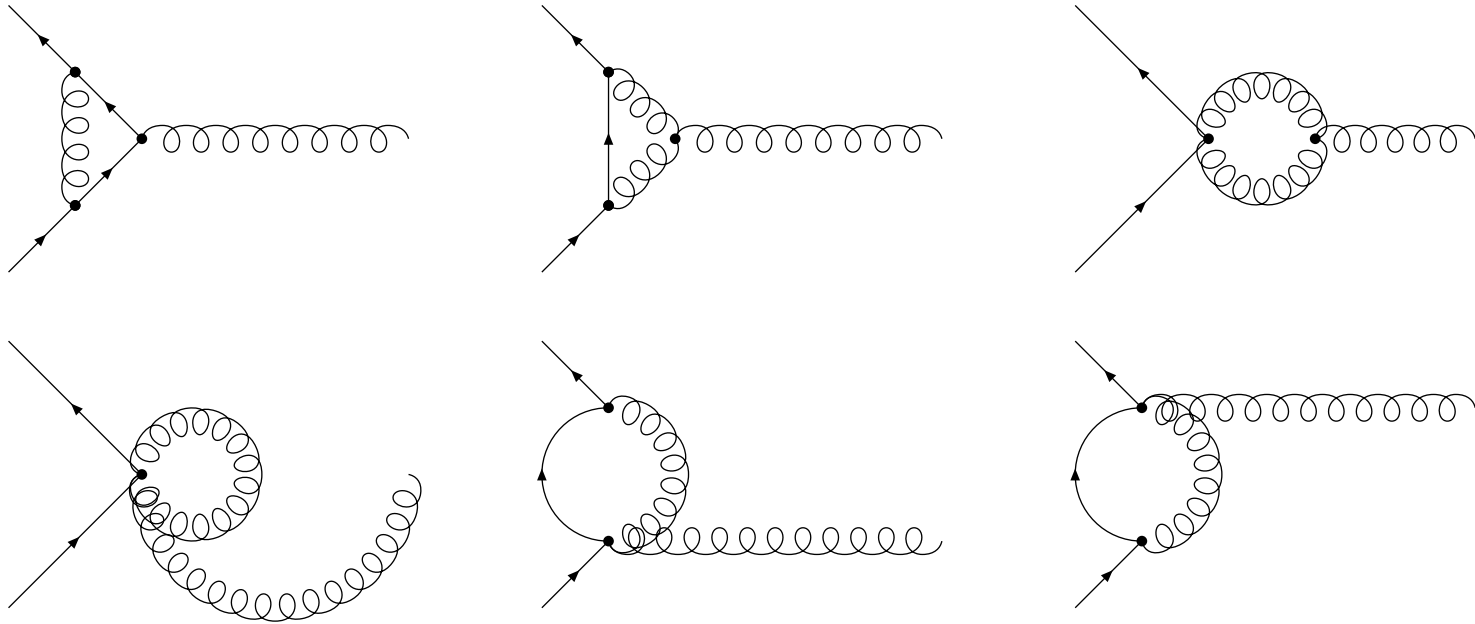


$$V_\mu(p, q) = -gT^A \left\{ i\gamma_\mu + r \left(\frac{p_\mu a + q_\mu a}{2} \right) \right\} \\ -gr \frac{c_{SW}}{2} T^A \sum_\nu \sigma_{\mu\nu} (p_\nu - q_\nu) a + O(a^2)$$

$$\bar{u}(q) V_\mu(p, q) u(p) = -gT^A \bar{u}(q) \left\{ i\gamma_\mu + \frac{r}{2} (1 - c_{SW}) (p_\mu + q_\mu) a \right\} u(p) + O(a^2)$$

→ $c_{SW} = 1$ at tree level

one-loop diagrams



$$\frac{11}{3}(1 - c_{\text{SW}}^{(0)})\frac{1}{16\pi^2} \ln \left| \frac{\pi^2}{\lambda^2 a^2} \right| + 0.26858825(1)$$

consistent with previous work (Wohlert, Lüscher-Weisz)

cancellation of infrared divergences is not trivial

determination of $c_B^{(1)}$

general form of off-shell vertex function at one-loop level
without space-time rotational symmetry

$$\begin{aligned}\Lambda_k^{(1)}(p, q, m) = & \gamma_k F_1^k + \gamma_k \{\not{p} F_2^k + \not{p}_0 F_3^k\} + \{\not{q} F_4^k + \not{q}_0 F_5^k\} \gamma_k \\ & + \not{q} \gamma_k \not{p} F_6^k + \not{q} \gamma_k \not{p}_0 F_7^k + \not{q}_0 \gamma_k \not{p} F_8^k + \gamma_k \not{p}_0 \not{p} F_9 + \not{q} \not{q}_0 \gamma_k F_{10}^k \\ & + (p_k + q_k) [H_1^k + \not{p} H_2^k + \not{q} H_3^k + \not{q} \not{p} H_4^k] \\ & + (p_k - q_k) [G_1^k + \not{p} G_2^k + \not{q} G_3^k + \not{q} \not{p} G_4^k] + O(a^2)\end{aligned}$$

with $\not{p}_0 = p_0 \gamma_0$, $\not{q}_0 = q_0 \gamma_0$

charge conjugation symmetry demands

$$\begin{aligned}F_2^k &= F_4^k, & F_3^k &= F_5^k, & F_7^k &= F_8^k, & F_9^k &= F_{10}^k \\ H_2^k &= H_3^k \\ G_1^k &= G_4^k = 0, & G_2^k &= -G_3^k\end{aligned}$$

