Staggered Chiral Perturbation Theory with Heavy-Light Mesons

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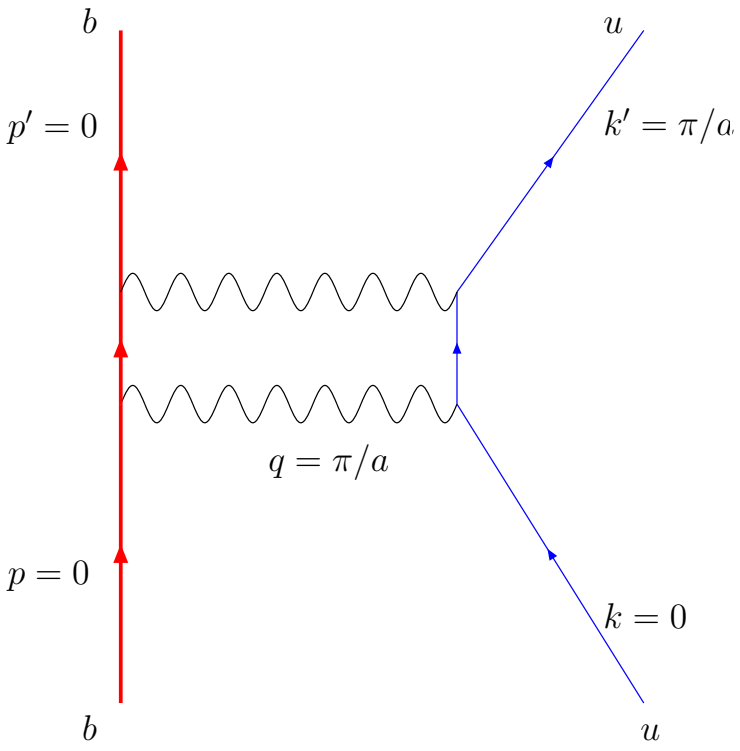
Outline

- Effective Theory of heavy-lights and pions
  - Heavy Quark Effective Theory (HQET)
  - Staggered Chiral Perturbation Theory (SχPT)
  - Taste breaking with heavy-lights
- f_B NLO Calculation
- Form factors for $B(D) \to \pi(K)\ell\nu$

Heavy Quarks on the Lattice

- m_Q is large: neglect $1/m_Q$ corrections
- $m_Q \neq \infty$ $\Rightarrow$Taste-changing interactions cause the heavy quark to become highly energetic, decoupling from the theory.

$a \approx .12\text{fm} \Rightarrow \pi/a \approx 5.2\text{GeV}$
b quark: $5\text{GeV} \longrightarrow 7.2\text{GeV}$



Heavy-Lights on the Lattice

- One light staggered quark, one heavy quark
- 16 tastes of pions (in $SO(4)$ representations: P, A, T, V, S); 4 tastes of heavy-lights.
- We are parametrizing only the taste violations in heavy-light quantities. Discretization errors that arise in the heavy sector are not considered.
- If we were to have a highly improved heavy quark, these errors could be small and unimportant.
- In practice, the discretization errors would have to be estimated/extrapolated away.

HQET & SχPT

- The heavy-lights are combined into a single field H:

$$H_a = \frac{1+\not{v}}{2}\left[\gamma^\mu B^*_{\mu a} - \gamma_5 B_a\right] \qquad \overline{H}_a \equiv \gamma_0 H_a^\dagger \gamma_0$$

- Light mesons: $\Sigma = \sigma^2 = \exp(i\Phi/f)$, with (for 3 flavors of light quarks)

$$\Phi = \begin{pmatrix} U & \pi^+ & K^+ \\ \pi^- & D & K^0 \\ K^- & \bar{K}^0 & S \end{pmatrix}, \quad U = U_i T_i\ , \quad K^+ = K_i^+ T_i\ , \quad \textit{etc.},$$