Sandwiching $\Lambda_k^{(1)}(p, q, m)$ by the on-shell quark states

$$\begin{aligned}
& \bar{u}(q)\Lambda_k^{(1)}(p, q, m)u(p)|^{\text{latt}} \\
= & \bar{u}(q)\gamma_k u(p) \left\{ F_1^k + im_p(F_2^k + F_4^k) - m_p^2 F_6^k \right\}^{\text{latt}} \\
& + (p_k + q_k)\bar{u}(q)u(p) \left\{ H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k \right\}^{\text{latt}} \\
& + (p_k - q_k)\bar{u}(q)u(p) \left\{ G_1^k + im_p(G_2^k + G_3^k) - m_p^2 G_4^k \right\}^{\text{latt}} + O(a^2)
\end{aligned}$$

withn $\not{p}u(p) = im_p u(p)$, $\bar{u}(q)\not{q} = im_p \bar{u}(q)$

Sandwiching $\Lambda_k^{(1)}(p, q, m)$ by the on-shell quark states

$$\begin{aligned}
 & \bar{u}(q)\Lambda_k^{(1)}(p, q, m)u(p)|^{\text{latt}} \\
 = & \bar{u}(q)\gamma_k u(p) \underbrace{\left\{ F_1^k + im_p(F_2^k + F_4^k) - m_p^2 F_6^k \right\}}_{Z_q^{(1)}}|^{\text{latt}} \\
 & + (p_k + q_k)\bar{u}(q)u(p) \underbrace{\left\{ H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k \right\}}_{c_B^{(1)}}|^{\text{latt}} \\
 & + (p_k - q_k)\bar{u}(q)u(p) \underbrace{\left\{ G_1^k + im_p(G_2^k + G_3^k) - m_p^2 G_4^k \right\}}_{= 0 \text{ by C.C.}}|^{\text{latt}} + O(a^2)
 \end{aligned}$$

withn $\not{p}u(p) = im_p u(p)$, $\bar{u}(q)\not{q} = im_p \bar{u}(q)$

$\{H_1^k + \dots\}^{\text{latt}}$ contains both $O(a)$ and $O(1/m)$

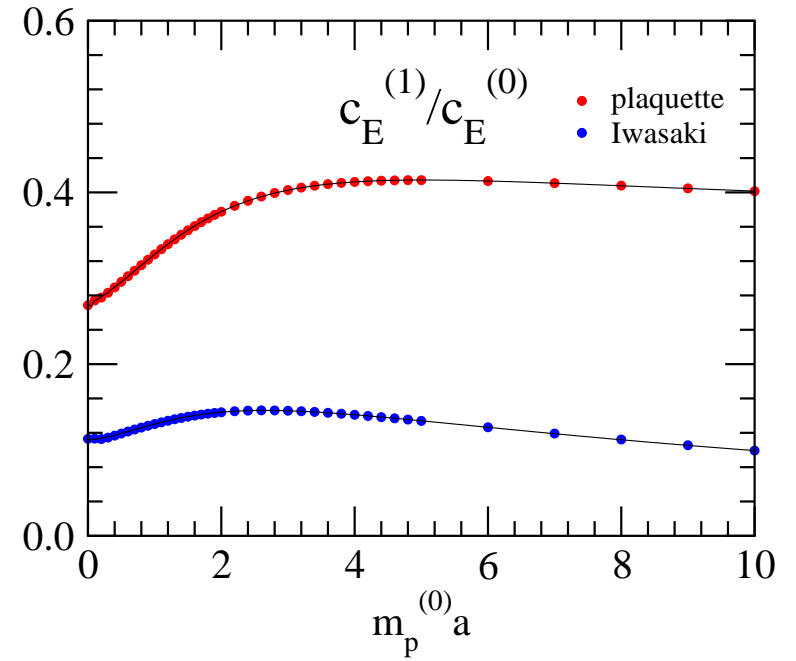
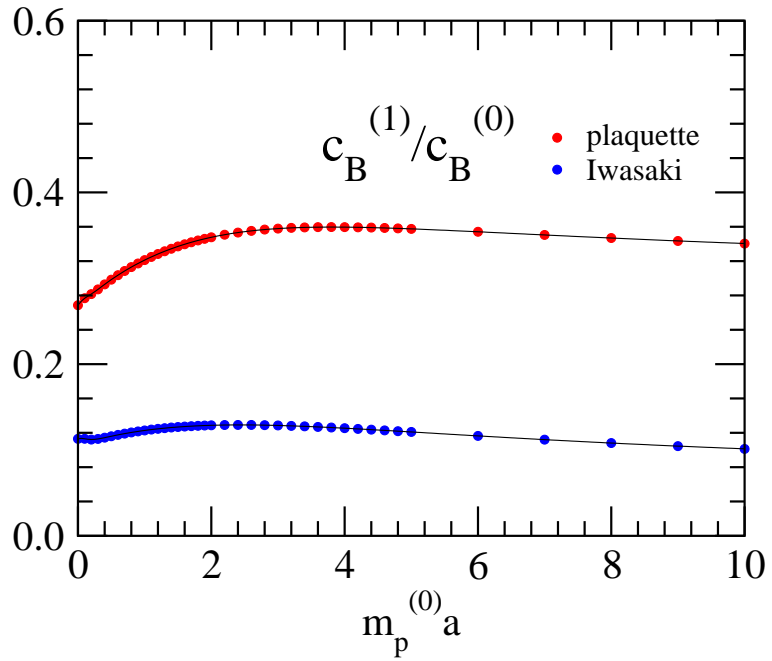
do the same calculation in the continuum

$$\begin{aligned}
& \bar{u}(q) \Lambda_k^{(1)}(p, q, m) u(p) |^{\text{cont}} \\
&= \bar{u}(q) \gamma_k u(p) \left\{ F_1^k + im_p(F_2^k + F_4^k) - m_p^2 F_6^k \right\}^{\text{cont}} \\
& \quad + (p_k + q_k) \bar{u}(q) u(p) \left\{ H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k \right\}^{\text{cont}}
\end{aligned}$$

subtract the physical contribution

$$\begin{aligned}
\frac{c_B^{(1)} - r_s^{(1)}}{2} &= \left[H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k \right]_{p=p^*, q=q^*}^{\text{latt}} \\
& \quad - Z_q^{(0)} \left[H_1^k + im_p(H_2^k + H_3^k) - m_p^2 H_4^k \right]_{p=p^*, q=q^*}^{\text{cont}}
\end{aligned}$$

with $p^* \equiv (p_0 = im_p, p_i = 0)$, $q^* \equiv (q_0 = im_p, q_i = 0)$

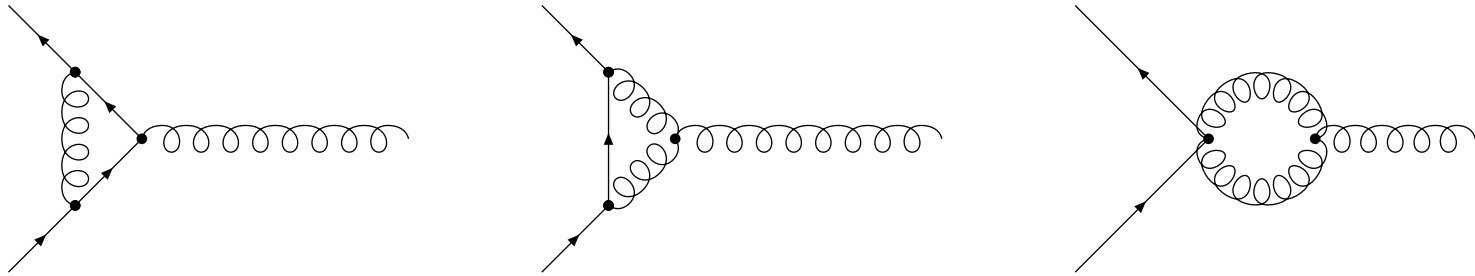


$c_B^{(1)}$ is increased \longrightarrow hyperfine splitting will be enhanced

infrared divergence

c_B, c_E should be free from infrared divergence

three divergent diagrams



infrared divergences are cancelled out after summation

if and only if $\nu^{(0)}, r_s^{(0)}, c_B^{(0)}, c_E^{(0)}$ are properly tuned

→ another evidence for validity of our formulation

4 parameters in the action should be properly tuned

§4. Improvement of the axial current

focus on the time component

form of improved operators

$$A_0^{\text{latt},R}(x) = Z_{A_0}^{\text{latt}} \left[\bar{q}(x) \gamma_0 \gamma_5 Q(x) - g^2 c_{A_0}^+ \partial_0^+ \{ \bar{q}(x) \gamma_5 Q(x) \} \right. \\ \left. - g^2 c_{A_0}^- \partial_0^- \{ \bar{q}(x) \gamma_5 Q(x) \} + O(g^4) \right]$$

$$\partial_0^+ \{ \bar{q}(x) \gamma_5 Q(x) \} = \bar{q}(x) \gamma_5 \{ \partial_0 Q(x) \} + \{ \partial_0 \bar{q}(x) \} \gamma_5 Q(x)$$