- Under chiral $SU(12)_L \times SU(12)_R$:

$$H_a \to H_b U^\dagger_{ba} \qquad \overline{H}_a \to U_{ab}\overline{H}_b$$

$$\sigma \to L\sigma U^\dagger = U\sigma R^\dagger \qquad \Sigma \to L\Sigma R^\dagger$$

HQET & SχPT

- $\mathcal{L}$ is an expansion in
  - $m_\pi \sim \sqrt{m_q}$; m_q is a light quark mass
  - a^2, the lattice spacing
  - k ($p_B = m_Q v + k$), the heavy-light residual momentum

$$\mathcal{L} = \mathcal{L}_{1,2} + \mathcal{L}_3$$

$$\begin{aligned}
\mathcal{L}_{1,2} &= i\mathrm{tr}_D[\overline{H}_a v^\mu(\delta_{ab}\partial_\mu + i\mathbb{V}_\mu^{ba})H_b] + g_\pi \mathrm{tr}_D(\overline{H}_a H_b \gamma^\nu \gamma_5 \mathbb{A}_\nu^{ba}) + \mathcal{L}_{S\chi PT} \\
\mathcal{L}_3 &= g_1 \mathrm{tr}_D(\overline{H}_a H_b)\mathcal{M}_{ba}^+ + g_2 \mathrm{tr}_D(\overline{H}_a H_a)\mathcal{M}_{bb}^+ + \mathcal{L}_{GL} - a^2 \mathcal{V}_H
\end{aligned}$$

$$\mathbb{V}_\mu = \frac{i}{2}\left[\sigma^\dagger \partial_\mu \sigma + \sigma \partial_\mu \sigma^\dagger\right] \qquad \mathbb{A}_\mu = \frac{i}{2}\left[\sigma^\dagger \partial_\mu \sigma - \sigma \partial_\mu \sigma^\dagger\right]$$

$$\mathcal{M}^+ = \sigma \mathcal{M} \sigma + \sigma^\dagger \mathcal{M} \sigma^\dagger$$

Discrete Symmetry

- Staggered quarks have a discrete symmetry, given by

$$q \to (1 \otimes \xi_\mu) q$$

- At the meson level, this takes the form

$$\Sigma \to \xi_\mu^{(3)} \Sigma \xi_\mu^{(3)} \qquad \sigma \to \xi_\mu^{(3)} \sigma \xi_\mu^{(3)}$$
$$H \to H \xi_\mu^{(3)} \qquad \overline{H} \to \xi_\mu^{(3)} \overline{H}$$

- $\mathcal{L}$ is invariant under this symmetry; It can be shown that quantities such as f_B are independent of the external light quark taste.

NLO calculation of f_B

- The B decay constant can be extracted from the matrix element

$$\left\langle 0 \left| L^{\mu}_{x,a} \right| B_{x,a}(v) \right\rangle = -i f_{B_x} m_{B_x} v^{\mu} \ ,$$

where $L^{\mu}_{x,a}$ is the axial current which destroys a $B_{x,a}$ meson. The corresponding chiral operator is

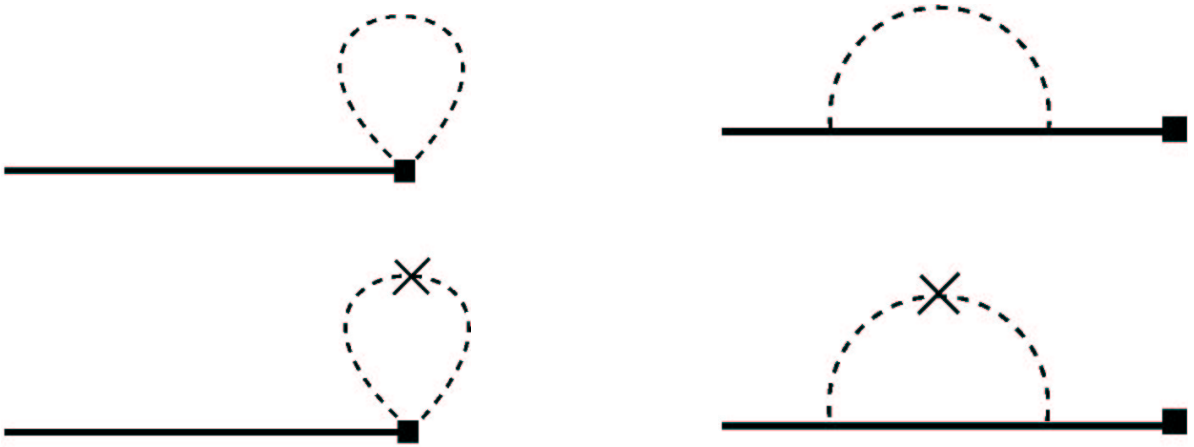
$$L^{\mu}_{x,a} = \frac{i\kappa}{2} \mathrm{tr}_D \left[\gamma^{\mu} (1 - \gamma_5) (\mathcal{P}_x H_b) \right] \sigma_{ba}$$

- We will write the decay constant as:

$$f_{B_x} = \frac{\kappa}{\sqrt{m_{B_x}}} \left(1 + \frac{1}{16\pi^2 f^2} \delta f_{B_x} \right) \ ,$$

NLO calculation of f_B

- The only non-zero diagrams are:



- Crosses are one or more hairpin insertions from $\mathcal{L}_{S\chi PT}$:

$$\boxed{-ia^2\delta'_V\,(\text{taste-vector});\quad -ia^2\delta'_A\,(\text{taste-axial});\quad -i4m_0^2/3,\,(\text{taste-singlet})}$$

- Moving from $4 \to 1$ tastes per flavor is no different than in the calculations for light meson quantities.

Partially quenched f_{B_x}

$$\delta f_{B_x} = -\frac{1+3g_\pi^2}{2}\left\{\frac{1}{16}\sum_{F,t}\ell(m^2_{(xF)_t}) + \frac{1}{3}\sum_{j_I\in M_I^{(3)}}\frac{\partial}{\partial m^2_{X_I}}\left[R^{[3,3]}_{j_I}(M_I^{(3)};\mu_I^{(3)})\ell(m^2_{j_I})\right]\right.$$

$$\left. +a^2\delta'_V\sum_{j_V\in M_V^{(4)}}\frac{\partial}{\partial m^2_{X_V}}\left[R^{[4,3]}_{j_V}(M_V^{(4)};\mu_V^{(3)})\ell(m^2_{j_V})\right] + [V\to A]\right\}$$

$$+c_1(m_u+m_d+m_s) + c_2 m_x + c_a a^2$$

$$\ell(m^2) = m^2\ln m^2 + \text{finite volume corrections}$$

$$\mu^{(3)} = \{m_U^2, m_D^2, m_S^2\} \qquad \mathcal{M}_I^{(3)} = \{m^2_{X_I}, m^2_{\pi^0_I}, m^2_{\eta_I}\}$$

$$\mathcal{M}_V^{(4)} = \{m^2_{X_V}, m^2_{\pi^0_V}, m^2_{\eta_V}, m^2_{\eta'_V}\}$$