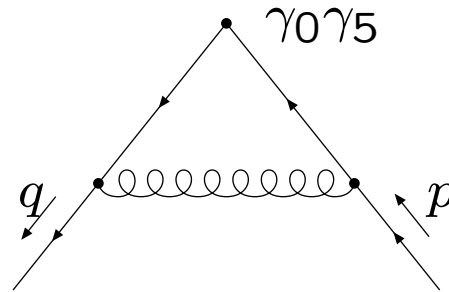
$$\partial_0^- \{ \bar{q}(x) \gamma_5 Q(x) \} = \bar{q}(x) \gamma_5 \{ \partial_0 Q(x) \} - \{ \partial_0 \bar{q}(x) \} \gamma_5 Q(x)$$

in case of $m_Q = m_q$, $c_{A_0}^- = 0$ from charge conjugation symmetry

determination of $c_{A_0}^\pm$ is quite similar to that of c_B , c_E

general form of off-shell vertex function at one-loop level

$$\begin{aligned}
 & \Lambda_{05}^{(1)}(p, q, m_{p1}, m_{p2}) \\
 = & \gamma_0 \gamma_5 F_1^{05} + \gamma_0 \gamma_5 \not{p} F_2^{05} + \not{q} \gamma_0 \gamma_5 F_3^{05} + \not{q} \gamma_0 \gamma_5 \not{p} F_4^{05} \\
 & + (p_0 - q_0) [\gamma_5 G_1^{05} + \gamma_5 \not{p} G_2^{05} + \not{q} \gamma_5 G_3^{05} + \not{q} \gamma_5 \not{p} G_4^{05}] \\
 & + (p_0 + q_0) [\gamma_5 H_1^{05} + \gamma_5 \not{p} H_2^{05} + \not{q} \gamma_5 H_3^{05} + \not{q} \gamma_5 \not{p} H_4^{05}] + O(a^2),
 \end{aligned}$$



Sandwiching $\Lambda_k^{(1)}(p, q, m)$ by the on-shell quark states

$$\begin{aligned}
& \bar{u}(q)\Lambda_{05}^{(1)}(p, q, m_{p1}, m_{p2})u(p) \\
= & \bar{u}(q)\gamma_0\gamma_5u(p) \left\{ F_1^{05} + im_{p1}F_2^{05} + im_{p2}F_3^{05} - m_{p1}m_{p2}F_4^{05} \right\} \\
& + (p_0 - q_0)\bar{u}(q)u(p) \left\{ G_1^{05} + im_{p1}G_2^{05} + im_{p2}G_3^{05} - m_{p1}m_{p2}G_4^{05} \right\} \\
& + (p_0 + q_0)\bar{u}(q)\gamma_5u(p) \left\{ H_1^{05} + im_{p1}H_2^{05} + im_{p2}H_3^{05} - m_{p1}m_{p2}H_4^{05} \right\} \\
& + O(a^2)
\end{aligned}$$

with $\not{p}u(p) = im_{p1}u(p)$, $\bar{u}(q)\not{q} = im_{p2}\bar{u}(q)$

Sandwiching $\Lambda_k^{(1)}(p, q, m)$ by the on-shell quark states

$$\begin{aligned}
& \bar{u}(q)\Lambda_{05}^{(1)}(p, q, m_{p1}, m_{p2})u(p) \\
= & \bar{u}(q)\gamma_0\gamma_5u(p) \left\{ \underline{F_1^{05} + im_{p1}F_2^{05} + im_{p2}F_3^{05} - m_{p1}m_{p2}F_4^{05}} \right\} \\
& \qquad \qquad \qquad \Delta_{\gamma_0\gamma_5} \text{ in } Z_{A_0} \\
& + (p_0 - q_0)\bar{u}(q)u(p) \left\{ \underline{G_1^{05} + im_{p1}G_2^{05} + im_{p2}G_3^{05} - m_{p1}m_{p2}G_4^{05}} \right\} \\
& \qquad \qquad \qquad c_{A_0}^+ \partial_0^+ \{ \bar{q}(x)\gamma_5 Q(x) \} \\
& + (p_0 + q_0)\bar{u}(q)\gamma_5u(p) \left\{ \underline{H_1^{05} + im_{p1}H_2^{05} + im_{p2}H_3^{05} - m_{p1}m_{p2}H_4^{05}} \right\} \\
& \qquad \qquad \qquad c_{A_0}^- \partial_0^- \{ \bar{q}(x)\gamma_5 Q(x) \} \\
& + O(a^2)
\end{aligned}$$