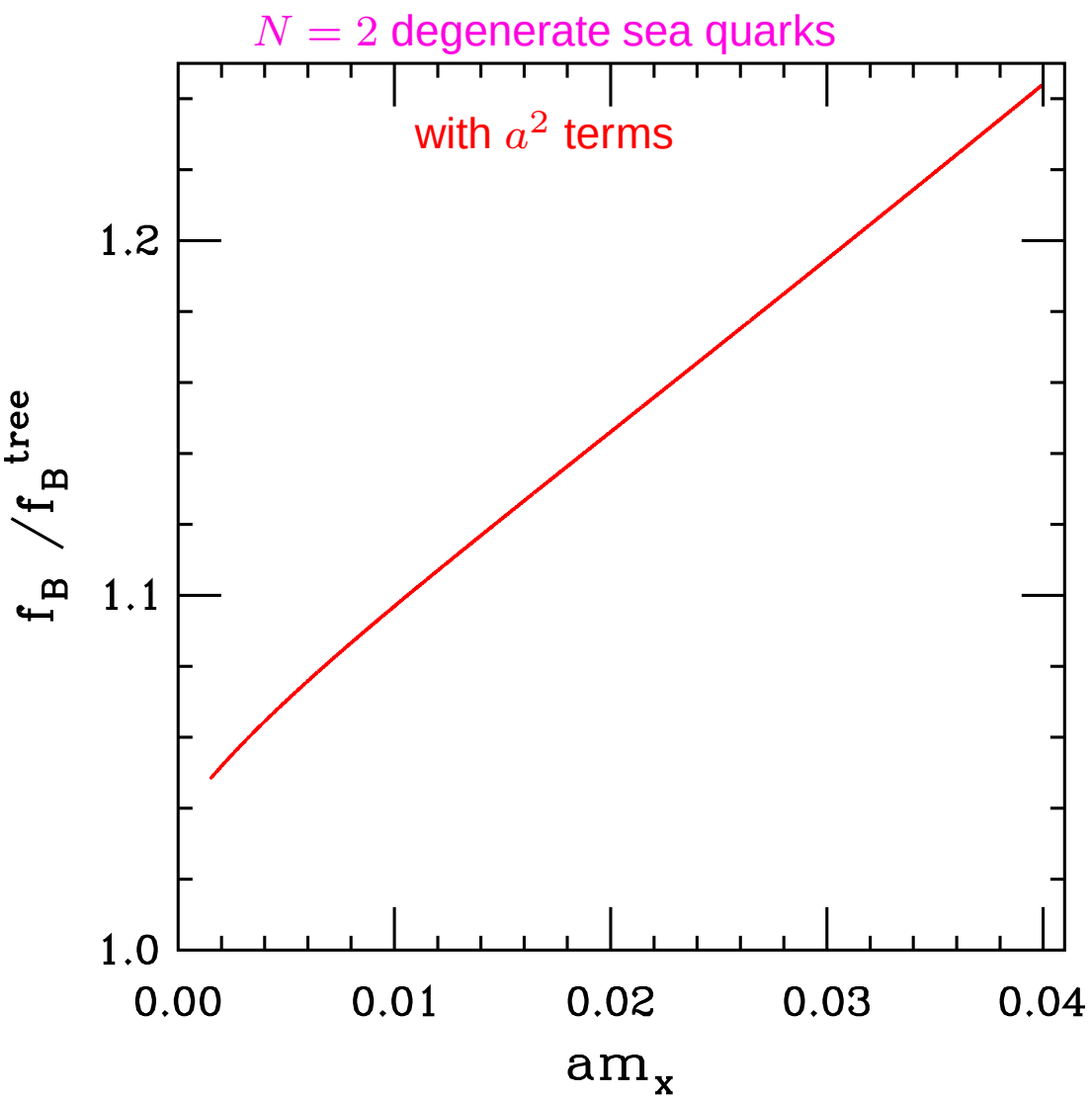
- The R_j are the residues of the poles of the disconnected flavor-neutral propagators.

f_B for $2+1$ flavors

$$m_u = m_d \equiv m_l$$

$$\begin{aligned}
\delta f_B \quad = \quad & -\left(\frac{1+3g_\pi^2}{2}\right)\left\{\frac{1}{16}\sum_t\left[2\ell(m^2_{\pi_t})+\ell(m^2_{K_t})\right]-\frac{1}{2}\ell(m^2_{\pi^0_I})+\frac{1}{6}\ell(m^2_{\eta_I})\right. \\
& +a^2\delta'_V\left[\frac{(m^2_{\pi^0_V}-m^2_{S_V})}{(m^2_{\pi^0_V}-m^2_{\eta_V})(m^2_{\pi^0_V}-m^2_{\eta'_V})}\ell(m^2_{\pi^0_V})+(\pi^0_V\to\eta_V\to\eta'_V)\right] \\
& \left.+[V\to A]\right\}+c_1(2m_l+m_s)+c_2m_l+c_aa^2
\end{aligned}$$

f_B with $am = 0.010$



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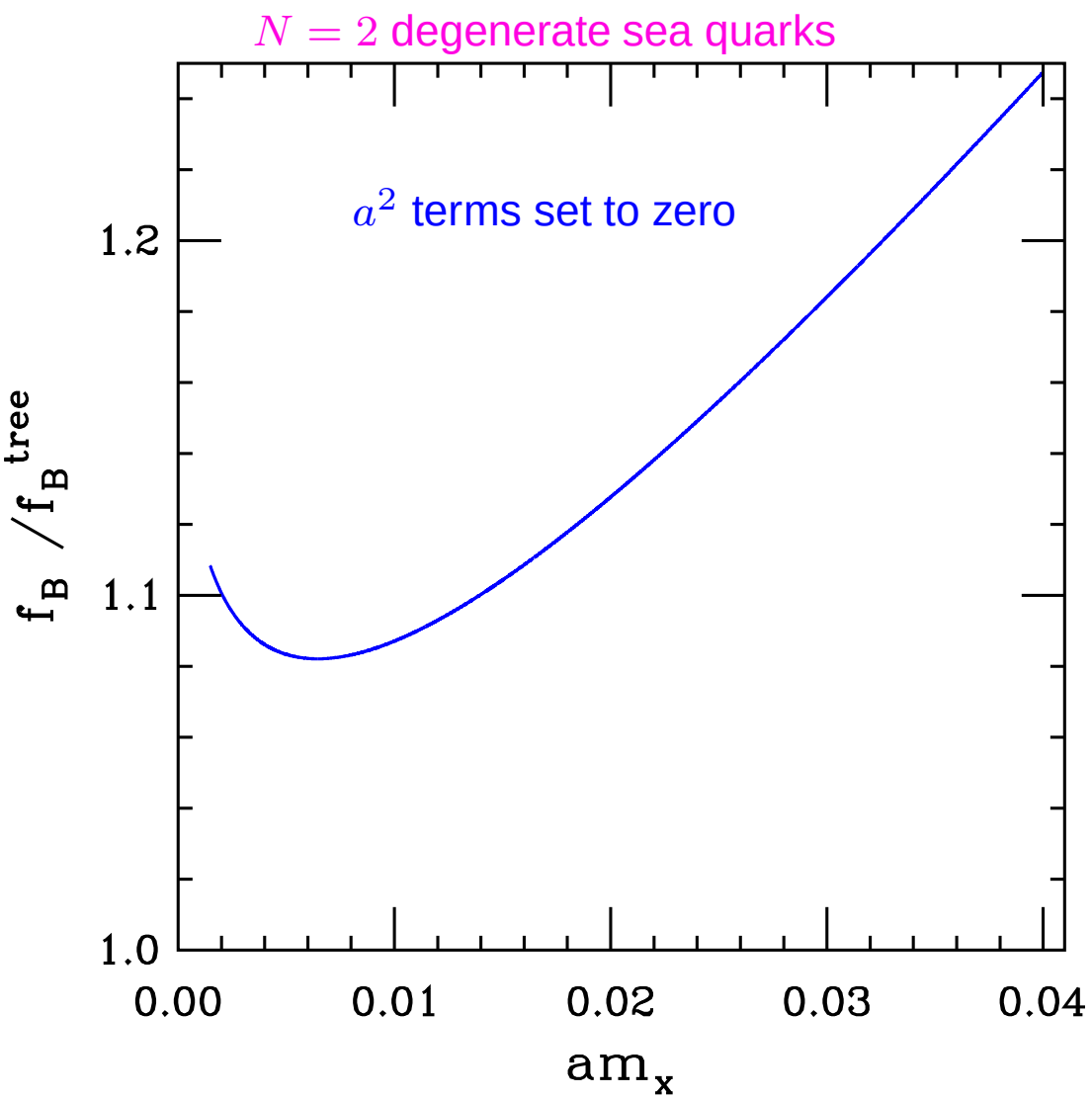
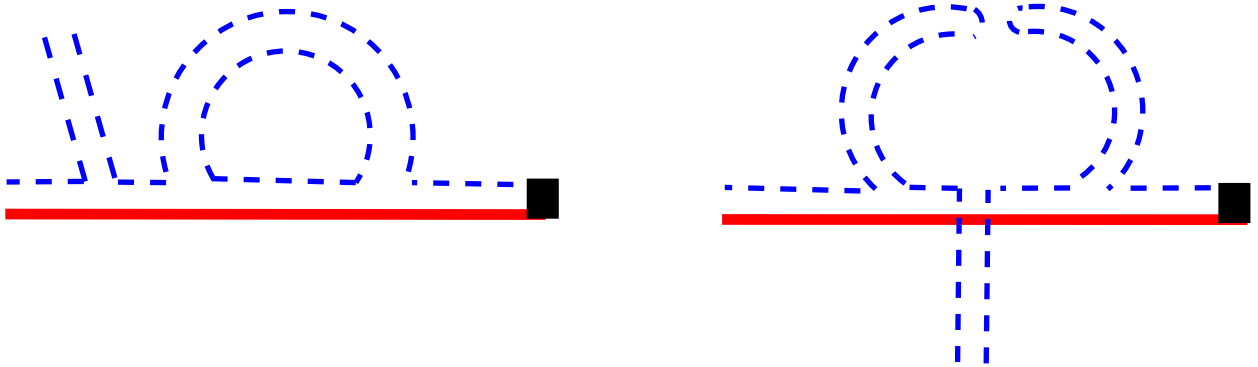