with $\not{p}u(p) = im_{p1}u(p)$, $\bar{u}(q)\not{q} = im_{p2}\bar{u}(q)$

$\{F_1^{05} + \dots\}$, $\{G_1^{05} + \dots\}$, $\{H_1^{05} + \dots\}$ contain both $O(a)$ and $O(1/m)$

subtract the continuum counter part

$$\begin{aligned}\Delta_{\gamma_0\gamma_5} &= \{F_1^{05} + \dots\}^{\text{latt}} - \{F_1^{05} + \dots\}^{\text{cont}} \\ ic_{A_0}^+ &= \{G_1^{05} + \dots\}^{\text{latt}} - \{G_1^{05} + \dots\}^{\text{cont}} \\ ic_{A_0}^- &= \{H_1^{05} + \dots\}^{\text{latt}} - \{H_1^{05} + \dots\}^{\text{cont}}\end{aligned}$$

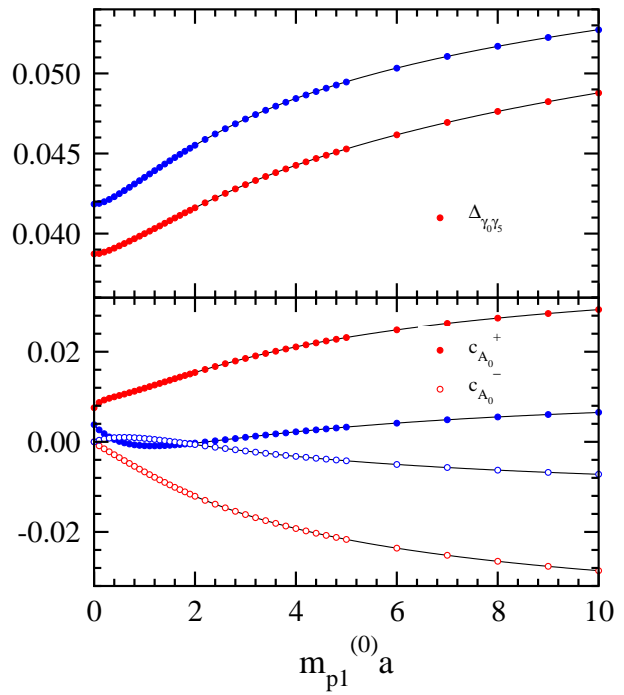
should be independent of infrared divergences

heavy-light case

red : plaquette

blue : Iwasaki

small values for $\Delta_{\gamma_0\gamma_5}$, $c_{A_0}^\pm$



$O(a)$ improvement of V_0 , V_k , A_0 , A_k is done for H-H and H-L cases
see hep-lat/0401030

§5. Numerical simulations

§5-1. test of the action in quenched QCD

check restoration of space-time symmetry

- dispersion relation of H-H and H-L mesons

$$c_{\text{eff}} = \sqrt{\frac{E(p_s)^2 - E(0)^2}{p_s^2}}$$

- H-L PS meson decay constant from A_0 and A_k

$$R \equiv i \frac{\langle 0 | A_k^R | PS \rangle}{\langle 0 | A_4^R | PS \rangle} \cdot \frac{E}{|p_s|}$$

c_{eff} and R should be 1 in the continuum limit

simulation parameters

quenched, $24^3 \times 48$, #conf.=300

plaquette gauge action @ $\beta = 5.9$, $a(r_0) \sim 0.1\text{fm}$

Iwasaki gauge action @ $\beta = 2.6$, $a(r_0) \sim 0.1\text{fm}$

light quarks : SW with nonperturbative c_{SW}

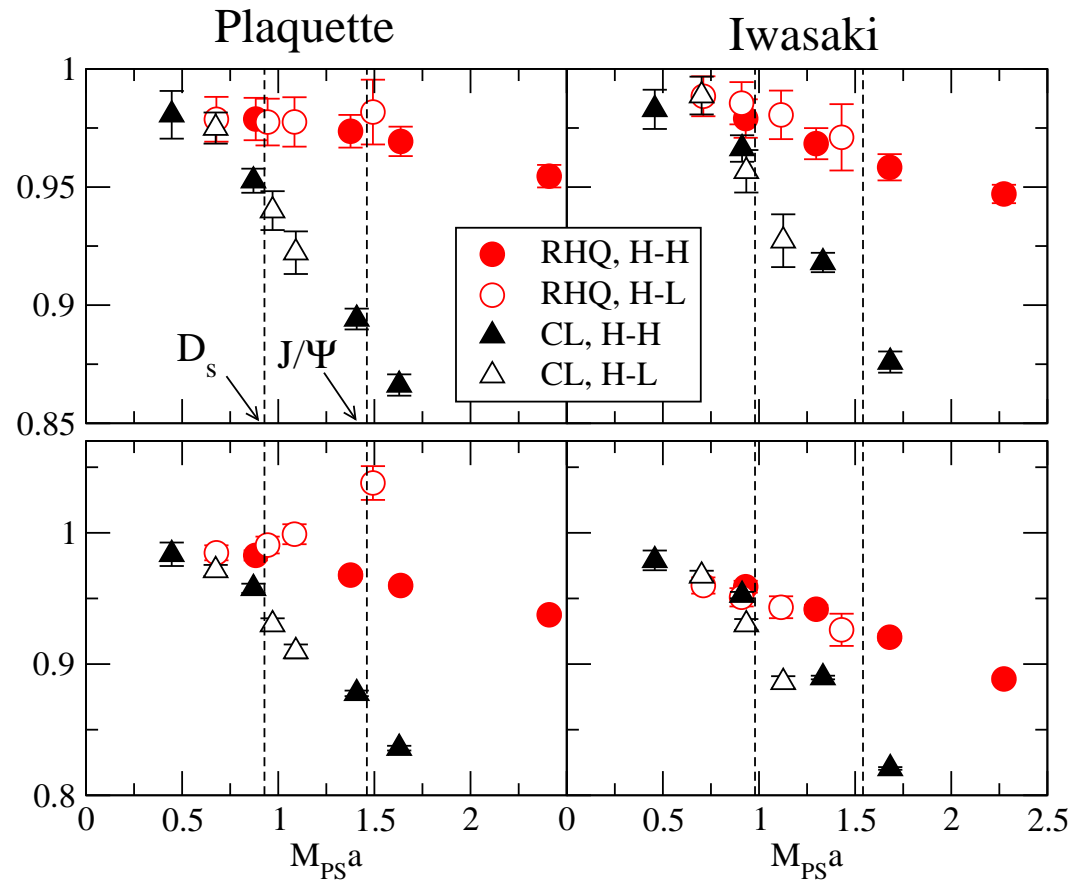
heavy quarks (A) : ν , r_s , c_B , c_E up to one-loop level

$$c_{B/E} = \{c_{B/E}^{\text{PT}}(m_Q a) - c_{B/E}^{\text{PT}}(0)\} + c_{\text{SW}}^{\text{NP}}$$

cover charm quark mass

heavy quarks (B) : SW with nonperturbative c_{SW} for comparison

axial currents : mass dependently improved up to one-loop level



clear improvement over the clover heavy quark action

§5-2. Preliminary results of $N_f = 2$ QCD

simulation parameters PRD65(2002)054505

Iwasaki gauge action

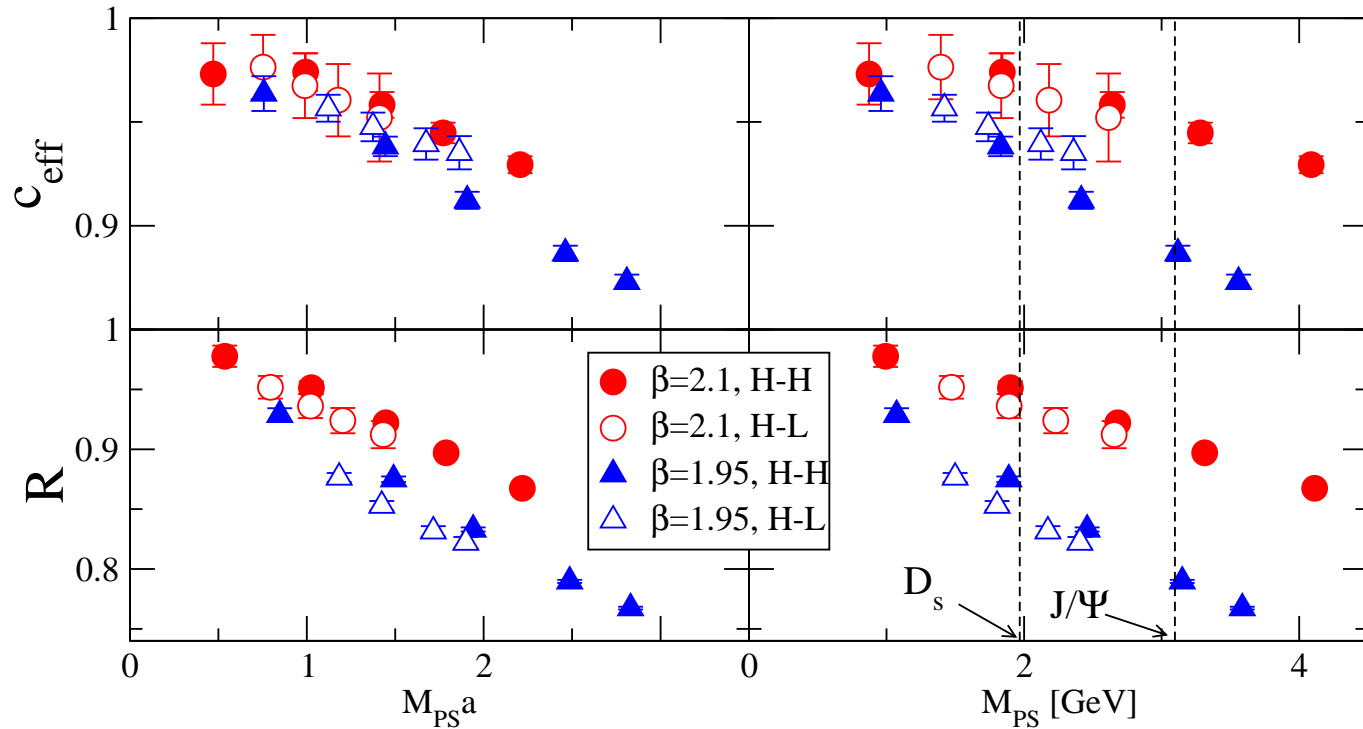
4 sea quark masses with mean-field improved SW