Form Factors

- Continuum PQχPT results with $N_{\rm sea}$ degenerate sea quarks can be easily generalized to SχPT:
  1. Terms $\propto N_{\rm sea} \rightarrow$ average over tastes.
  2. Terms $\propto \dfrac{1}{N_{\rm sea}}$ are disconnected; in SχPT we have $S, V,$ and A pieces.
  3. One must be careful of possible sign changes when commuting taste matrices.
- With this in mind, it is straightforward to write down the SχPT expressions for the form factors for $B(D) \rightarrow \pi(K)\ell\nu$ decays.



Taste-breaking Four-quark operators

- There are three types of four-quark operators that can possibly contribute to $\mathcal{V}_H$:

1. $\overline{Q}(\gamma_S \otimes I)Q\overline{Q}(\gamma_{S'} \otimes I)Q$
   Taste singlet: No taste violations from these operators

2. $\overline{Q}(\gamma_S \otimes I)Q\overline{q}_j(\gamma_{S'} \otimes \xi_I)q_j$
   Correct the masses of the heavy-lights at $\mathcal{O}(a^2)$, do not involve the pions at this order.

3. $\overline{q}_i(\gamma_S \otimes \xi_T)q_i\overline{q}_j(\gamma_{S'} \otimes \xi_{T'})q_j$
   Give rise to the terms in $\mathcal{V}_H$.

Potential with Heavy-lights

- The terms in $\mathcal{V}_H$ have one of two forms:

$$\overline{H}_a H_b O_{ba}$$
$$\overline{H}_a H_a O_{bb}$$

- There are 8 operators O_{ba} which contribute, giving 16 new terms.
- For example:

$$\begin{aligned} O^1_{ba} &= (\sigma \xi^{(3)}_5 \Sigma^\dagger \xi^{(3)}_5 \sigma)_{ba} \\ O^7_{ba} &= (\sigma \xi^{(3)}_\nu \sigma)_{ba} \, \mathrm{Tr}(\xi^{(3)\nu} \Sigma^\dagger) \end{aligned}$$

$$\xi^{(3)}_\mu = \begin{pmatrix} \xi_\mu & 0 & 0 \\ 0 & \xi_\mu & 0 \\ 0 & 0 & \xi_\mu \end{pmatrix}$$

Conclusions

- We can now include heavy-light mesons in SχPT calculations.
- $\mathcal{V}_H$ is not necessary for many important quantities at one-loop order.
- Calculations for f_B and form factors have been completed.
- Other calculations, such as B parameters or other relevant quantities are now straightfoward.