$\beta = 1.95$ $16^3 \times 32$ $a(r_0) \sim 0.15\text{fm}$ #conf.=400

$\beta = 2.1$ $24^3 \times 48$ $a(r_0) \sim 0.11\text{fm}$ #conf.=200

light quarks : mean-field improved SW

heavy quarks : same as quenched calculation

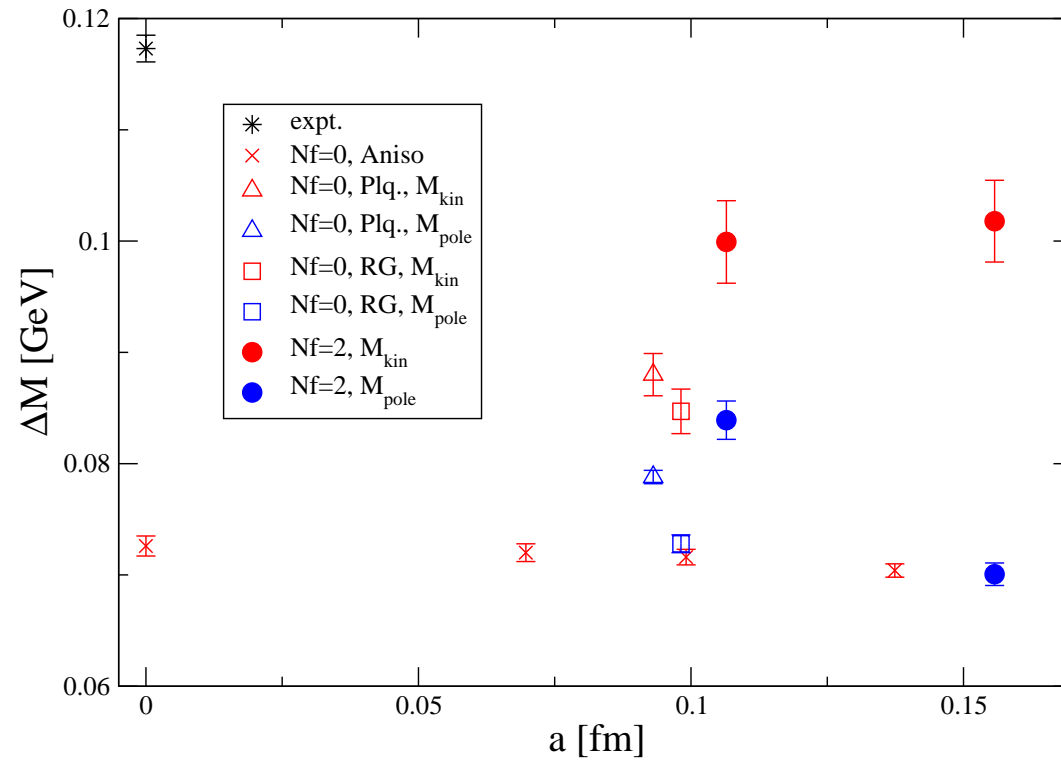


expected leading scaling violation:

$$O(f_2(m_Q a)(a\Lambda_{\text{QCD}})^2) \text{ or } O(f_1(m_Q a)g^4 a\Lambda_{\text{QCD}})$$

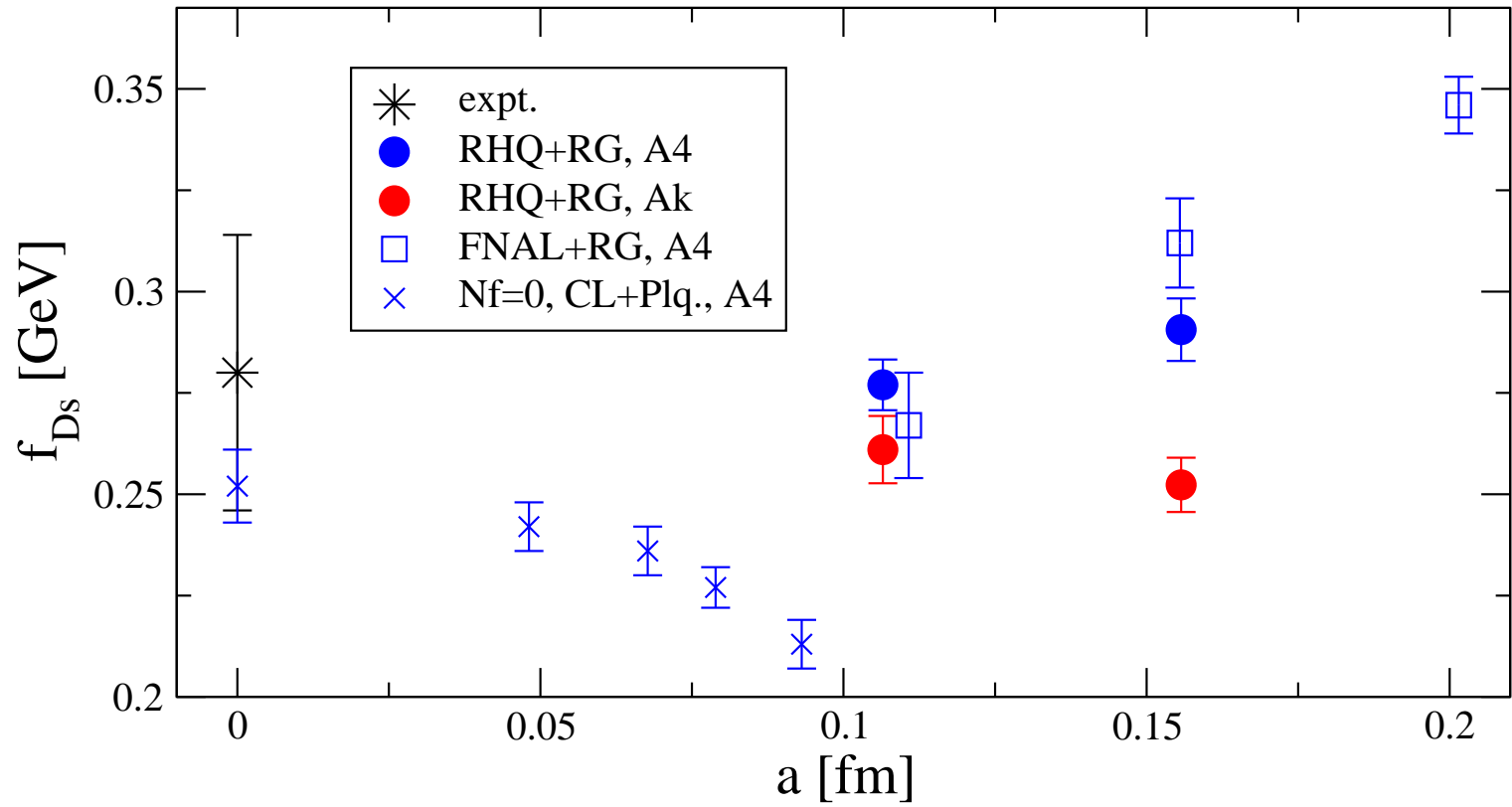
a bit surprising is $f_{1,2}$ look linear beyond $m_Q a = 1$

Charmonium S-state hyperfine splitting



dynamical quark effects?

→ need more scaling study for $N_f = 0$ and 2



possibility of reliable continuum extrapolation with $f_{D_s}(A_4)$ and $f_{D_s}(A_k)$

§5. Summary

- general form of quark action

$$S_q^{\text{imp}} = \sum_x \left[m_0 \bar{q}(x) q(x) + \bar{q}(x) \gamma_0 D_0 q(x) + \nu \sum_i \bar{q}(x) \gamma_i D_i q(x) \right. \\ \left. - \frac{r_t a}{2} \bar{q}(x) D_0^2 q(x) - \frac{r_s a}{2} \sum_i \bar{q}(x) D_i^2 q(x) \right. \\ \left. - \frac{iga}{2} c_E \sum_i \bar{q}(x) \sigma_{0i} F_{0i} q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{q}(x) \sigma_{ij} F_{ij} q(x) \right],$$

r_t is redundant

ν, r_s, c_E, c_B should be adjusted

- 4 parameters in the action are determined up to one-loop level
massless c_{SW} using the ordinary PT with fictitious gluon mass
→ extend to c_B, c_E in the massive case
- $O(a)$ improvement of the vector and axial vector currents
up to one-loop level
- numerical study shows encouraging results
dynamical quark effects? → need more scaling study

ongoing project

- renormalization and improvement of four-fermi ops.
- numerical calculation of H-H and H-L meson spectrum, decay constants, form factors in $N_f = 2$ QCD

next plan

- detailed scaling study in quenched QCD
- nonperturbative determination of ν , r_s , c_E , c_B
- numerical calculation in $N_f = 3$ QCD